

P7 EKVIVALENTNA FUNCIJA PRENOSA

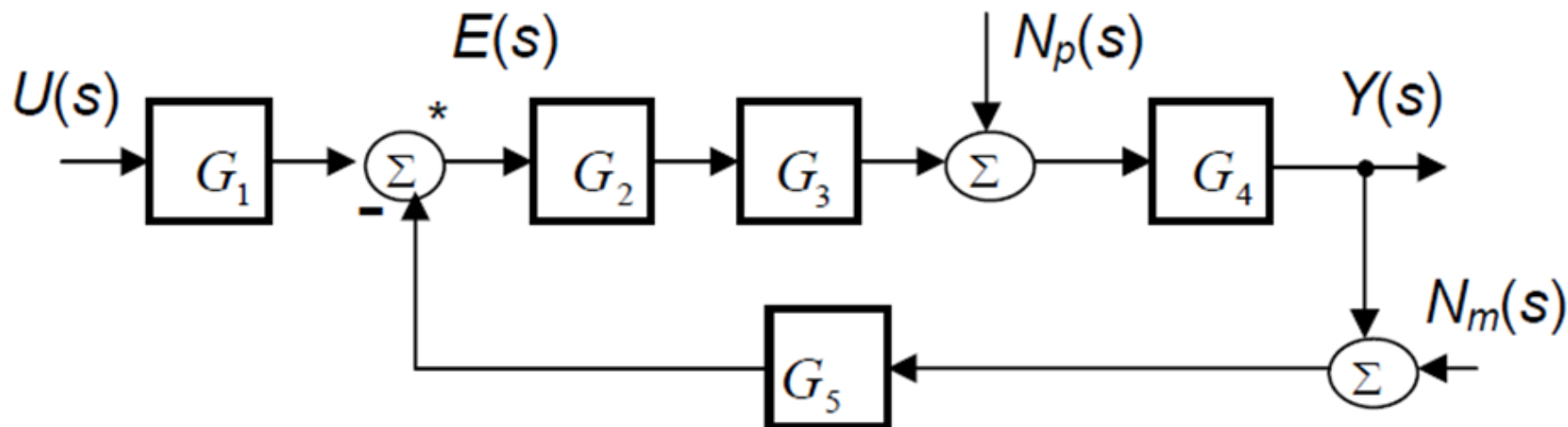
PREDMET: SIGNALI I SISTEMI

Funkcija prenosa

- Za analizu sistema automatskog upravljanja koriste se strukturni blok dijagrami
- Omogućava se prikaz promenjivih veličina sistema i interakcija između njih
- Omogućava se predstavljanje elemenata sistema u vidu blokova sa odgovarajućim funkcijama prenosa $(G_1, G_2, G_3, \dots, G_n,)$
- Povezivanjem blokova omogućava se uvid u tok informacije od ulaza prema izlazu sistema

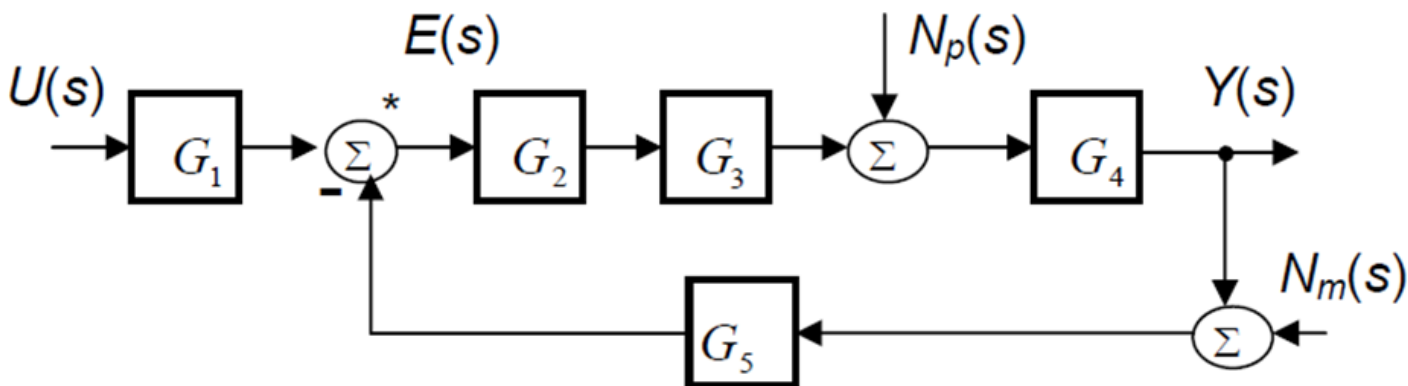
Strukturni blok dijagram

- Orijentisane linije označavaju smerove protoka informacija
- Kružići sa upisanim znakom za sumu označavaju diskriminatore koji vrše funkcije sabiranja i oduzimanja



Strukturni blok dijagram

- Ulaz sistema je $U(s)$, a izlaz sistema je $Y(s)$
- Povratna sprega sa izlaza na ulaz vrši se preko funkcije povratne sprega G_5
- Diskriminator označen * na svom izlazu generiše signal greške $E(s)$
- G_2 i G_3 opisuju zakom upravljanja
- G_4 predstavlja objekat upravljanja
- Neželjeni poremećaj i šum obeleženi su sa $N_p(s)$ i $N_m(s)$



Strukturni blok dijagram

- Strukturni blok dijagram omogućava kompletan uvid u sistem i određivanje ekvivalentne funkcije prenosa
- Ekvivalentna funkcija prenosa G_e predstavlja odnos kompleksnih likova izlaza $Y(s)$ i ulaza $X(s)$
- $G_e = \frac{Y(s)}{X(s)}$
- Elementi sistema između ulaza i izlaza predstavljaju direktnu granu (Primer: $(G_1, G_2, G_3$ i $G_4)$)
- Elementi koji omogućavaju povratak informacije sa izlaza na ulaz formiraju povratnu granu (G_5)
- Elementi koji formiraju zatvorenu konturu formiraju petku povratne sprege (G_2, G_3, G_4 i G_5)

Algebra funkcije prenosa

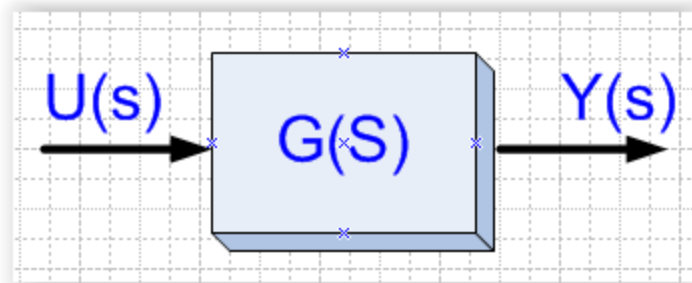
- Složeni sistemi AU imaju više povratnih sprega i veći broj ulaza
- Ovakvi sistemi imaju složeni strukturni blok dijagram
- Za pogodnu analizu neophodno je sisteme prevesti u jednostavne strukture
- Primjenjuju se metode superpozicije i pravila algebre funkcije prenosa
- Najčešće se računa ekvivalentna funkcija prenosa

Algebra funkcije prenosa

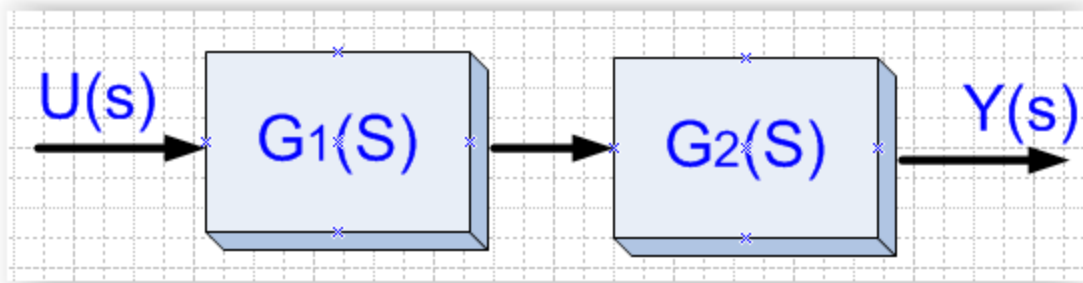
□ Osnovni element – funkcija prenosa

□ $Y(s) = G(s) \cdot U(s)$

□ $G(s) = \frac{Y(s)}{U(s)}$



□ Redna (serijska) veza elemenata sistema upravljanja

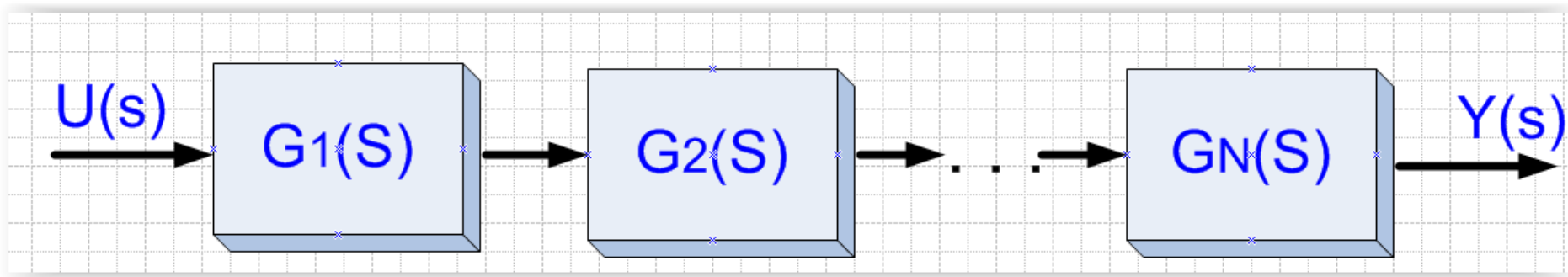


□ $Y(s) = G_1(s) \cdot G_2(s) \cdot U(s)$

□ $G_e(s) = \frac{Y(s)}{U(s)} = G_1(s) \cdot G_2(s)$

Algebra funkcije prenosa

□ Redna (serijska) veza elemenata sistema upravljanja

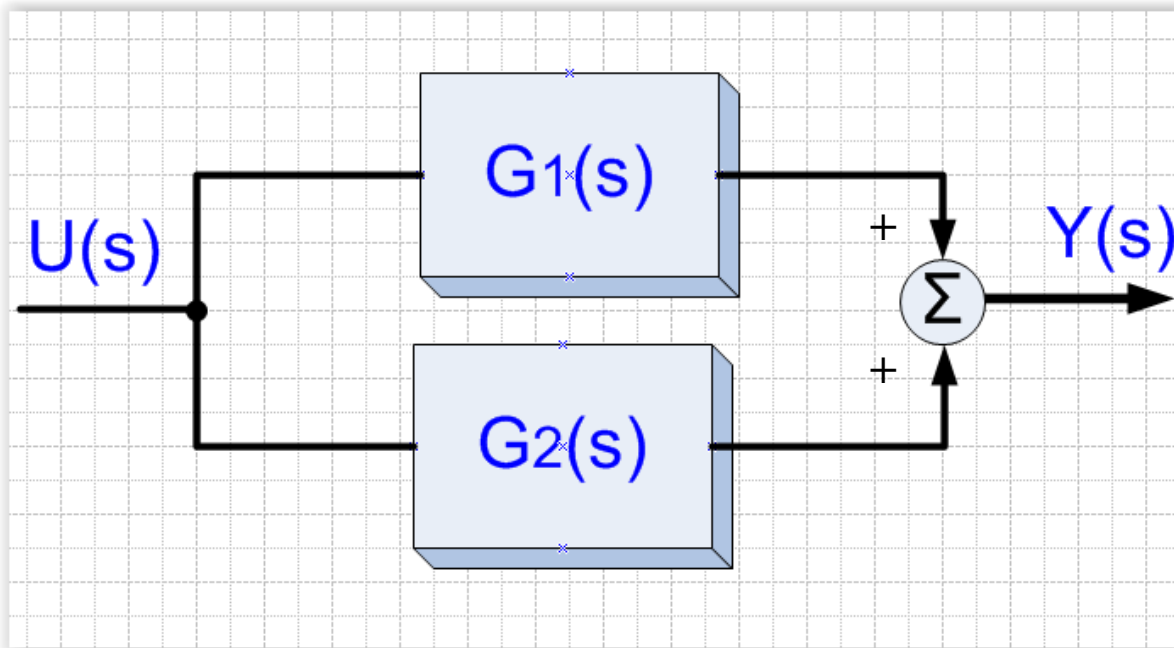


□ $Y(s) = G_1(s) \cdot G_2(s) \dots G_N(s) \cdot U(s)$

□ $G_e(s) = \frac{Y(s)}{U(s)} = G_1(s) \cdot G_2 \dots G_N(s)$

Algebra funkcije prenosa

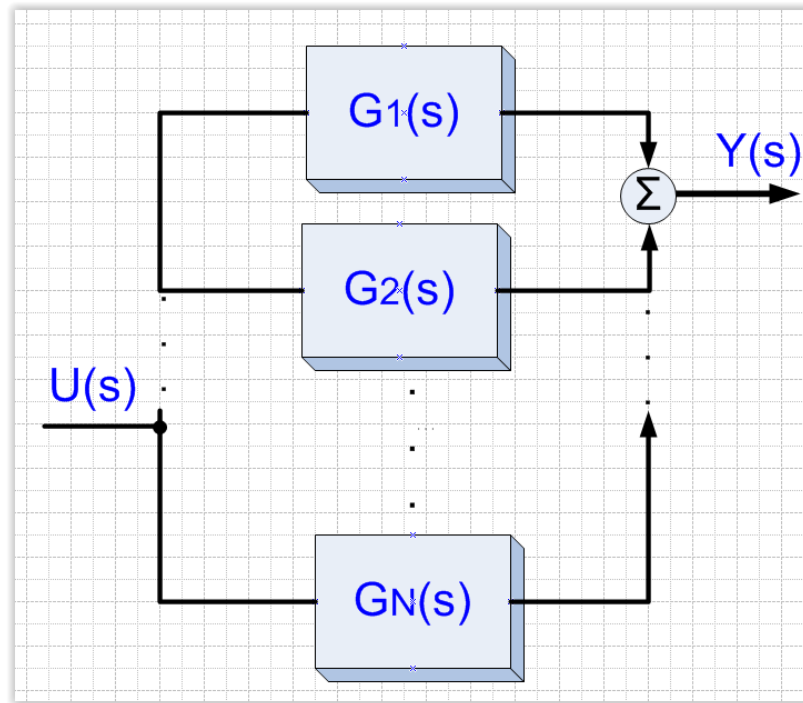
- Oznaka za diskriminator Σ
- Paralelna veza elemenata sistema upravljanja



- $Y(s) = (G_1(s) + G_2(s)) U(s)$
- $G_e(s) = \frac{Y(s)}{U(s)} = G_1(s) + G_2(s)$

Algebra funkcije prenosa

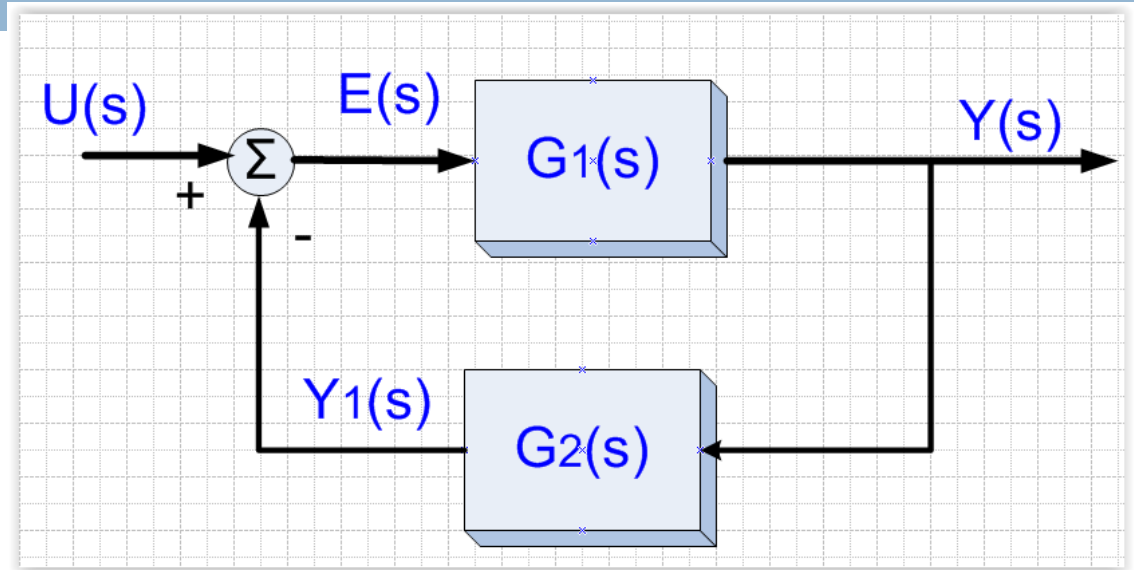
□ Paralelna veza elemenata sistema upravljanja



□ $Y(s) = (G_1(s) + \dots + G_N(s)) \cdot U(s)$

□ $G_e(s) = \frac{Y(s)}{U(s)} = G_1(s) + G_2(s) + \dots + G_N(s)$

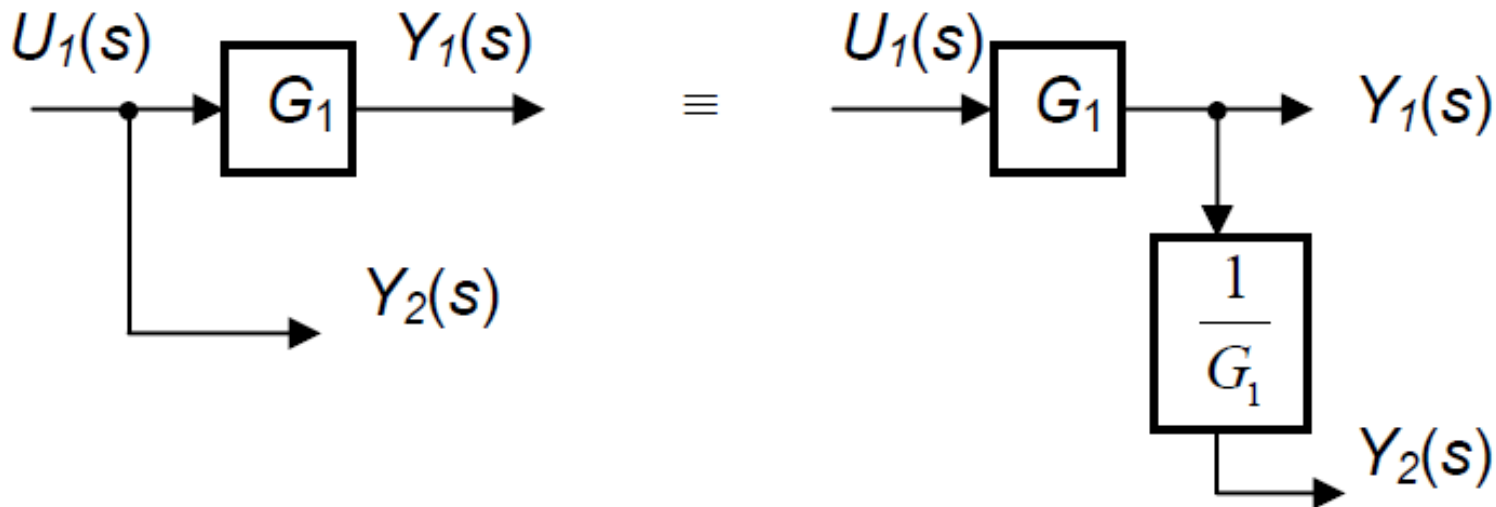
Negativna povratna sprega



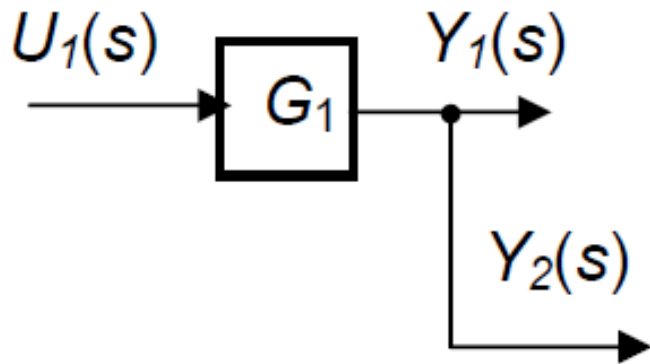
- $E(s) = U(s) - Y_1(s)$
- $Y_1(s) = G_2(s)Y(s)$
- $Y(s) = G_1(s)E(s) = G_1(s)(U(s) - Y_1(s)) = G_1(s)(U(s) - G_2(s)Y(s))$
- $Y(s) = G_1(s)U(s) - G_1(s)G_2(s)Y(s)$
- $Y(s) + G_1(s)G_2(s)Y(s) = G_1(s)U(s)$
- $Y(s)(1 + G_1(s)G_2(s)) = G_1(s)U(s)$
- $G(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)}$

Pomeranje tačke grananja iza bloka

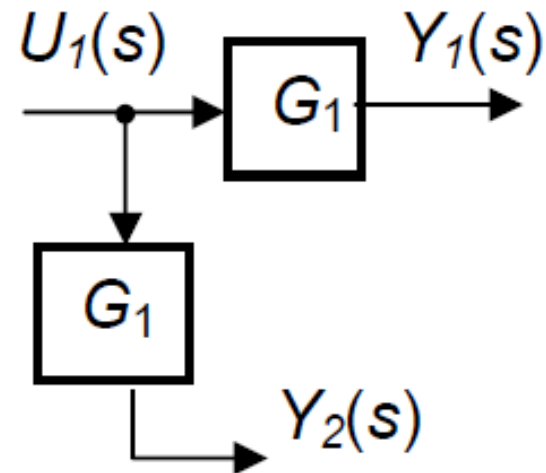
- Primenom principa superpozicije i algebre funkcije prenosa, složeni strukturni dijagrami se lako prevode u osnovne strukture



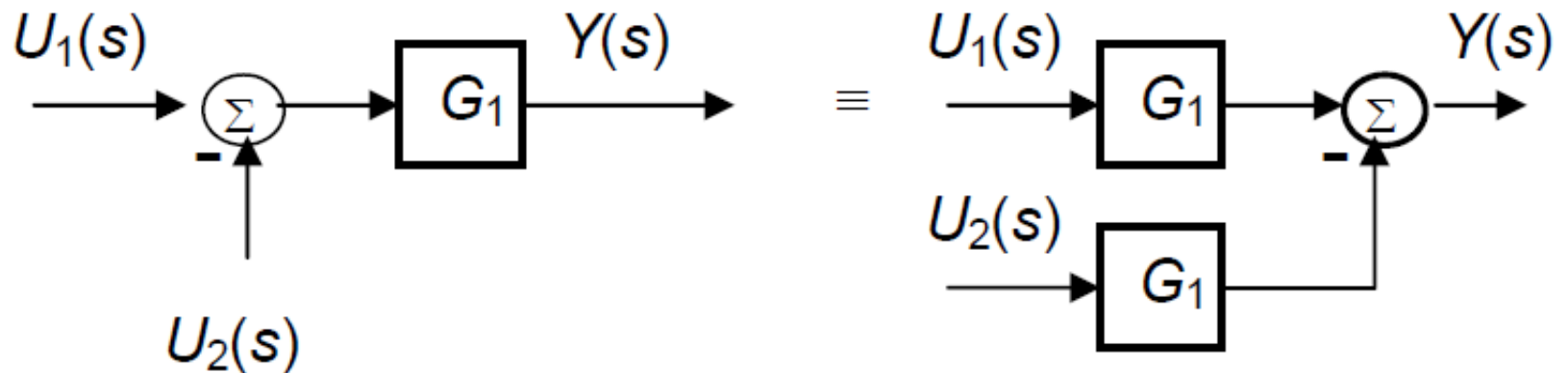
Pomeranje tačke grananja ispred bloka



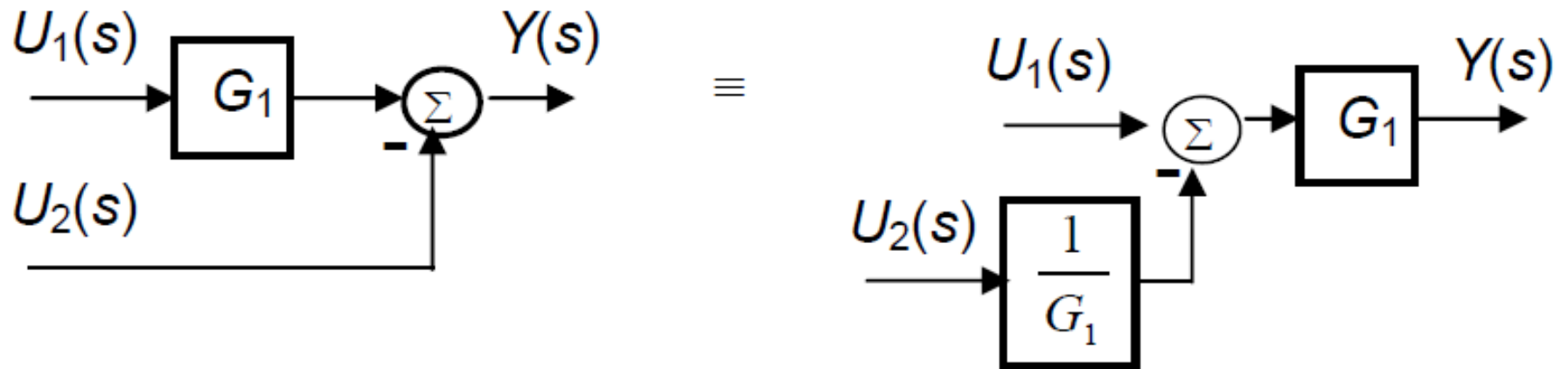
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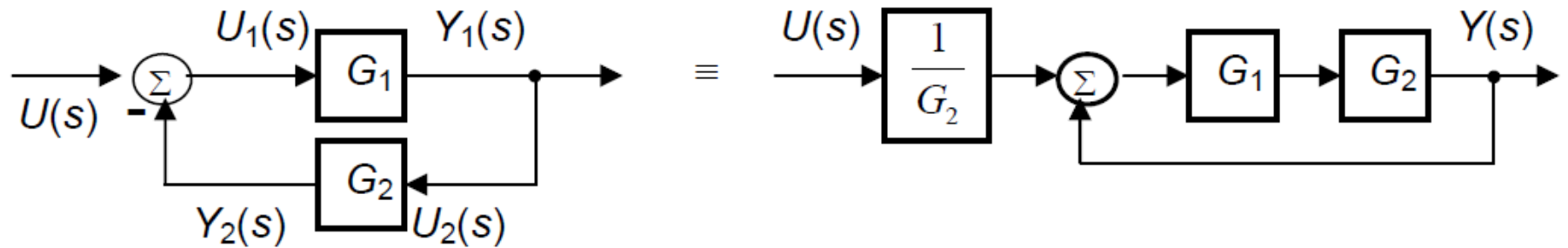
Premeštanje diskriminatora iza bloka



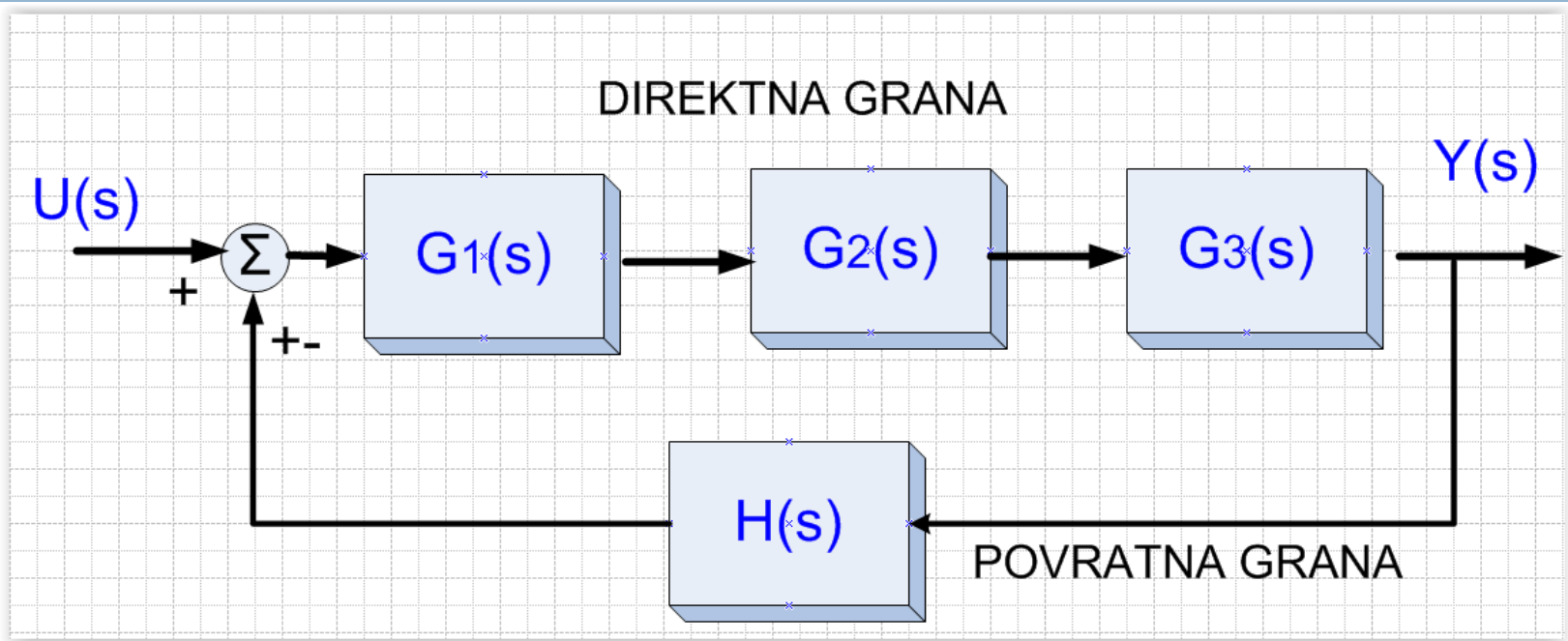
Premeštanje diskriminatora ispred bloka



Premeštanje diskriminatora iz povratnog kola

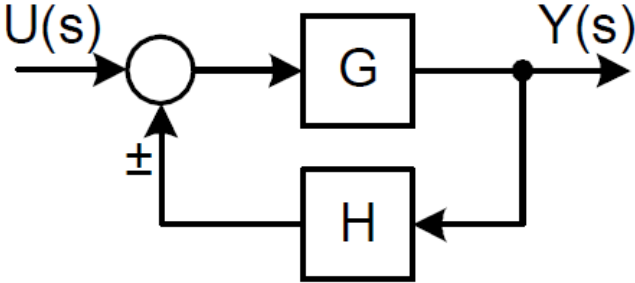
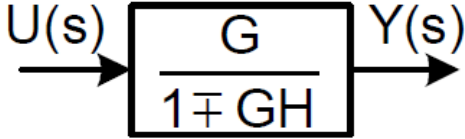
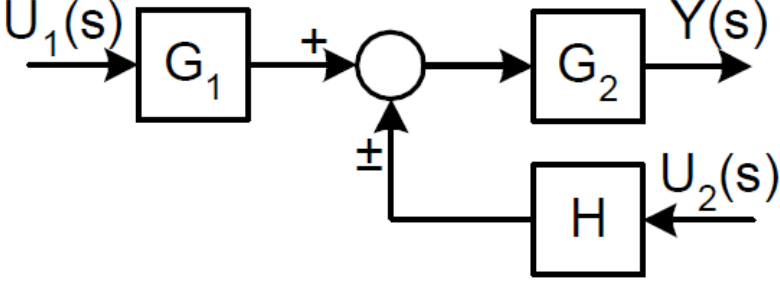
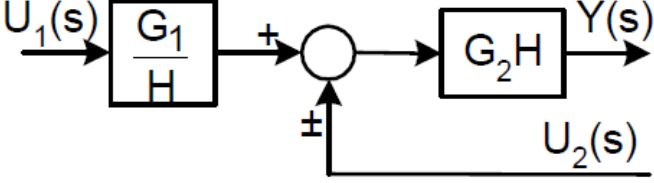


Primer 1. Za kolo sa slike izračunati ekvivalentnu funkciju prenosa



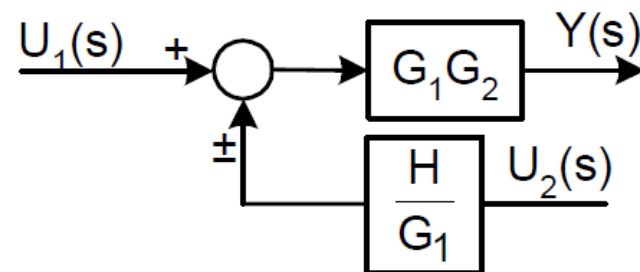
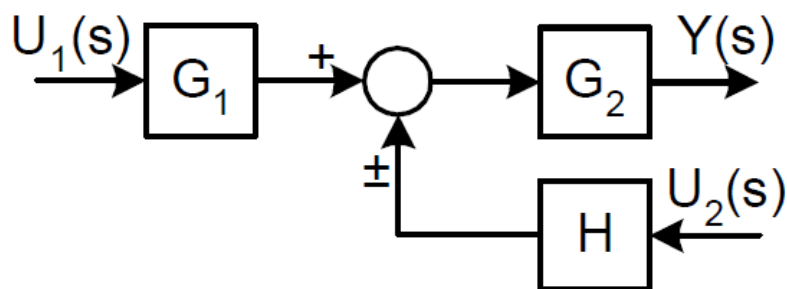
$$\square G(s) = \frac{G_D(s)}{1 + H(s)G_D(s)}$$
$$\square G(s) = \frac{G_1(s)G_2(s)G_3(s)}{1 + H(s)G_1(s)G_2(s)G_3(s)}$$

Pravila algebre funkcije prenosa

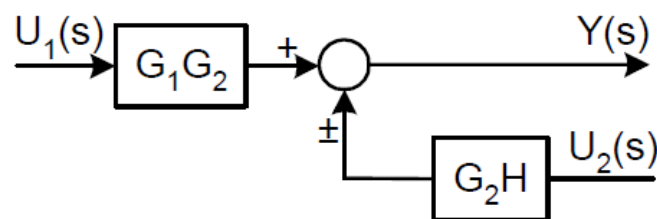
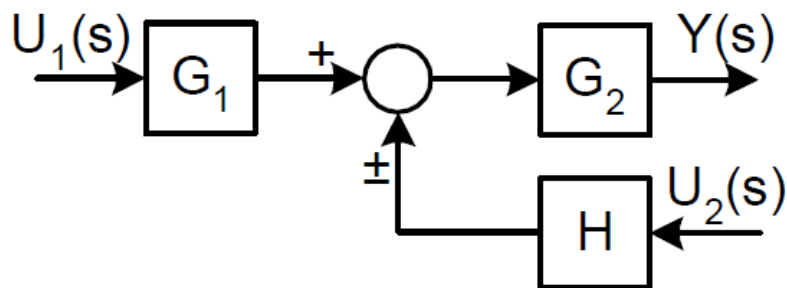
Svođenje povratne sprege na jedan blok		
Premeštanje bloka H iz povratne grane		

Pravila algebre funkcije prenosa

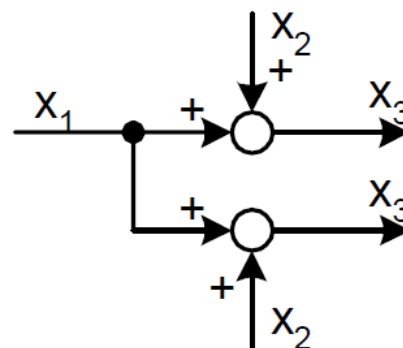
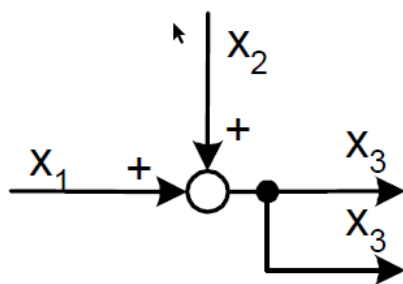
Premeštanje
diskriminatora
ispred bloka
 G_1



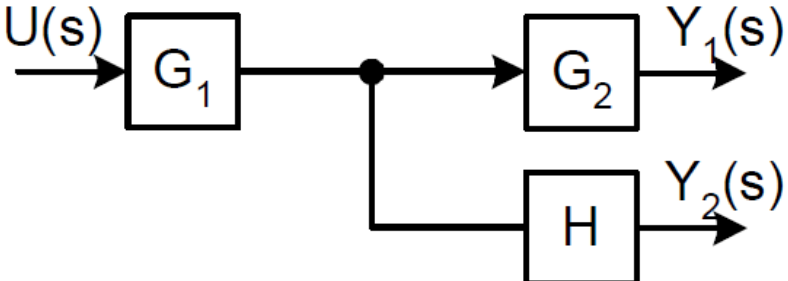
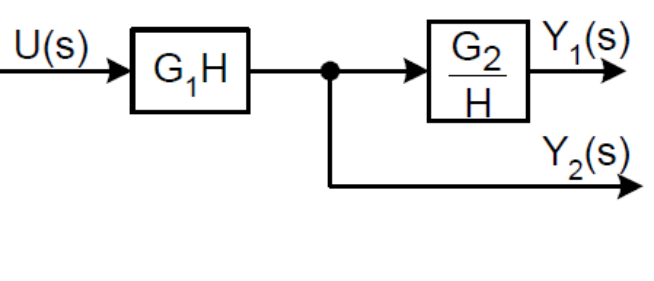
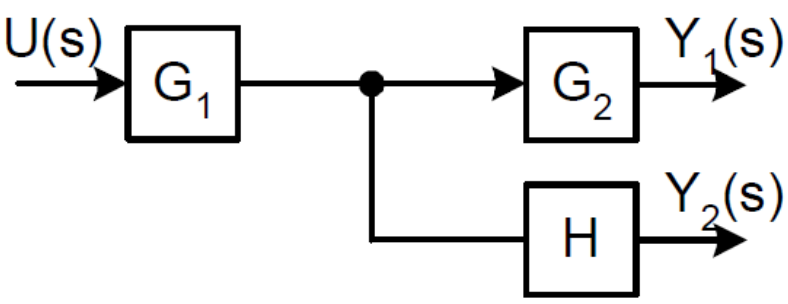
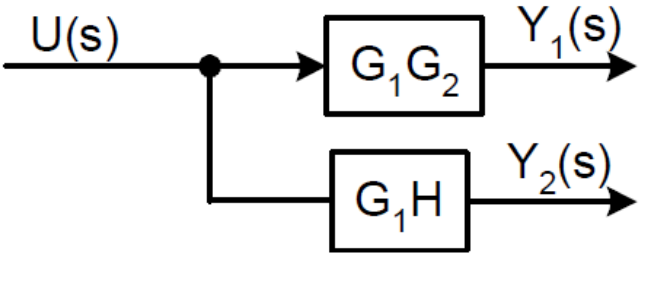
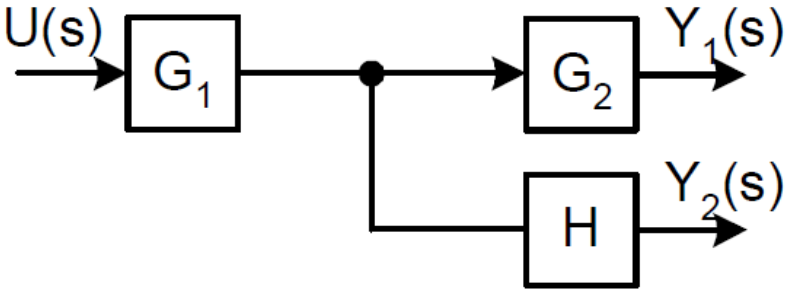
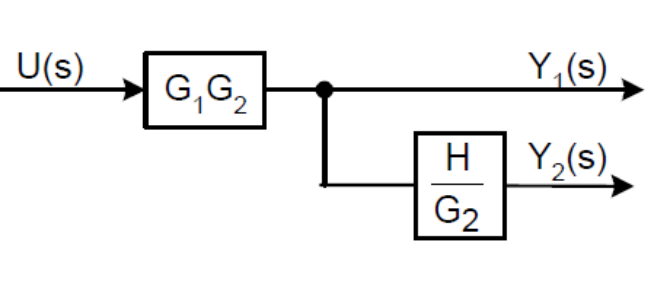
Premeštanje
diskriminatora
iza bloka G_2



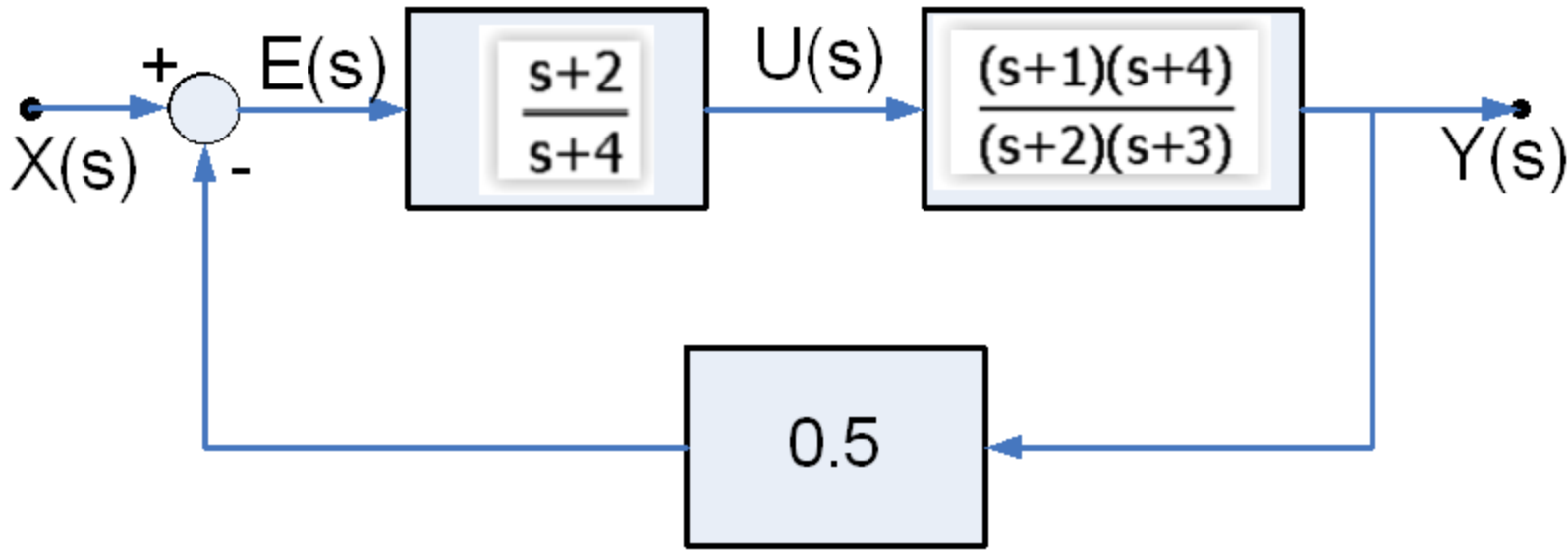
Deljenje
diskriminatora
na dva dela



Pravila algebre funkcije prenosa

Premeštanje bloka H iz direktne grane		
Premeštanje tačke račvanja (čvora) ispred bloka G_1		
Premeštanje tačke račvanja (čvora) iza bloka G_2		

Primer 2. Za sistem sa slike odrediti funkciju prenosa direktne grane (otvorenog sistema) i ukupnu funkciju prenosa (zatvorenog sistema)

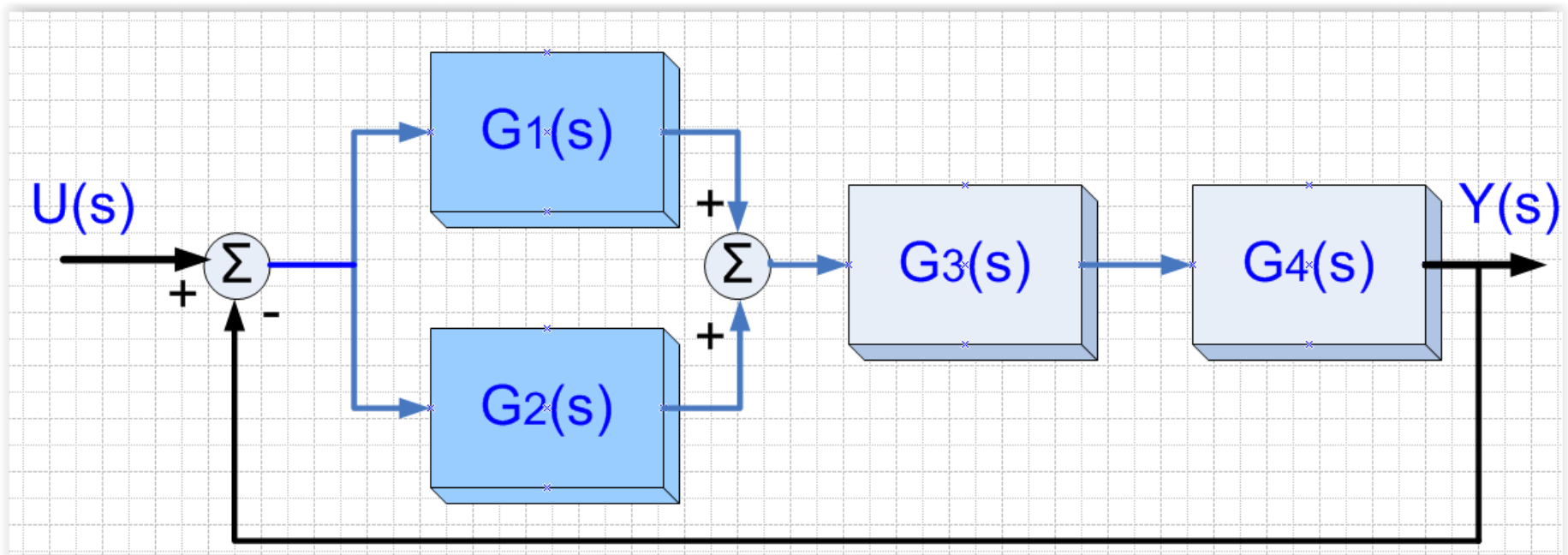


$$\square G_D(s) = \frac{Y(s)}{E(s)} = \frac{s+2}{s+4} \frac{(s+1)(s+4)}{(s+2)(s+3)} = \frac{(s+1)(s+2)(s+4)}{(s+2)(s+3)(s+4)} = \frac{s+1}{s+3}$$

$$\square G_H(s) = 0.5$$

$$\square G_e(s) = \frac{Y(s)}{X(s)} = \frac{G_D(s)}{1 + G_D(s)G_H(s)} = \frac{\frac{s+1}{s+3}}{1 + 0.5 \frac{s+1}{s+3}} = \frac{s+1}{1.5s + 3.5}$$

Primer 3: Za sistem sa slike odrediti funkciju prenosa direktne grane i ukupnu funkciju prenosa

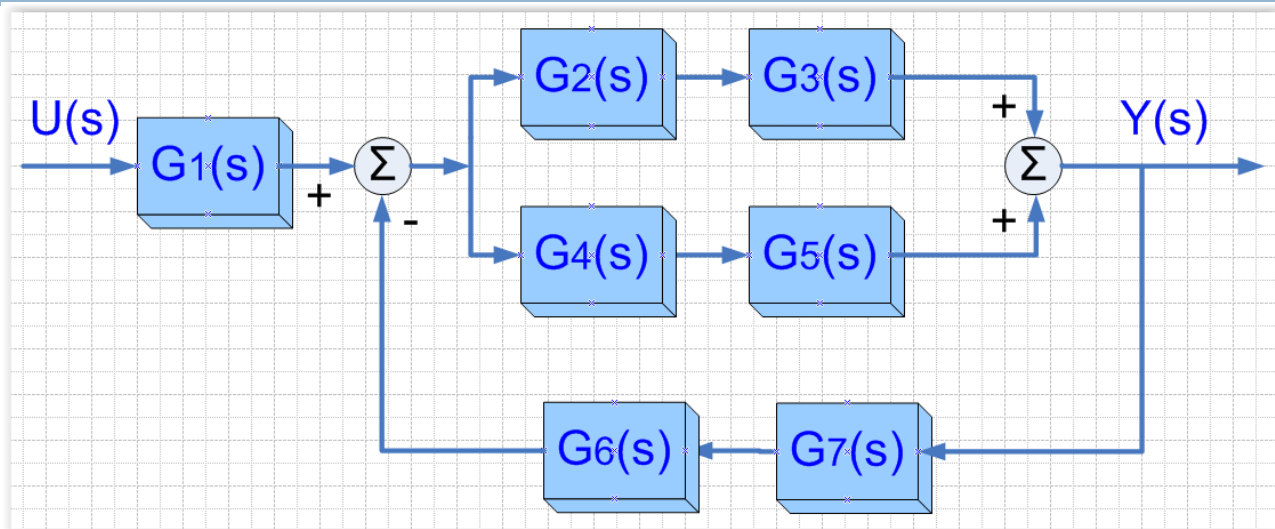


$$\square G_D(s) = (G_1(s) + G_2(s))G_3(s)G_4(s)$$

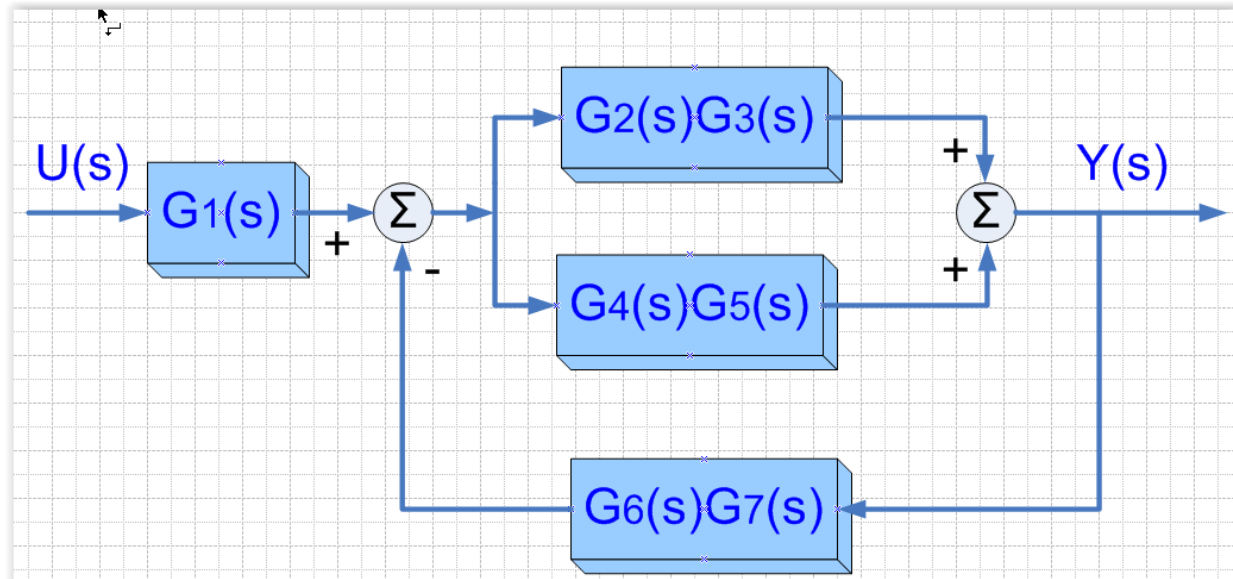
$$\square G(s) = \frac{G_D(s)}{1 + H(s)G_D(s)} ; H(s) = 1$$

$$\square G(s) = \frac{(G_1(s) + G_2(s))G_3(s)G_4(s)}{1 + (G_1(s) + G_2(s))G_3(s)G_4(s)}$$

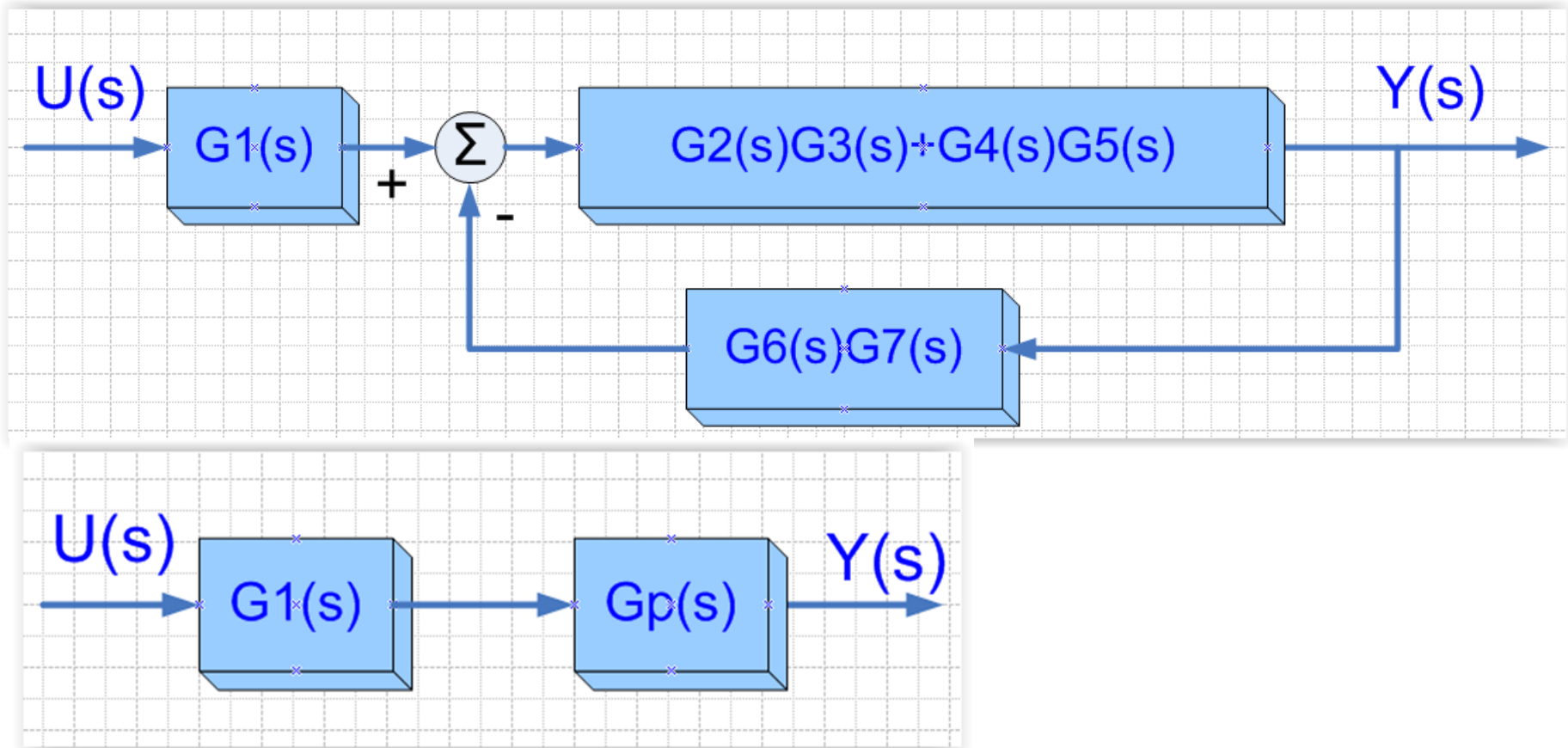
Primer 4: Za sistem sa slike odrediti funkciju prenosa direktne grane i ukupnu funkciju prenosa



□ Rešenje:

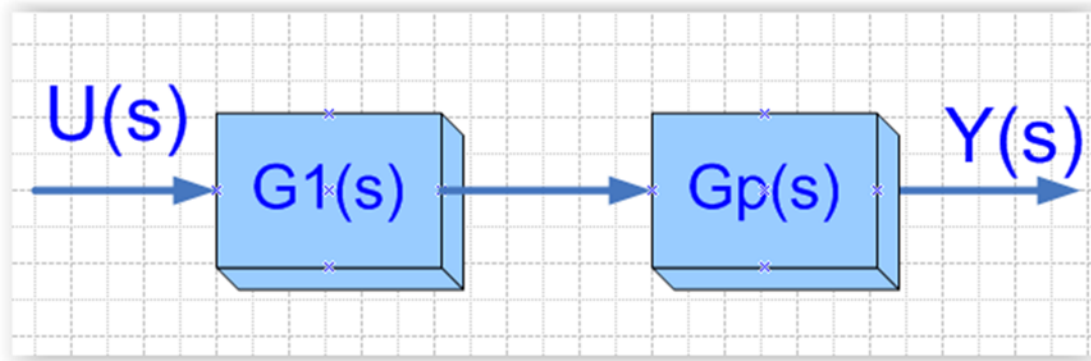


Primer 4: Za sistem sa slike odrediti funkciju prenosa direktne grane i ukupnu funkciju prenosa



$$G_p(s) = \frac{(G_2(s)G_3(s) + G_4(s)G_5(s))}{1 + G_6(s)G_7(s)(G_2(s)G_3(s) + G_4(s)G_5(s))}$$

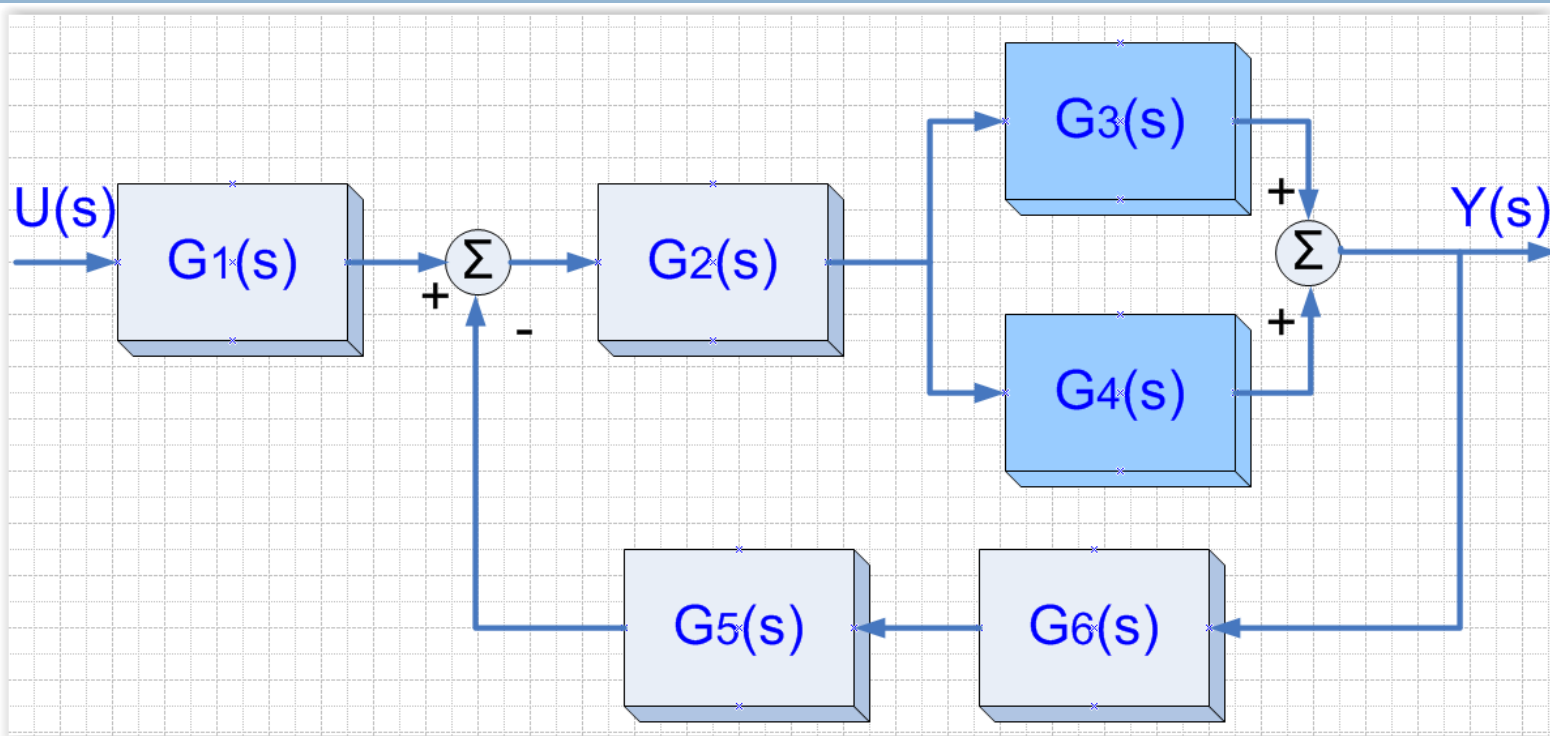
Primer 4: Za sistem sa slike odrediti funkciju prenosa direktne grane i ukupnu funkciju prenosa



$$G(s) = \frac{Y(s)}{X(s)} = G_1(s)G_p(s)$$

$$G(s) = \frac{G_1(s)(G_2(s)G_3(s) + G_4(s)G_5(s))}{1 + G_6(s)G_7(s)(G_2(s)G_3(s) + G_4(s)G_5(s))}$$

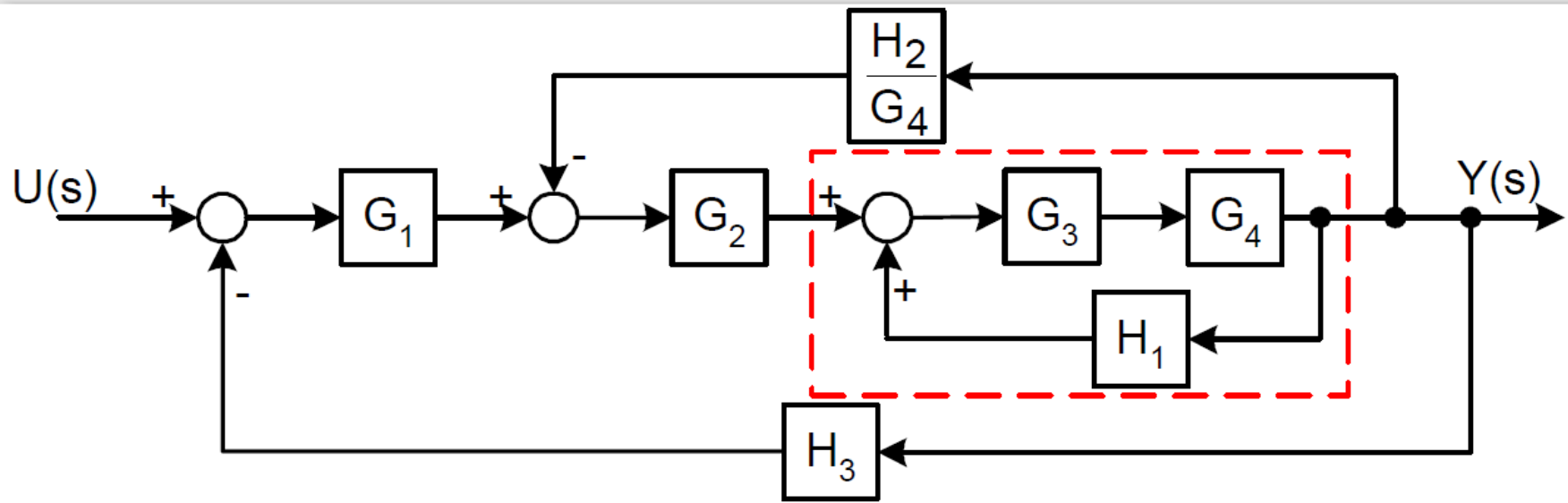
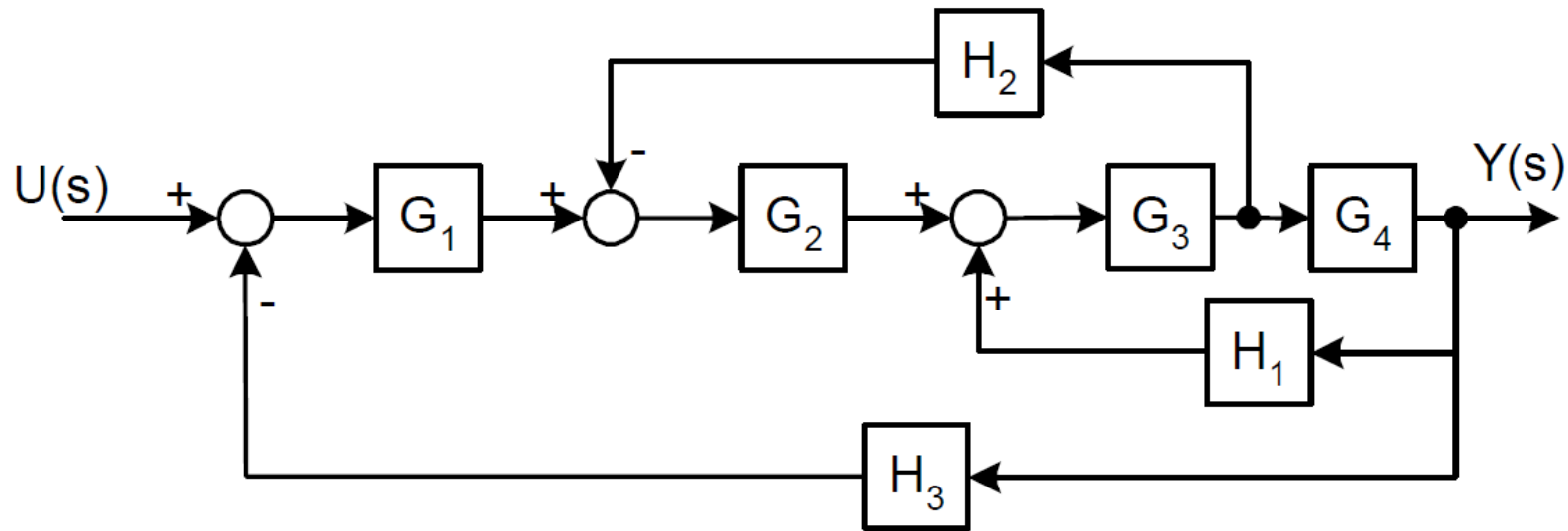
Primer 5: Za sistem sa slike ekvivalentnu funkciju prenosa



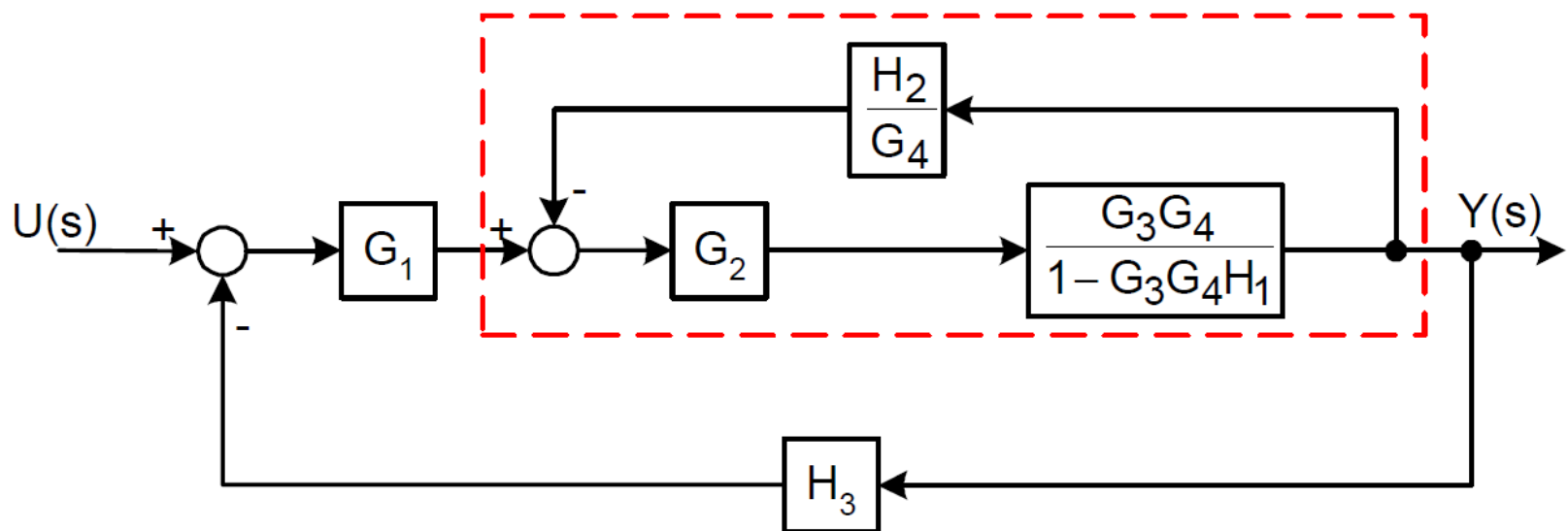
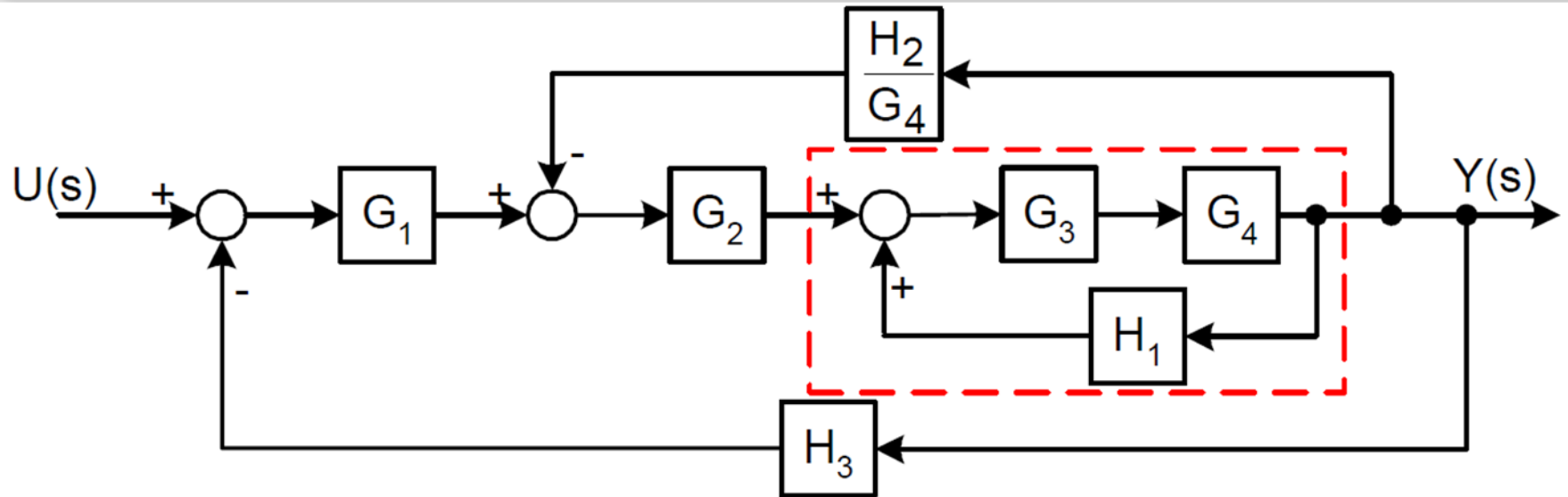
$$\square \quad G(s) = \frac{Y(s)}{U(s)} = G_1(s)G_p(s)$$

$$\square \quad G(s) = \frac{G_1(s)G_2(s)(G_3(s)+G_4(s))}{1 + G_5(s)G_6(s)G_2(s)(G_3(s)+G_4(s))}$$

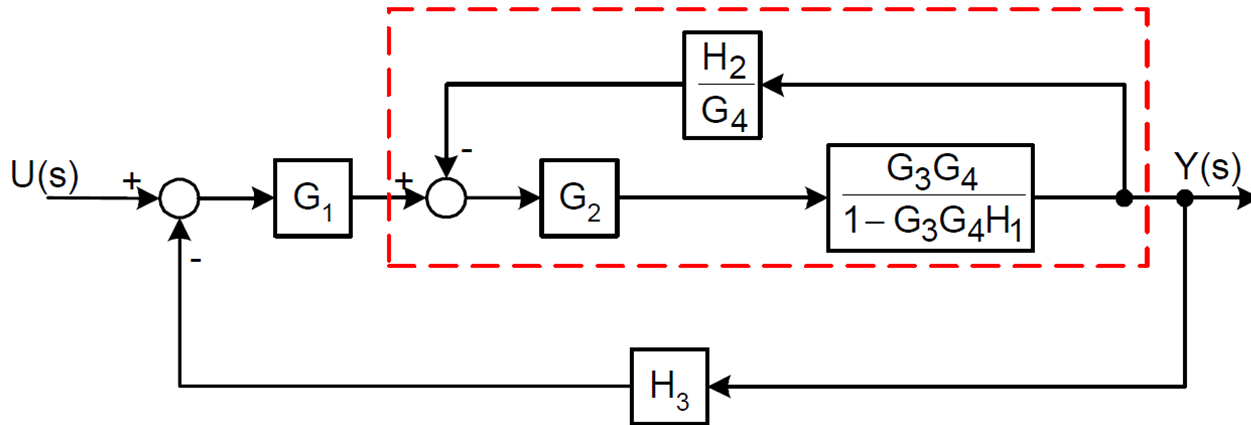
Primer 6: Primenom algebre funkcije prenosa odrediti funkciju prenosa $G(s) = Y(s)/U(s)$ sistema sa slike



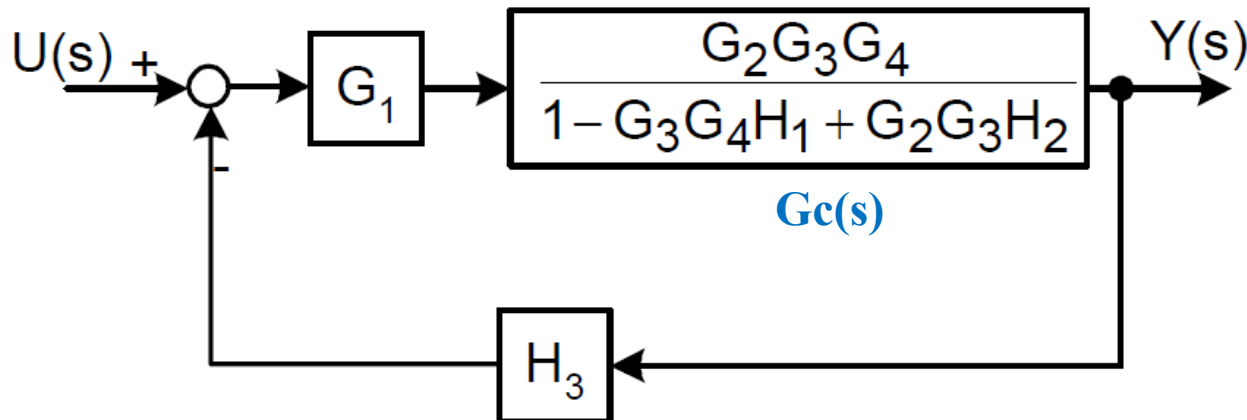
Primer 6:



Primer 6:

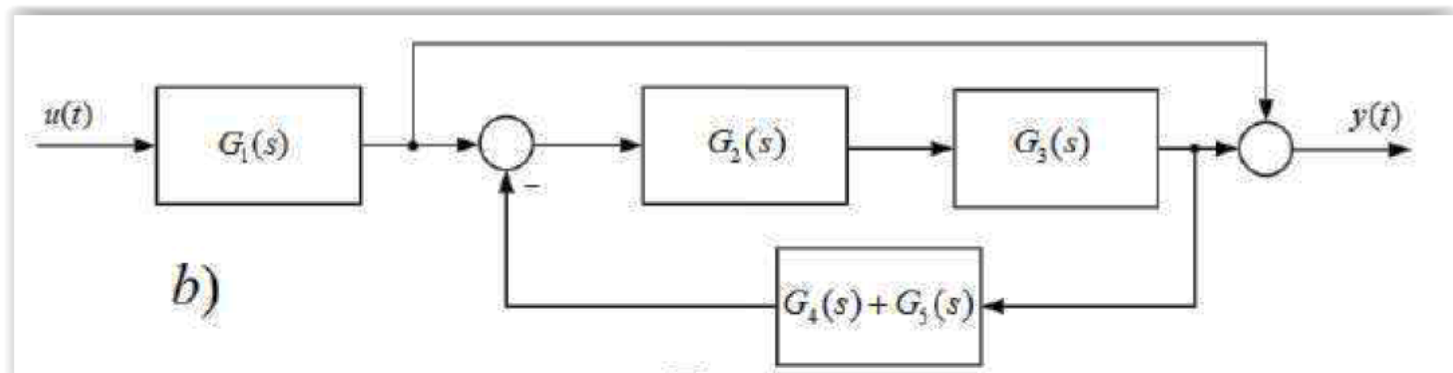
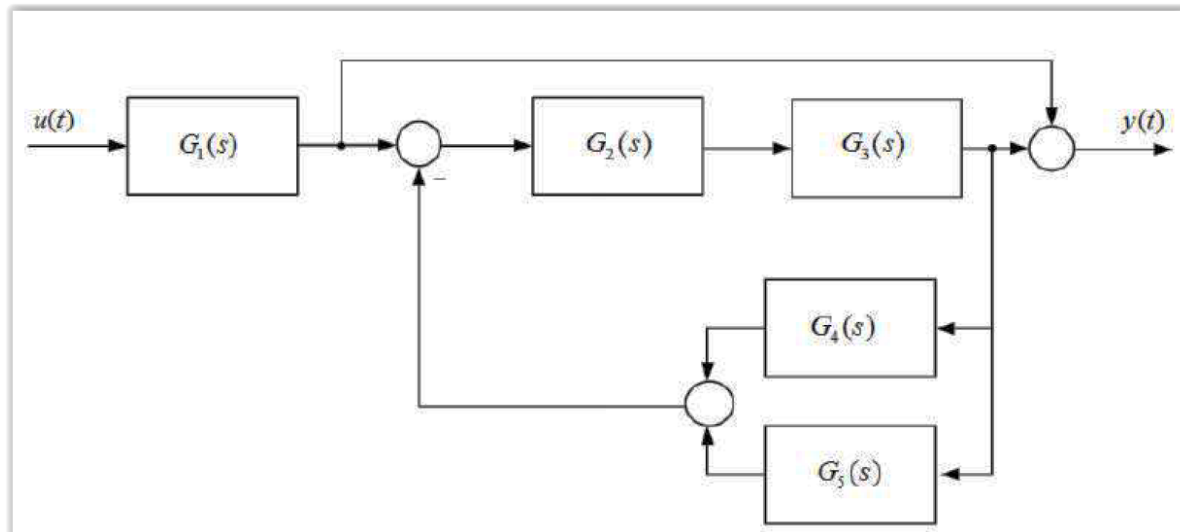


$$G_c(s) = \frac{\frac{G_2 G_3 G_4}{1 - G_2 G_3 G_4}}{1 + \frac{H_2}{G_4} \frac{G_2 G_3 G_4}{1 - G_2 G_3 G_4}}$$



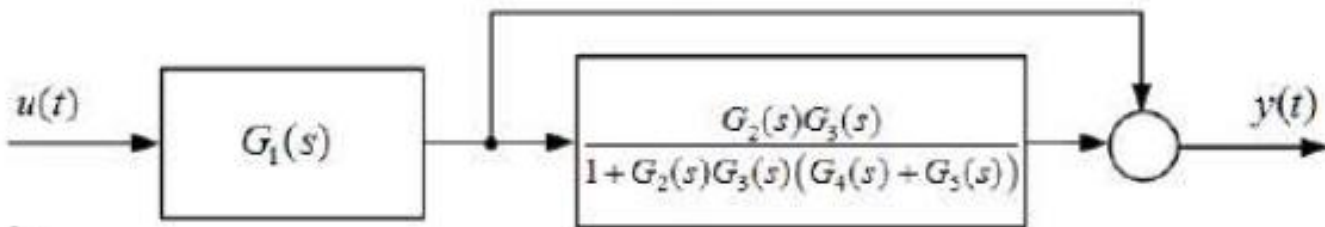
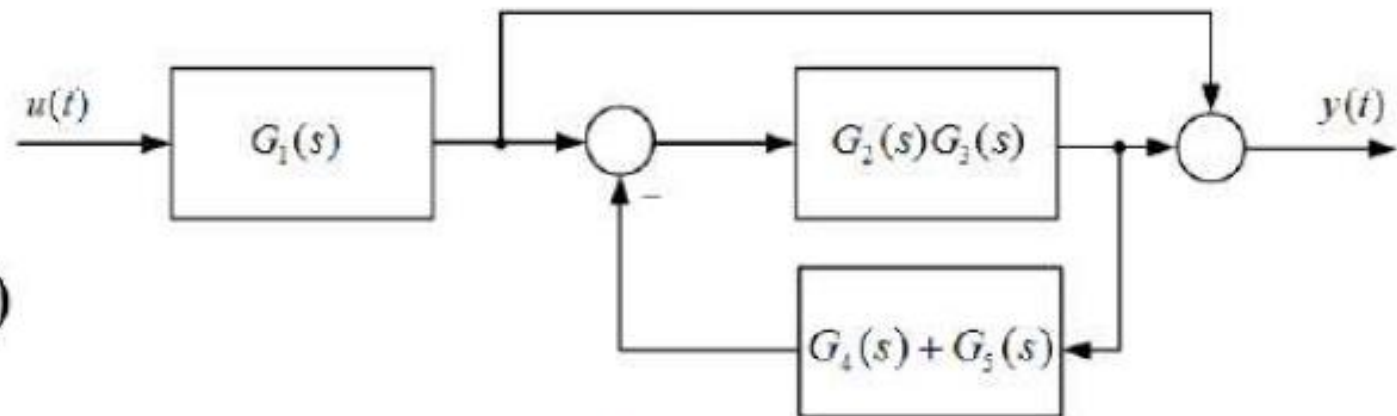
$$U(s) \rightarrow \frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3} Y(s)$$

Primer 7: Odrediti prenosnu funkciju za sistem sa slike

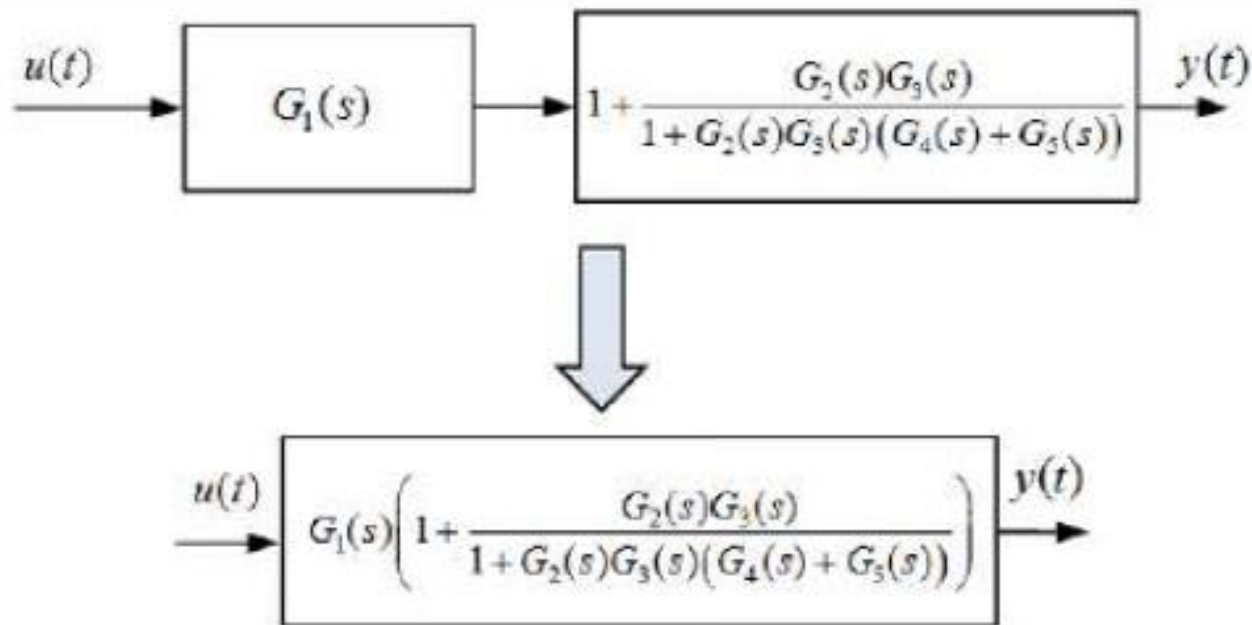


Primer 7:

c)



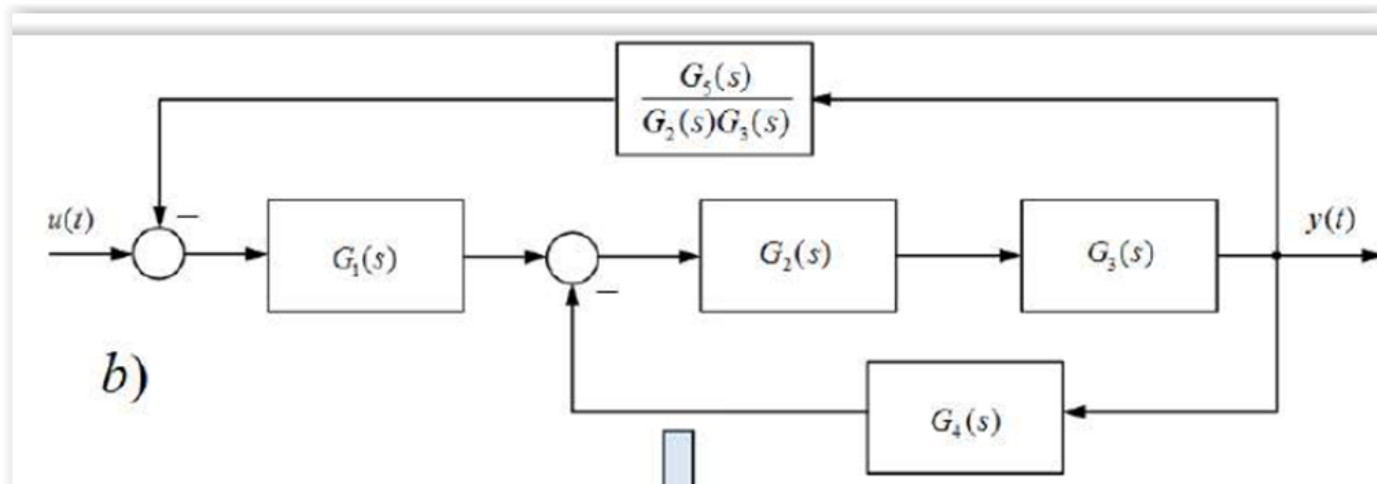
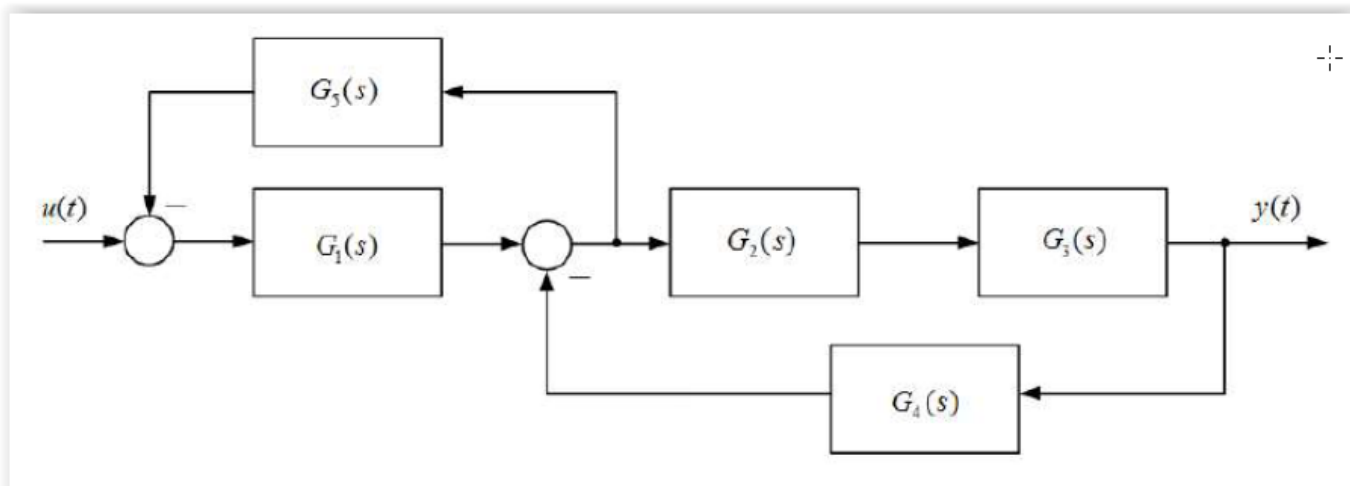
Primer 7:



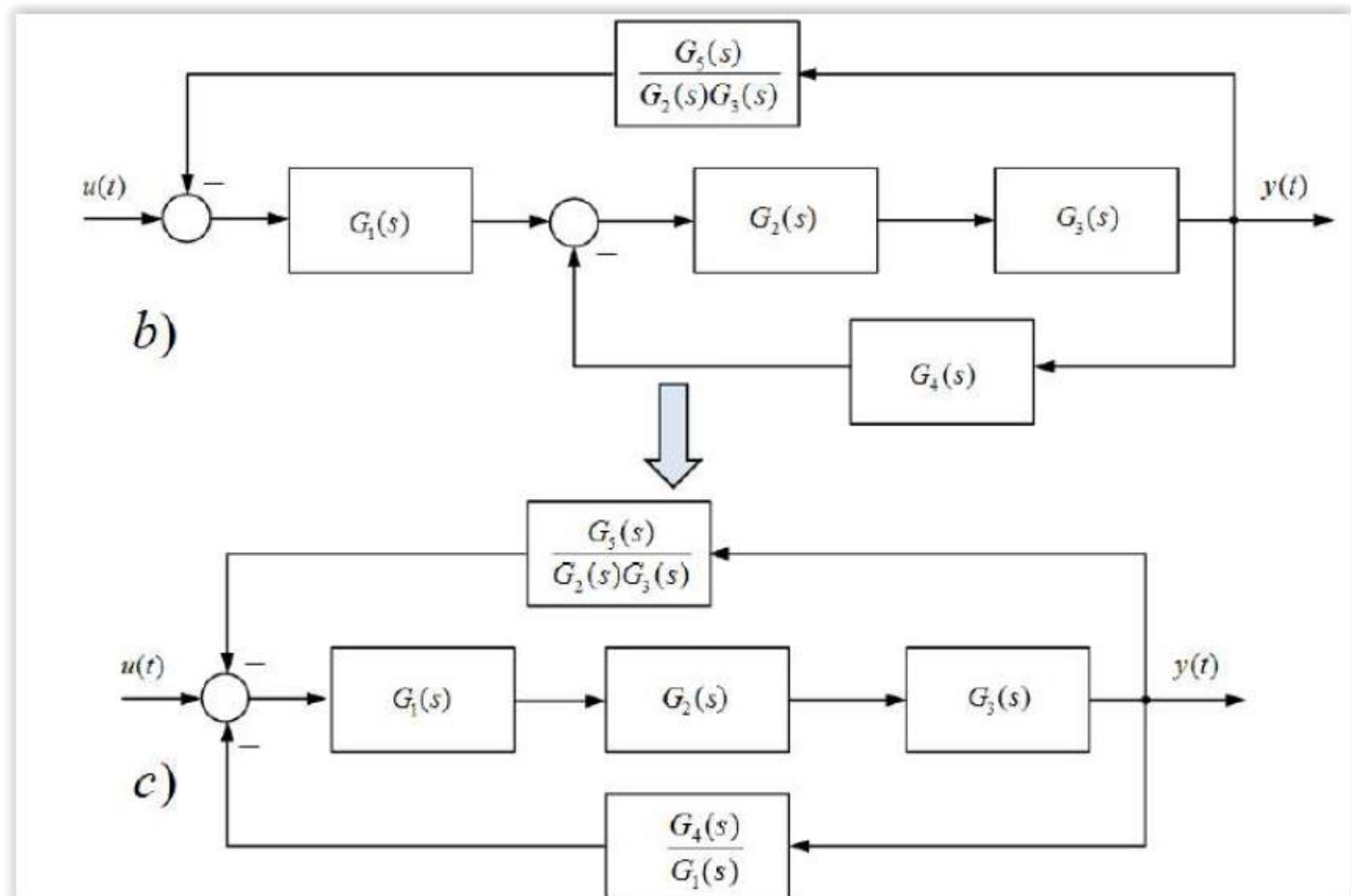
$$G(s) = G_1(s) \left(1 + \frac{G_2(s)G_3(s)}{1 + G_2(s)G_3(s)(G_4(s) + G_5(s))} \right).$$

$$G(s) = \frac{G_1(s)G_2(s)G_3(s)}{1 + G_1(s)G_5(s) + G_2(s)G_3(s)G_4(s)}.$$

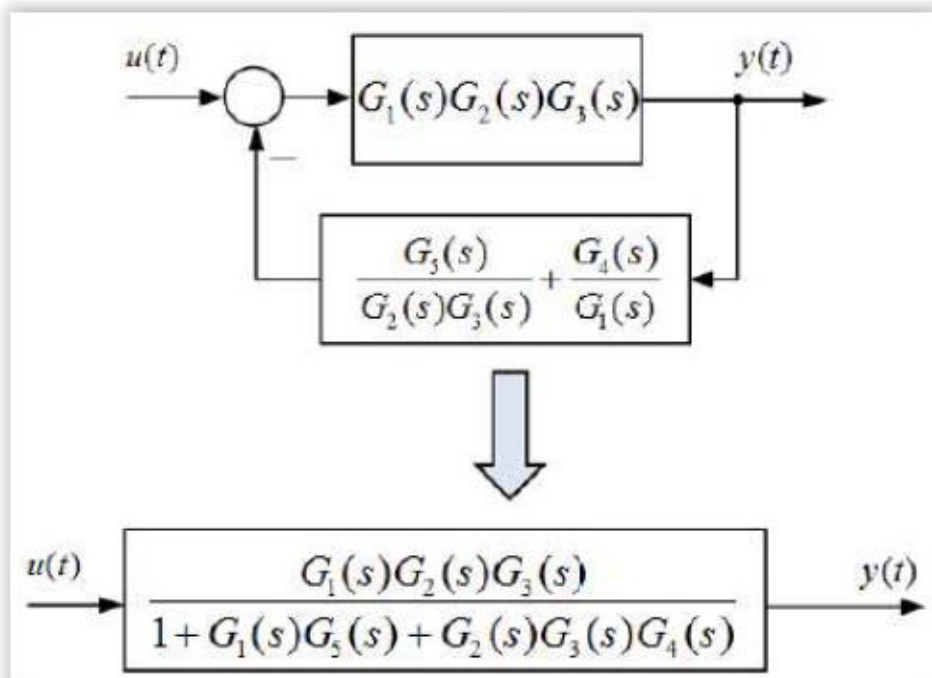
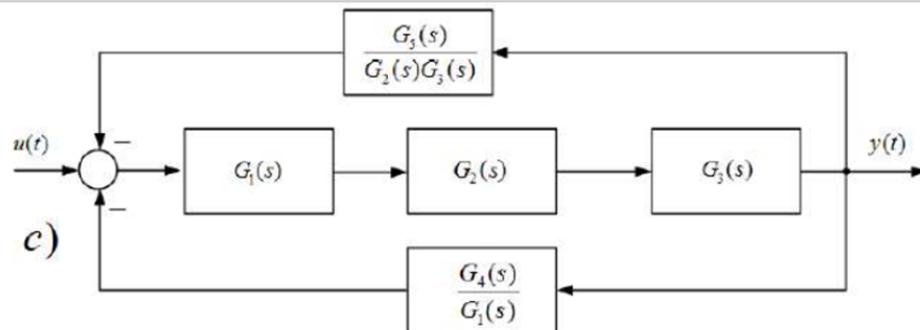
Primer 8: Odrediti prenosnu funkciju za sistem sa slike



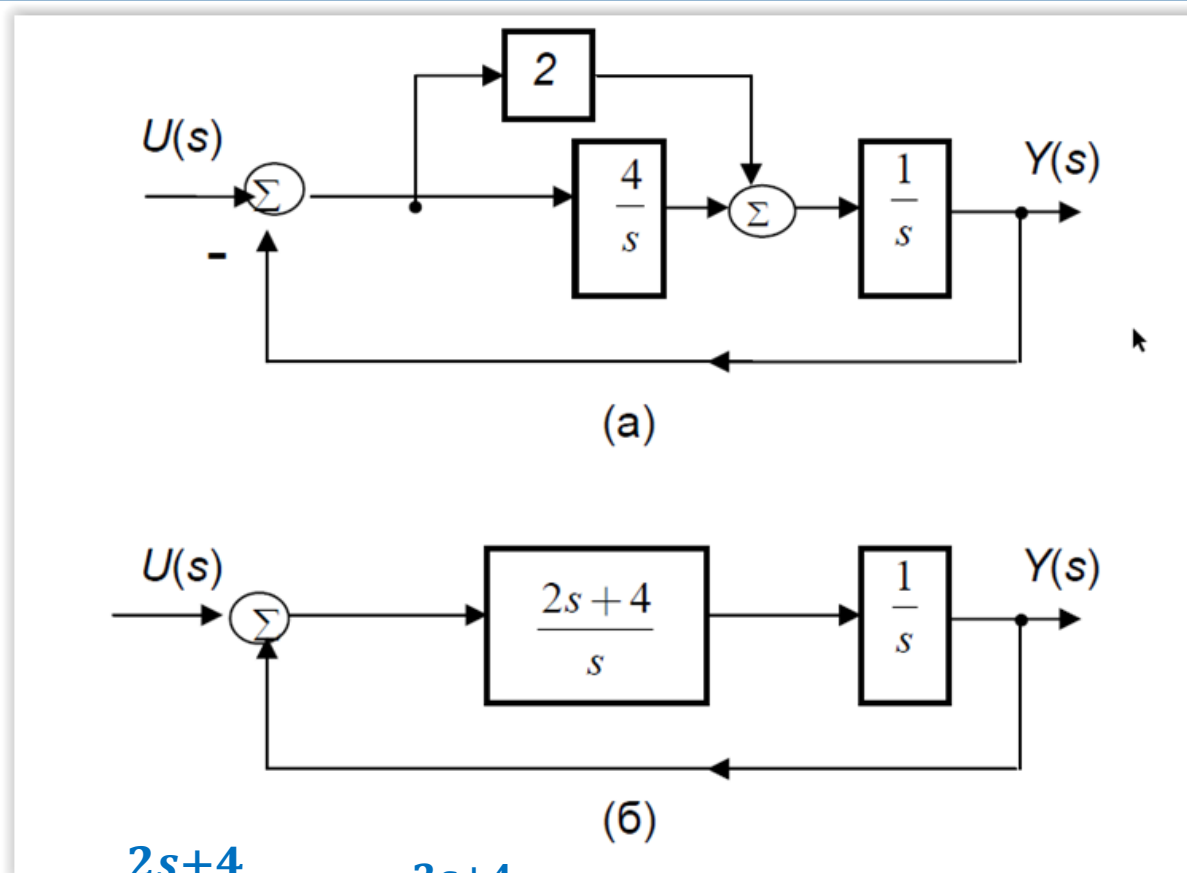
Primer 8:



Primer 8:

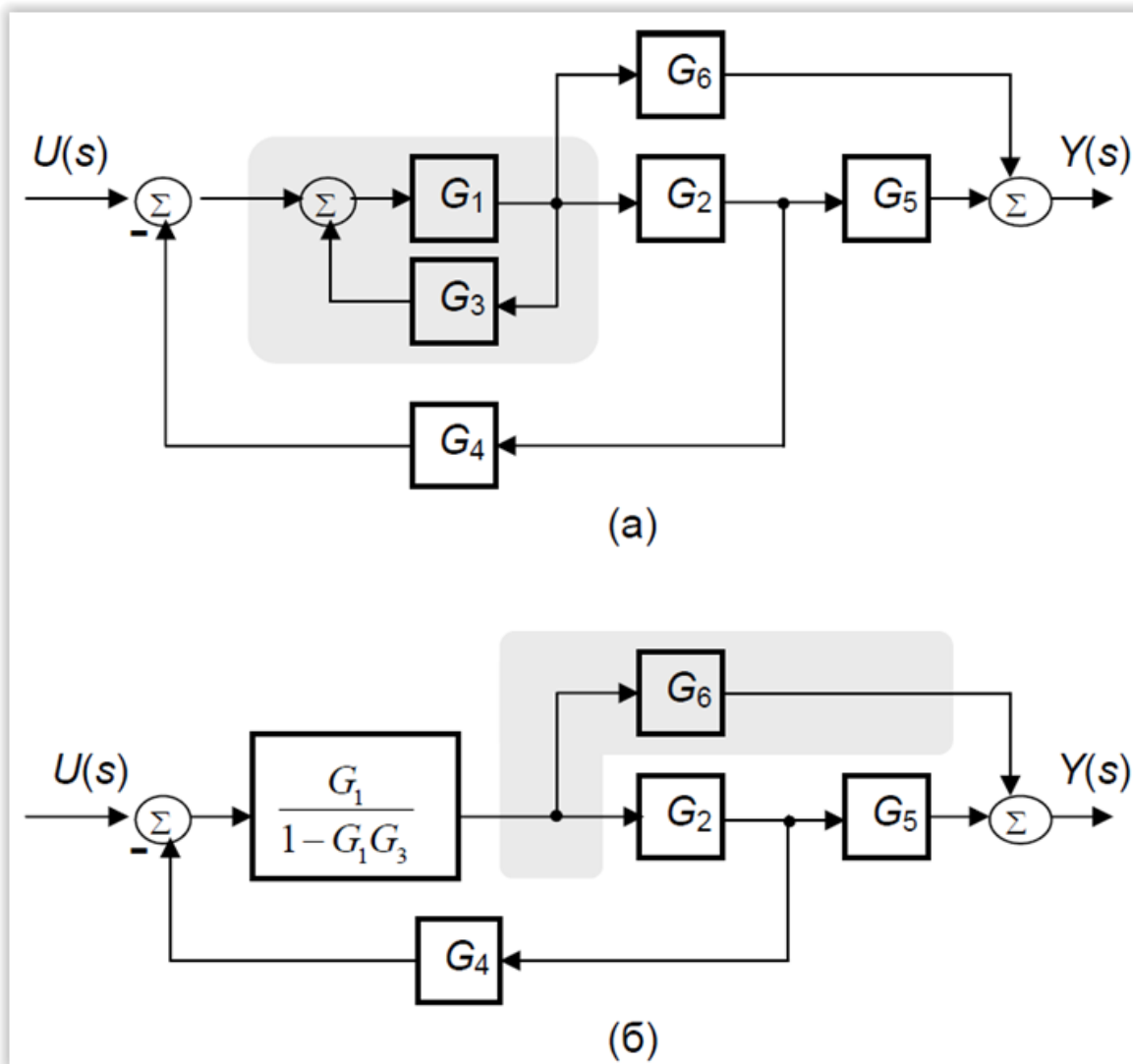


Primer 9. Odrediti funkciju prenosa za sistem sa slike

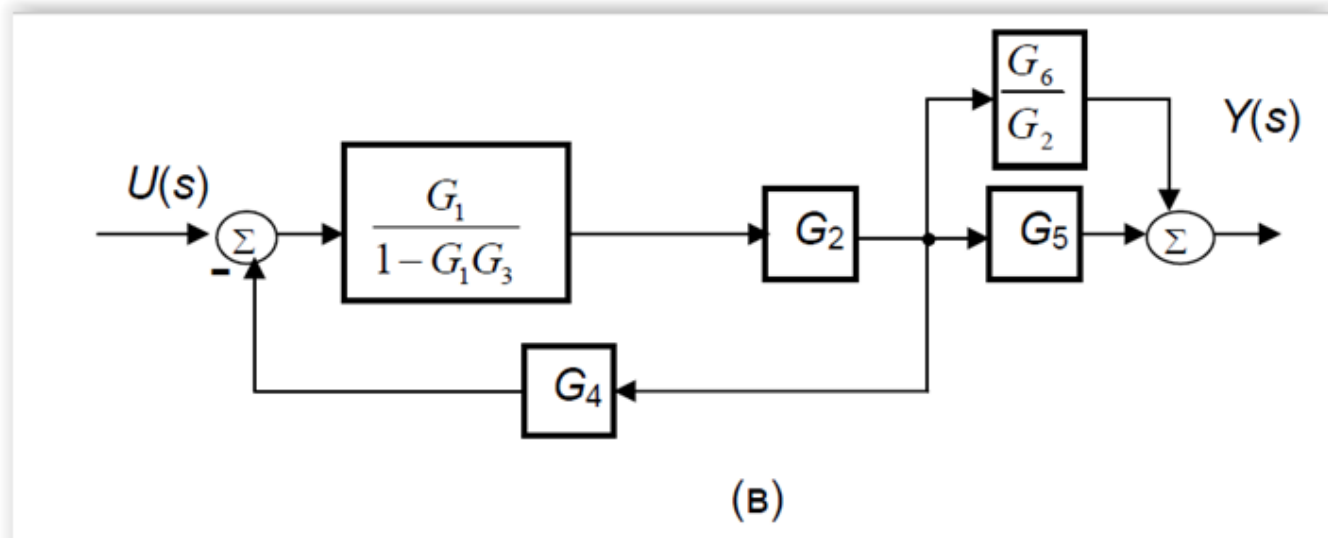
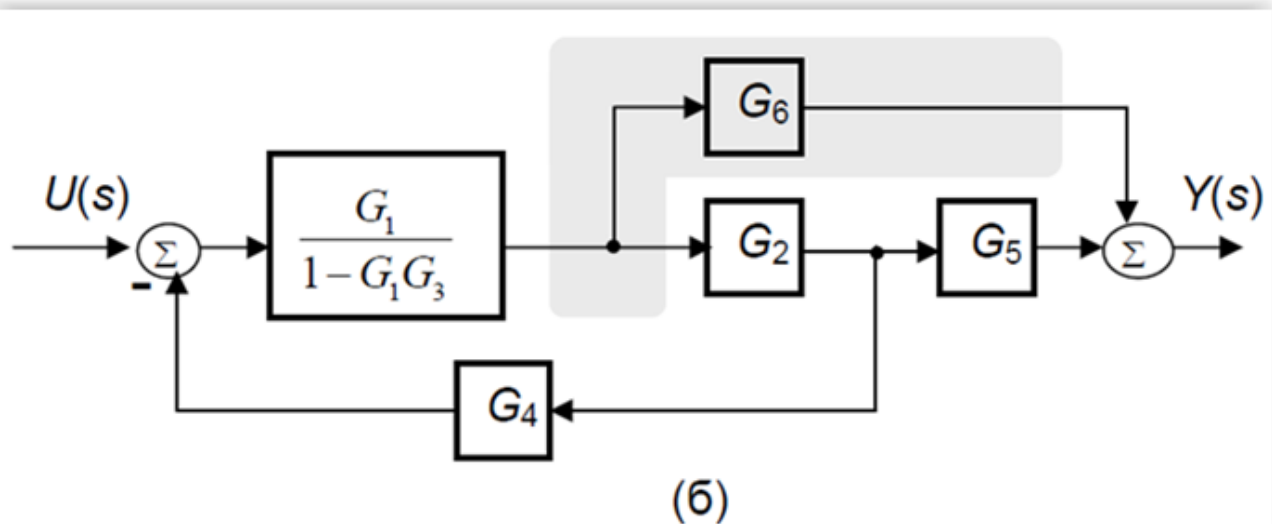


$$\square \quad G(s) = \frac{\frac{2s+4}{s^2}}{1 + \frac{2s+4}{s^2}} = \frac{\frac{2s+4}{s^2}}{\frac{s^2+2s+4}{s^2}} = \frac{2s+4}{s^2+2s+4}$$

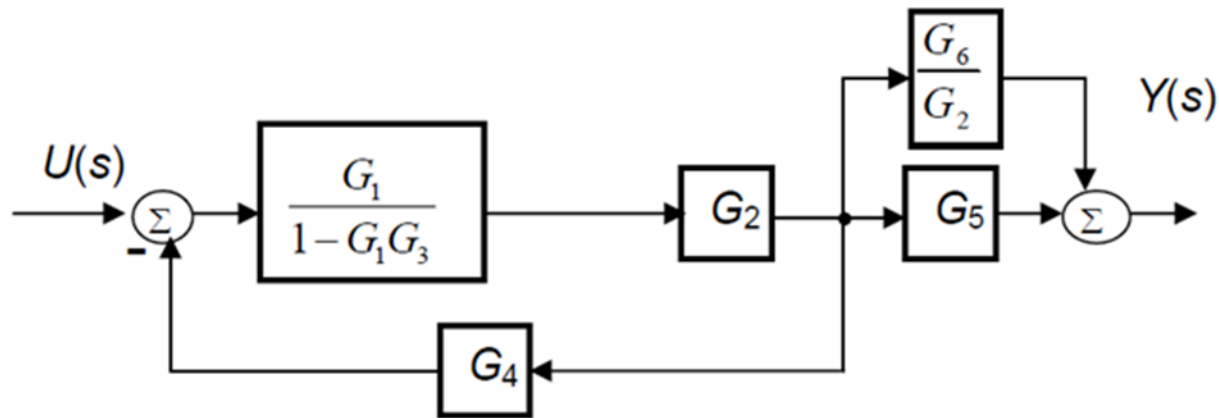
Primer 10. Odrediti funkciju prenosa za sistem sa slike



Primer 10:



Primer 10:

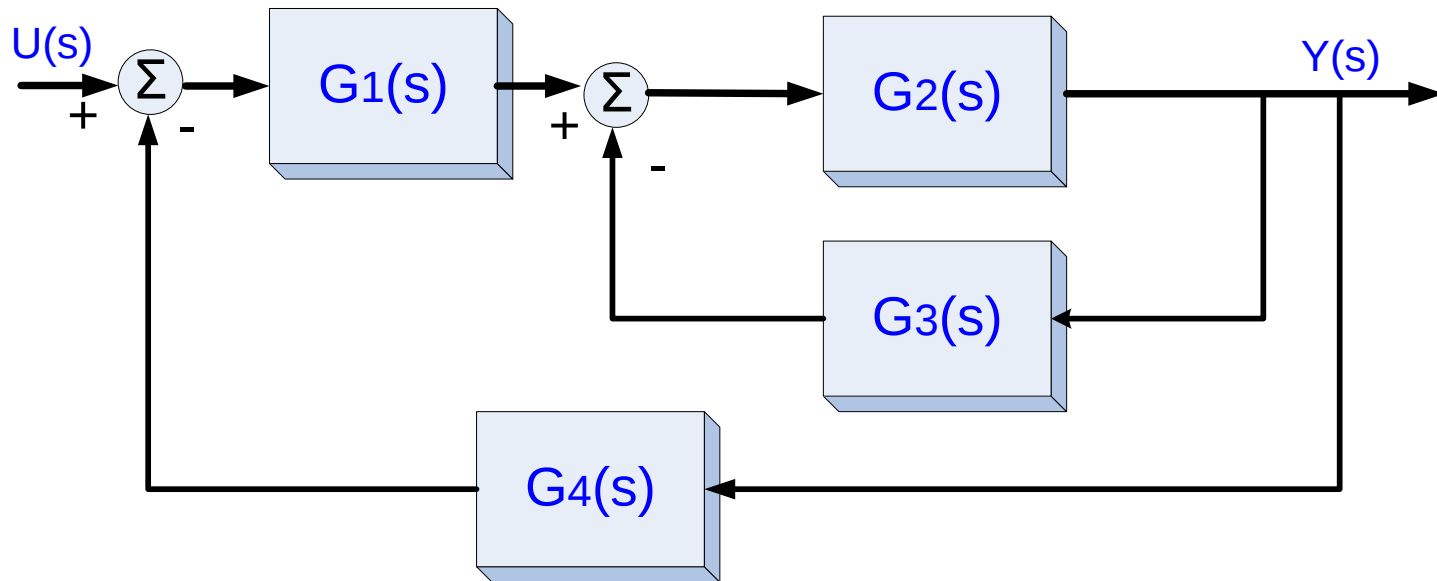


(B)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\frac{G_1 G_2}{1 - G_1 G_3}}{1 + \frac{G_1 G_2 G_4}{1 - G_1 G_3}} \left(G_5 + \frac{G_6}{G_2} \right)$$

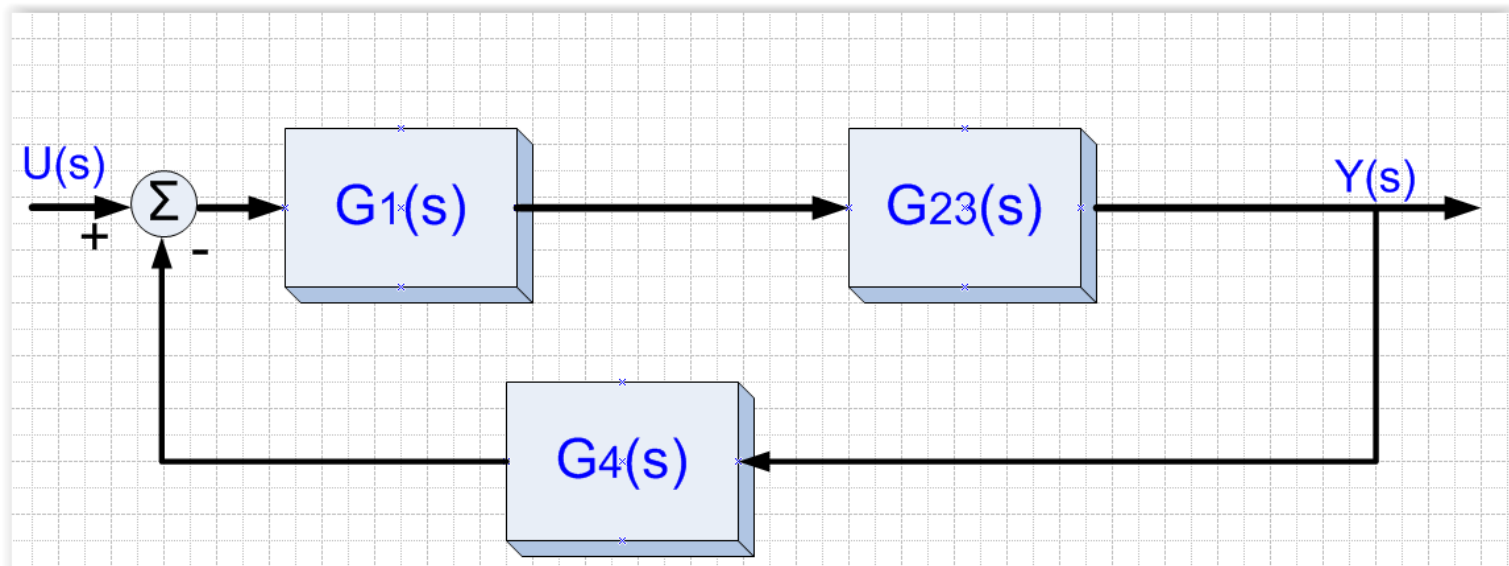
$$G(s) = \frac{G_1 G_2 G_5 + G_1 G_6}{1 - G_1 G_3 + G_1 G_2 G_4}.$$

Primer 11. Odrediti funkciju prenosa za sistem sa slike



$$G_{23}(s) = \frac{G_2(s)}{1 + G_2(s)G_3(s)}$$

Primer 11. Odrediti funkciju prenosa za sistem sa slike



$$\square \quad G_{23}(s) = \frac{G_2(s)}{1 + G_2(s)G_3(s)}$$

$$\square \quad G(s) = \frac{Y(s)}{X(s)} = \frac{G_1(s)G_{23}(s)}{1 + G_4(s)G_1(s)G_{23}(s)} = \frac{\frac{G_1(s)G_2(s)}{1 + G_2(s)G_3(s)}}{1 + G_4(s)G_1(s)\frac{G_2(s)}{1 + G_2(s)G_3(s)}}$$

$$\square \quad G(s) = \frac{Y(s)}{X(s)} = \frac{G_1(s)G_2(s)}{1 + G_2(s)G_3(s) + G_4(s)G_1(s)G_2(s)}$$