



Elektrotehnika

Vežbe 12

Naizmenične struje



Naizmenične struje

IV.1.9.1 Dva prijemnika koja se sastoje od redne veze elemenata, $R_1 = 80 \, \Omega$, $L_1 = 1 \, \text{mH}$ i $C_1 = 250 \, \text{nF}$, i $R_2 = 60 \, \Omega$, $L_2 = 1,2 \, \text{mH}$ i $C_2 = 50 \, \text{nF}$, vezana su redno i priključena u kolo naizmenične struje kružne učestanosti $\omega = 10^5 \, \text{s}^{-1}$. Odrediti:

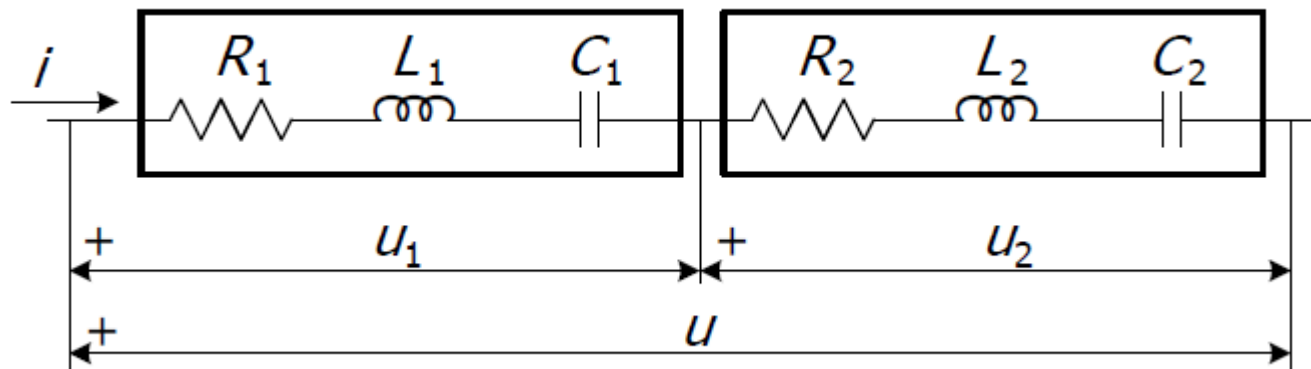
- a) impedanse i razlike faza napona i struje pojedinih prijemnika,
- b) impedansu, razliku faza napona i struje i karakter redne veze,
- c) efektivnu vrednost napona na svakom od prijemnika, kao i efektivnu vrednost napona redne veze
- d) ekvivalentne paralelne parametre i admitansu cele veze,
- e) aktivnu, reaktivnu i prividnu snagu svakog prijemnika i redne veze, ako je efektivna vrednost struje $I = 1 \, \text{A}$.



Naizmenične struje

Zadatak IV.1.9.1

Rešenje a):



$$X_1 = \omega L_1 - \frac{1}{\omega C_1} = 10^5 \text{ s}^{-1} \cdot 1 \cdot 10^{-3} \text{ H} - \frac{1}{10^5 \text{ s}^{-1} \cdot 250 \cdot 10^{-9} \text{ F}} = 100 \Omega - 40 \Omega = 60 \Omega$$

$$Z_1 = \sqrt{R_1^2 + X_1^2} = \sqrt{(80 \Omega)^2 + (60 \Omega)^2} = \sqrt{1000 \Omega^2} = 100 \Omega$$

$$\varphi_1 = \arctg \frac{X_1}{R_1} = \arctg \frac{60 \Omega}{80 \Omega} = \arctg 0,75$$

Izraz $\arctg 0,75$ ima dva rešenja: 0,64 radijana i -2,5 radijana,

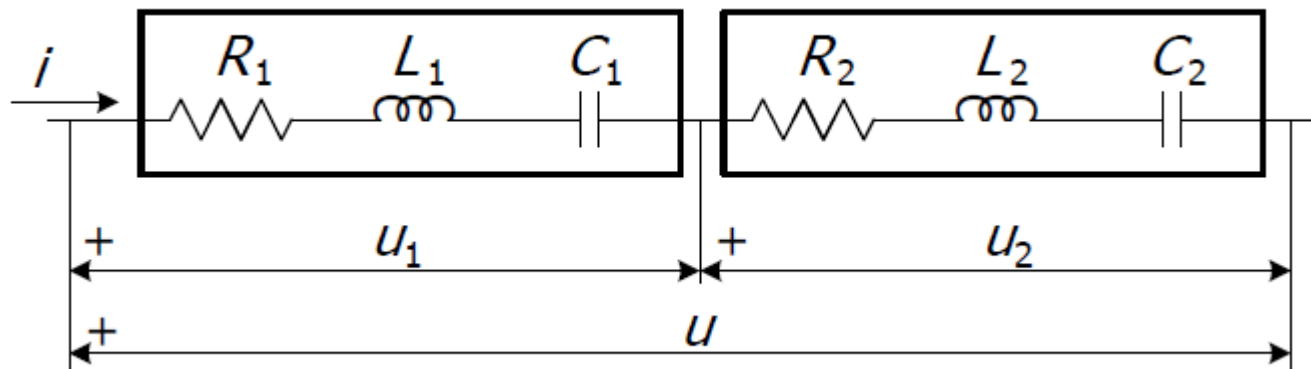
$$\varphi_1 = 0,64$$



Naizmenične struje

Zadatak IV.1.9.1

Rešenje a):



$$X_2 = \omega L_2 - \frac{1}{\omega C_2} = 10^5 \text{ s}^{-1} \cdot 1,2 \cdot 10^{-3} \text{ H} - \frac{1}{10^5 \text{ s}^{-1} \cdot 50 \cdot 10^{-9} \text{ F}} = 120 \Omega - 200 \Omega = -80 \Omega$$

$$Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{(60 \Omega)^2 + (-80 \Omega)^2} = \sqrt{10000 \Omega^2} = 100 \Omega$$

$$\varphi_2 = \arctg \frac{X_2}{R_2} = \arctg \frac{-80 \Omega}{60 \Omega} = \arctg (-1,33) \text{ Izraz } \arctg (-1,33) \text{ ima dva rešenja: } -0,93 \text{ radijana i } 2,21 \text{ radijana}$$

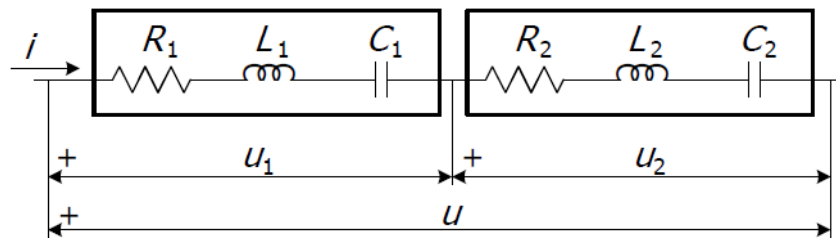
$$\varphi_2 = -0,93$$



Naizmenične struje

Zadatak IV.1.9.1

Rešenje b):



$$R_e = R_1 + R_2 = 80 \, \Omega + 60 \, \Omega = 140 \, \Omega$$

$$X_e = X_1 + X_2 = 60 \, \Omega + (-80 \, \Omega) = -20 \, \Omega$$

$$Z_e = \sqrt{R_e^2 + X_e^2} = \sqrt{(140 \, \Omega)^2 + (-20 \, \Omega)^2} = \sqrt{20000 \, \Omega^2} = 141,4 \, \Omega$$

$$\varphi_e = \arctg \frac{X_e}{R_e} = \arctg \frac{-20 \, \Omega}{140 \, \Omega} = \arctg (-0,14)$$

Rešenja ovog izraza su: - 0,07 radijana i 3 radijana

$$\varphi_e = -0,14$$



Naizmenične struje

Zadatak IV.1.9.1

Rešenje c):

$$U_1 = Z_1 \cdot I = 100 \, \Omega \cdot 1 \, \text{A} = 100 \, \text{V}$$

$$U_2 = Z_2 \cdot I = 100 \, \Omega \cdot 1 \, \text{A} = 100 \, \text{V}$$

$$U = Z_e \cdot I = 141,4 \, \Omega \cdot 1 \, \text{A} = 141,4 \, \text{V}$$

$$U_1 + U_2 = 100 \, \text{V} + 100 \, \text{V} = 200 \, \text{V}$$

$$U \neq U_1 + U_2$$

$$U = Z_e \cdot I \neq U_1 + U_2 = Z_1 \cdot I + Z_2 \cdot I \quad ; \quad Z_e \neq Z_1 + Z_2$$



Naizmenične struje

Zadatak IV.1.9.1

Rešenje d):

$$G_e = \frac{R_e}{R_e^2 + X_e^2} = \frac{140 \, \Omega}{(140 \, \Omega)^2 + (-20 \, \Omega)^2} = \frac{140}{20000 \, \Omega} = 0,007 \, \text{S} = 7 \, \text{mS}$$

$$B_e = -\frac{X_e}{R_e^2 + X_e^2} = -\frac{-20 \, \Omega}{(140 \, \Omega)^2 + (-20 \, \Omega)^2} = \frac{-20 \, \Omega}{20000 \, \Omega} = 0,001 \, \text{S} = 1 \, \text{mS}$$

$$Y_e = \sqrt{G_e^2 + B_e^2} = \frac{1}{Z} = \frac{1}{141,4 \, \Omega} = 0,0071 \, \text{S} = 7,1 \, \text{mS}$$



Naizmenične struje

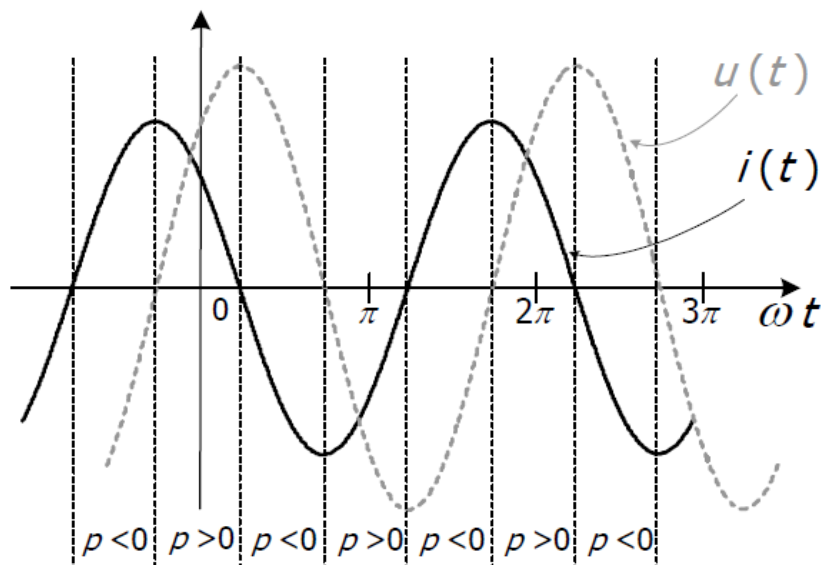
Zadatak IV.1.9.1

Rešenje e):

$$P = UI \cos \varphi = S \cos \varphi$$

$$Q = UI \sin \varphi = S \sin \varphi$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{(UI \cos \varphi)^2 + (UI \sin \varphi)^2} = UI \sqrt{\cos^2 \varphi + \sin^2 \varphi} = UI$$





Naizmenične struje

Zadatak IV.1.9.1

Rešenje e):

$$P = UI \cos \varphi = Z \cdot I \cdot I \cdot \cos \varphi = Z \cdot I^2 \cdot \frac{R}{Z} = I^2 R$$

$$P = UI \cos \varphi = U \cdot \frac{U}{Z} \cdot \cos \varphi = U^2 \cdot \frac{R}{Z^2} = U^2 G$$

$$Q = UI \sin \varphi = Z \cdot I \cdot I \cdot \sin \varphi = Z \cdot I^2 \cdot \frac{X}{Z} = I^2 X$$

$$Q = UI \sin \varphi = U \cdot \frac{U}{Z} \cdot \sin \varphi = U^2 \cdot \frac{X}{Z^2} = -U^2 B$$

$$S = UI = Z \cdot I \cdot I = Z \cdot I^2$$

$$S = UI = U \cdot U \cdot Y = Y \cdot U^2$$



Naizmenične struje

Zadatak IV.1.9.1

Rešenje e):

$$P_1 = I^2 R_1 = (1 \text{ A})^2 \cdot 80 \Omega = 80 \text{ W} ,$$

$$Q_1 = I^2 X_1 = (1 \text{ A})^2 \cdot 60 \Omega = 60 \text{ VAr} ,$$

$$S_1 = \sqrt{P_1^2 + Q_1^2} = \sqrt{(80 \text{ W})^2 + (60 \text{ W})^2} = 100 \text{ VA} \text{ ili } S_1 = U_1 I_1 = 100 \text{ V} \cdot 1 \text{ A} = 100 \text{ VA} ,$$

$$P_2 = I^2 R_2 = (1 \text{ A})^2 \cdot 60 \Omega = 60 \text{ W} ,$$

$$Q_2 = I^2 X_2 = (1 \text{ A})^2 \cdot (-80 \Omega) = -80 \text{ VAr} .$$

$$S_2 = \sqrt{P_2^2 + Q_2^2} = \sqrt{(60 \text{ W})^2 + (-80 \text{ W})^2} = 100 \text{ VA} \text{ ili } S_2 = U_2 I_2 = 100 \text{ V} \cdot 1 \text{ A} = 100 \text{ VA}$$

$$P_e = I^2 R_e = I^2 (R_1 + R_2) = I^2 R_1 + I^2 R_2 = P_1 + P_2 = 80 \text{ W} + 60 \text{ W} = 140 \text{ W} ,$$

$$Q_e = I^2 X_e = I^2 (X_1 + X_2) = I^2 X_1 + I^2 X_2 = Q_1 + Q_2 = 60 \text{ VAr} + (-80 \text{ VAr}) = -20 \text{ VAr} .$$



Naizmenične struje

Zadatak IV.1.9.1

Rešenje e):

$$S_e = \sqrt{P_e^2 + Q_e^2} = \sqrt{(140 \text{ W})^2 + (-20 \text{ VAR})^2} = 141,4 \text{ VA} \text{ ili } S_e = UI = 141,1 \text{ V} \cdot 1 \text{ A} = 141,4 \text{ VA}$$

$$S_e = \sqrt{P_e^2 + Q_e^2} = \sqrt{(P_1 + Q_2)^2 + (P_1 + Q_2)^2} \neq S_1 + S_2 = \sqrt{P_1^2 + Q_1^2} + \sqrt{P_2^2 + Q_2^2}$$

$$S_e \neq S_1 + S_2.$$



Naizmenične struje

IV.1.9.2 Dva prijemnika vezana su redno i priključena su na napon efektivne vrednosti $U = 50 \text{ V}$ i kružne učestanosti $f = 50 \text{ Hz}$. Ako je $R_1 = 20 \text{ } \Omega$, $L_1 = 100 \text{ mH}$ i $R_2 = 15 \text{ } \Omega$, $C_2 = 100 \text{ } \mu\text{F}$, odrediti:

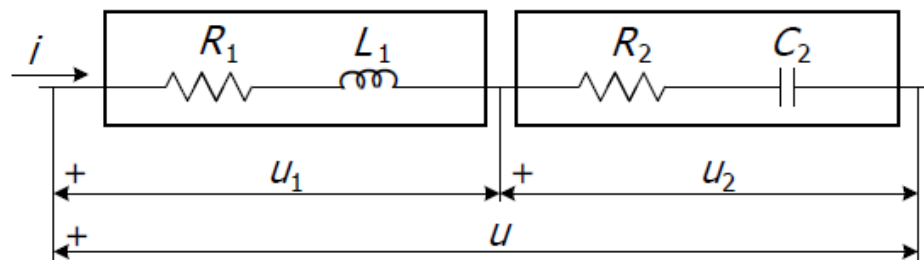
- a) impedanse pojedinih prijemnika, Z_1 i Z_2 ,
- b) faktore snage pojedinih prijemnika,
- c) impedansu redne veze,
- d) faktor snage redne veze,
- e) napon na pojedinim prijemnicima,
- f) aktivnu, reaktivnu i prividnu snagu svakog prijemnika i redne veze.



Naizmenične struje

Zadatak IV.1.9.2

Rešenje:a)



$$\omega = 2\pi f = 2\pi \cdot 50 \text{ Hz} = 100\pi \text{ s}^{-1} = 314 \text{ s}^{-1}$$

$$X_1 = \omega L_1 = 314 \text{ s}^{-1} \cdot 100 \cdot 10^{-3} \text{ H} = 31,4 \Omega,$$

$$Z_1 = \sqrt{R_1^2 + X_1^2} = \sqrt{(20 \Omega)^2 + (31,4 \Omega)^2} = 37,2 \Omega,$$

$$\varphi_1 = \arctg \frac{X_1}{R_1} = \arctg \frac{31,4 \Omega}{20 \Omega} = \arctg 1,57 = 1.$$

$$X_2 = -\frac{1}{\omega C_2} = \frac{1}{314 \text{ s}^{-1} \cdot 100 \cdot 10^{-6} \text{ F}} = -31,80 \Omega.$$

$$Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{(15 \Omega)^2 + (-31,8 \Omega)^2} = 35,2 \Omega,$$

$$\varphi_2 = \arctg \frac{X_2}{R_2} = \arctg \frac{-31,8 \Omega}{15 \Omega} = \arctg (-2,12) = -1,13.$$



Naizmenične struje

Zadatak IV.1.9.2

Rešenje: b)

$$\cos \varphi_1 = \cos 1 = 0,54,$$

$$\cos \varphi_2 = \cos (-1,13) = 0,43$$

c) $R_e = R_1 + R_2 = 20 \, \Omega + 15 \, \Omega = 35 \, \Omega$

$$X_e = X_1 + X_2 = 31,4 \, \Omega + (-31,8 \, \Omega) = -0,4 \, \Omega$$

$$Z_e = \sqrt{R_e^2 + X_e^2} = \sqrt{(35 \, \Omega)^2 + (-0,4 \, \Omega)^2} \approx 35 \, \Omega.$$

$$\varphi_e = \arctg \frac{X_e}{R_e} = \arctg \frac{-0,4 \, \Omega}{35 \, \Omega} = \arctg (-0,01) = -0,01$$



Naizmenične struje

Zadatak IV.1.9.2

Rešenje:d)

$$\cos \varphi_e = \cos (-0,01) \approx 1$$

e)
$$I = \frac{U}{Z_e} = \frac{50 \text{ V}}{35 \text{ A}} = 1,43 \text{ A}$$

$$U_1 = Z_1 \cdot I = 37,2 \, \Omega \cdot 1,43 \text{ A} = 53,2 \text{ V}$$

$$U_2 = Z_2 \cdot I = 35,2 \, \Omega \cdot 1,43 \text{ A} = 50,3 \text{ V}$$



Naizmenične struje

Zadatak IV.1.9.2

Rešenje:f)

$$P_1 = I^2 R_1 = (1,43 \text{ A})^2 \cdot 20 \Omega = 40,9 \text{ W} ,$$

$$Q_1 = I^2 X_1 = (1,43 \text{ A})^2 \cdot 31,4 \Omega = 64,2 \text{ VAr} ,$$

$$S_1 = \sqrt{P_1^2 + Q_1^2} = \sqrt{(40,9 \text{ W})^2 + (64,2 \text{ W})^2} = 76,1 \text{ VA} \text{ ili } S_1 = U_1 I_1 = 53,2 \text{ V} \cdot 1,43 \text{ A} = 76,1 \text{ VA} ,$$

$$P_2 = I^2 R_2 = (1,43 \text{ A})^2 \cdot 15 \Omega = 30,7 \text{ W} ,$$

$$Q_2 = I^2 X_2 = (1,43 \text{ A})^2 \cdot (-31,8 \Omega) = -65 \text{ VAr} .$$

$$S_2 = \sqrt{P_2^2 + Q_2^2} = \sqrt{(30,7 \text{ W})^2 + (-65 \text{ W})^2} = 71,9 \text{ VA} \text{ ili } S_2 = U_2 I_2 = 50,3 \text{ V} \cdot 1,43 \text{ A} = 71,9 \text{ VA}$$

$$P_e = I^2 R_e = (1,43 \text{ A})^2 \cdot 35 \Omega = 71,6 \text{ W} \text{ ili } P_e = P_1 + P_2 = 40,9 \text{ W} + 30,7 \text{ W} = 71,6 \text{ W} ,$$

$$Q_e = I^2 X_e = (1,43 \text{ A})^2 \cdot (-0,4 \Omega) = -0,8 \text{ VAr} \text{ ili } Q_e = Q_1 + Q_2 = 64,2 \text{ VAr} + (-65 \text{ VAr}) = -0,8 \text{ VAr} ,$$

$$S_e = \sqrt{P_e^2 + Q_e^2} = \sqrt{(71,6 \text{ W})^2 + (-0,8 \text{ VAr})^2} = 71,6 \text{ VA} \text{ ili } S_e = UI = 50 \text{ V} \cdot 1,43 \text{ A} = 71,5 \text{ VA} ,$$