



Elektrotehnika

Vežbe 3

Kulonova sila

Elektrostatičko polje

Elektrostatički potencijal

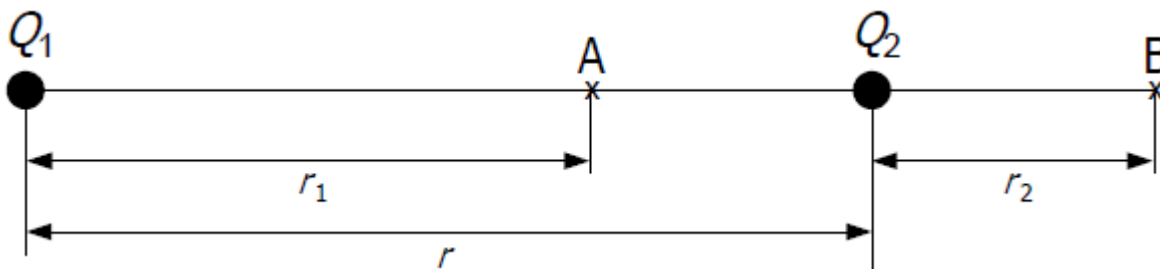
Kondenzatori



Elektrostatika

I.3.6 Dva mala naelektrisana tela naelektrisanja $Q_1 = 4 \text{ pC}$ i $Q_2 = 2 \text{ pC}$, nalaze se u vazduhu na rastojanju $r = 30 \text{ cm}$, kao na slici.

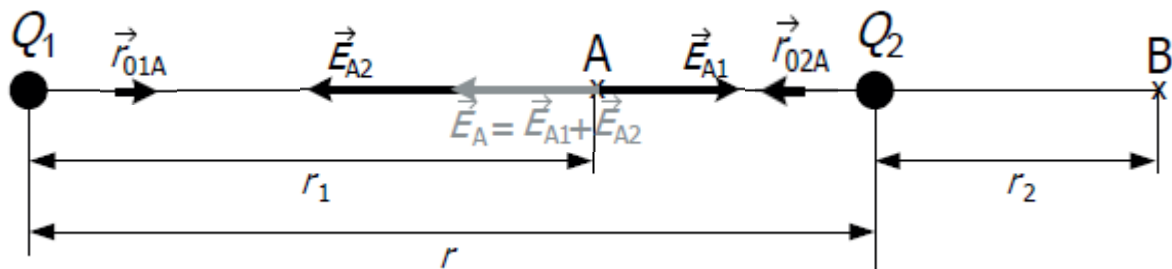
- a) Odrediti vektor jačine elektrostatičkog polja u tački A koja se nalazi na pravoj između ova dva naelektrisanja, a udaljena je od naelektrisanja Q_1 za $r_1 = 20 \text{ cm}$.
- b) Odrediti vektor jačine elektrostatičkog polja u tački B koja se nalazi na pravoj koju određuju ova dva naelektrisanja, sa strane naelektrisanja Q_2 , a udaljena je od njega za $r_2 = 10 \text{ cm}$.
- c) Odrediti potencijale tačaka A i B.
- d) Odrediti napon U_{AB} . Koliki je napon U_{BA} ?
- e) Odrediti silu (njen pravac, smer i intenzitet) koja bi delovala na naelektrisanje $Q_{pA} = 1 \text{ pC}$ kada bi se postavilo u tačku A.
- f) Odrediti silu (njen pravac, smer i intenzitet) koja bi delovala na naelektrisanje $Q_{pB} = -1 \text{ pC}$ kada bi se postavilo u tačku B.





Elektrostatika

a)



$$\vec{E}_{A1} = k \cdot \frac{Q_1}{r_1^2} \cdot \vec{r}_{01A} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{4 \cdot 10^{-12} \text{C}}{(20 \cdot 10^{-2} \text{m})^2} \vec{r}_{01A} = \frac{9 \cdot 10^9 \cdot 4 \cdot 10^{-12}}{4 \cdot 10^{-2}} \frac{\text{N}}{\text{C}} \vec{r}_{01A} = 0,9 \frac{\text{N}}{\text{C}} \vec{r}_{01A}$$

$$\vec{E}_{A2} = k \cdot \frac{Q_2}{(r - r_1)^2} \cdot \vec{r}_{02A} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{2 \cdot 10^{-12} \text{C}}{(10 \cdot 10^{-2} \text{m})^2} \vec{r}_{02A} = \frac{9 \cdot 10^9 \cdot 2 \cdot 10^{-12}}{1 \cdot 10^{-2}} \frac{\text{N}}{\text{C}} \vec{r}_{02A} = 1,8 \frac{\text{N}}{\text{C}} \vec{r}_{02A}$$

$$\vec{E}_A = \vec{E}_{A1} + \vec{E}_{A2} = 0,9 \frac{\text{N}}{\text{C}} \vec{r}_{01A} + 1,8 \frac{\text{N}}{\text{C}} \vec{r}_{02A}$$

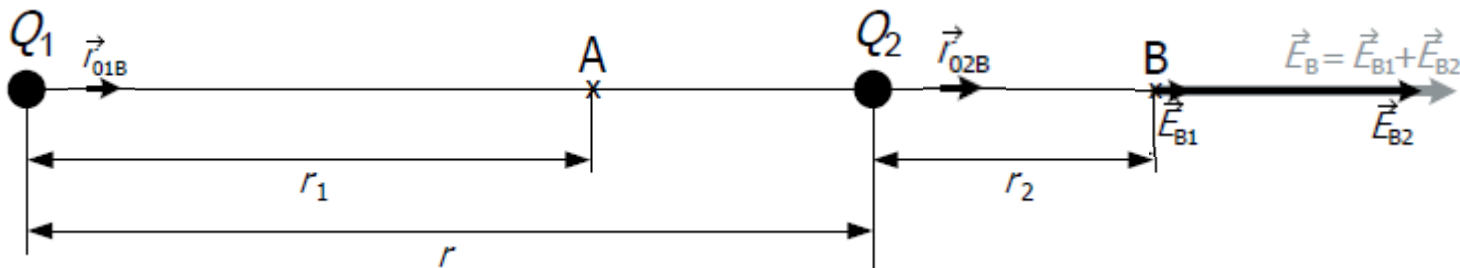
$$\vec{r}_{01A} = -\vec{r}_{02A}$$

$$\vec{E}_A = 0,9 \frac{\text{N}}{\text{C}} (-\vec{r}_{02A}) + 1,8 \frac{\text{N}}{\text{C}} \vec{r}_{02A} = 0,9 \frac{\text{N}}{\text{C}} \vec{r}_{02A} = -0,9 \frac{\text{N}}{\text{C}} \vec{r}_{01A}$$



Elektrostatika

b)



$$\vec{E}_{B1} = k \cdot \frac{Q_1}{(r + r_2)^2} \cdot \vec{r}_{01B} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{4 \cdot 10^{-12} \text{C}}{(40 \cdot 10^{-2} \text{m})^2} \vec{r}_{01B} = \frac{9 \cdot 10^9 \cdot 4 \cdot 10^{-12}}{16 \cdot 10^{-2}} \frac{\text{N}}{\text{C}} \vec{r}_{01B} = 0,225 \frac{\text{N}}{\text{C}} \vec{r}_{01B}$$

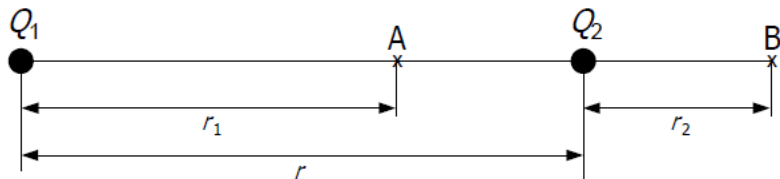
$$\vec{E}_{B2} = k \cdot \frac{Q_2}{r_2^2} \cdot \vec{r}_{02B} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{2 \cdot 10^{-12} \text{C}}{(10 \cdot 10^{-2} \text{m})^2} \vec{r}_{02B} = \frac{9 \cdot 10^9 \cdot 2 \cdot 10^{-12}}{1 \cdot 10^{-2}} \frac{\text{N}}{\text{C}} \vec{r}_{02B} = 1,8 \frac{\text{N}}{\text{C}} \vec{r}_{02B}$$

$$\vec{E}_B = \vec{E}_{B1} + \vec{E}_{B2} = 0,225 \frac{\text{N}}{\text{C}} \vec{r}_{01B} + 1,8 \frac{\text{N}}{\text{C}} \vec{r}_{02B} = 0,225 \frac{\text{N}}{\text{C}} \vec{r}_{01B} + 1,8 \frac{\text{N}}{\text{C}} \vec{r}_{01B} = 2,025 \frac{\text{N}}{\text{C}} \vec{r}_{01B} = 2,025 \frac{\text{N}}{\text{C}} \vec{r}_{02B}$$



Elektrostatika

c) Odrediti potencijale tačaka A i B.



$$V_{A1} = k \frac{Q_1}{r_1} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{4 \cdot 10^{-12} \text{C}}{20 \cdot 10^{-2} \text{m}} = 0,18 \text{ V}$$

$$V_{A2} = k \frac{Q_2}{r - r_1} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{2 \cdot 10^{-12} \text{C}}{10 \cdot 10^{-2} \text{m}} = 0,18 \text{ V}$$

$$V_A = V_{A1} + V_{A2} = 0,18 \text{ V} + 0,18 \text{ V} = 0,36 \text{ V}$$

$$V_{B1} = k \frac{Q_1}{r + r_2} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{4 \cdot 10^{-12} \text{C}}{40 \cdot 10^{-2} \text{m}} = 0,09 \text{ V}$$

$$V_{B2} = k \frac{Q_2}{r_2} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{2 \cdot 10^{-12} \text{C}}{10 \cdot 10^{-2} \text{m}} = 0,18 \text{ V}$$

$$V_B = V_{B1} + V_{B2} = 0,09 \text{ V} + 0,18 \text{ V} = 0,27 \text{ V}$$



Elektrostatika

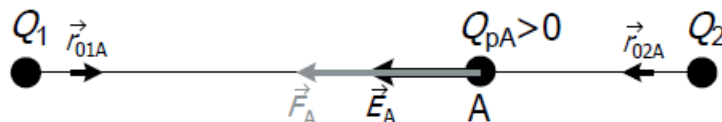
d) Odrediti napon U_{AB} . Koliki je napon U_{BA} ?

$$U_{AB} = V_A - V_B = 0,36 \text{ V} - 0,27 \text{ V} = 0,09 \text{ V}$$

$$U_{AB} = V_A - V_B = -U_{BA} = -0,09 \text{ V}$$

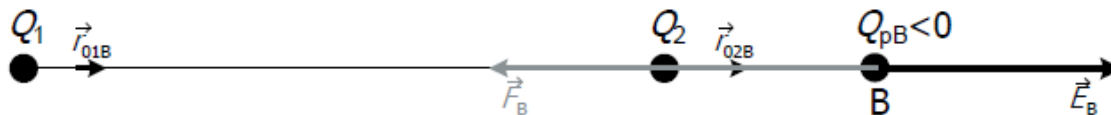
e) Odrediti silu (njen pravac, smer i intenzitet) koja bi delovala na naelektrisanje $Q_{pA} = 1 \text{ pC}$ kada bi se postavilo u tačku A.

$$\vec{F}_A = \vec{E}_A \cdot Q_{pA} = 0,9 \frac{\text{N}}{\text{C}} \vec{r}_{02A} \cdot 1 \cdot 10^{-12} \text{ C} = 0,9 \cdot 10^{-12} \text{ N} \vec{r}_{02A} = 0,9 \text{ pN} \vec{r}_{02A} = -0,9 \text{ pN} \vec{r}_{01A}$$



f) Odrediti silu (njen pravac, smer i intenzitet) koja bi delovala na naelektrisanje $Q_{pB} = -1 \text{ pC}$ kada bi se postavilo u tačku B.

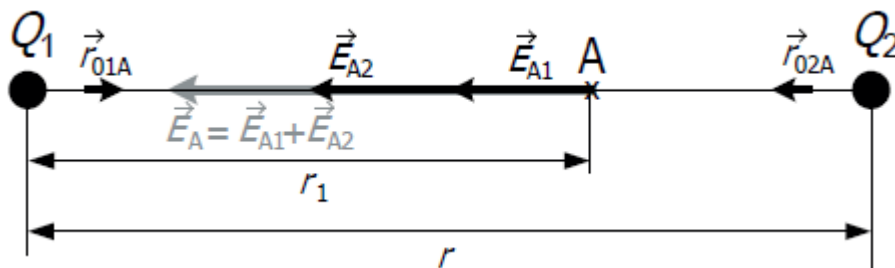
$$\vec{F}_B = \vec{E}_B \cdot Q_{pB} = 2,025 \frac{\text{N}}{\text{C}} \vec{r}_{01B} \cdot (-1 \cdot 10^{-12} \text{ C}) = -2,025 \cdot 10^{-12} \text{ N} \vec{r}_{01B} = -2,025 \text{ pN} \vec{r}_{01B} = -2,025 \text{ pN} \vec{r}_{02B}$$





Elektrostatika

I.3.7 Uraditi prethodni zadatak za vrednosti naelektrisanja $Q_1 = -4 \text{ pC}$ i $Q_2 = 2 \text{ pC}$.



$$\vec{E}_{A1} = k \cdot \frac{Q_1}{r_1^2} \cdot \vec{r}_{01A} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{-4 \cdot 10^{-12} \text{C}}{(20 \cdot 10^{-2} \text{m})^2} \vec{r}_{01A} = -0,9 \frac{\text{N}}{\text{C}} \vec{r}_{01A}$$

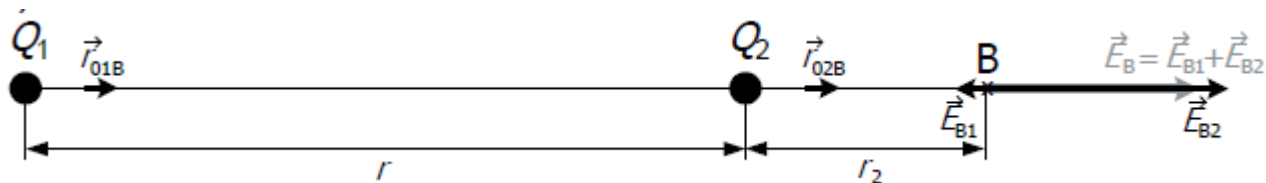
$$\vec{E}_{A2} = k \cdot \frac{Q_2}{(r - r_1)^2} \cdot \vec{r}_{02A} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{2 \cdot 10^{-12} \text{C}}{(10 \cdot 10^{-2} \text{m})^2} \vec{r}_{02A} = 1,8 \frac{\text{N}}{\text{C}} \vec{r}_{02A}$$

$$\vec{E}_A = \vec{E}_{A1} + \vec{E}_{A2} = -0,9 \frac{\text{N}}{\text{C}} \vec{r}_{01A} + 1,8 \frac{\text{N}}{\text{C}} \vec{r}_{02A} = -0,9 \frac{\text{N}}{\text{C}} (-\vec{r}_{02A}) + 1,8 \frac{\text{N}}{\text{C}} \vec{r}_{02A} = 2,7 \frac{\text{N}}{\text{C}} \vec{r}_{02A} = -2,7 \frac{\text{N}}{\text{C}} \vec{r}_{01A}$$



Elektrostatika

b)



$$\vec{E}_{B1} = k \cdot \frac{Q_1}{(r + r_2)^2} \cdot \vec{r}_{01B} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{-4 \cdot 10^{-12} \text{C}}{(40 \cdot 10^{-2} \text{m})^2} \vec{r}_{01B} = -0,225 \frac{\text{N}}{\text{C}} \vec{r}_{01B}$$

$$\vec{E}_{B2} = k \cdot \frac{Q_2}{r_2^2} \cdot \vec{r}_{02B} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{2 \cdot 10^{-12} \text{C}}{(10 \cdot 10^{-2} \text{m})^2} \vec{r}_{02B} = 1,8 \frac{\text{N}}{\text{C}} \vec{r}_{02B}$$

$$\vec{E}_B = \vec{E}_{B1} + \vec{E}_{B2} = -0,225 \frac{\text{N}}{\text{C}} \vec{r}_{01B} + 1,8 \frac{\text{N}}{\text{C}} \vec{r}_{02B} = -0,225 \frac{\text{N}}{\text{C}} \vec{r}_{01B} + 1,8 \frac{\text{N}}{\text{C}} \vec{r}_{01B} = 1,575 \frac{\text{N}}{\text{C}} \vec{r}_{01B} = 1,575 \frac{\text{N}}{\text{C}} \vec{r}_{02B}$$



Elektrostatika

$$c) \quad V_{A1} = k \frac{Q_1}{r_1} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{-4 \cdot 10^{-12} \text{C}}{20 \cdot 10^{-2} \text{m}} = -0,18 \text{ V}$$

$$V_{A2} = k \frac{Q_2}{r - r_1} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{2 \cdot 10^{-12} \text{C}}{10 \cdot 10^{-2} \text{m}} = 0,18 \text{ V}$$

$$V_A = V_{A1} + V_{A2} = -0,18 \text{ V} + 0,18 \text{ V} = 0 \text{ V}$$

$$V_{B1} = k \frac{Q_1}{r + r_2} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{-4 \cdot 10^{-12} \text{C}}{40 \cdot 10^{-2} \text{m}} = -0,09 \text{ V}$$

$$V_{B2} = k \frac{Q_2}{r_2} = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{2 \cdot 10^{-12} \text{C}}{10 \cdot 10^{-2} \text{m}} = 0,18 \text{ V}$$

$$V_B = V_{B1} + V_{B2} = -0,09 \text{ V} + 0,18 \text{ V} = 0,09 \text{ V}$$

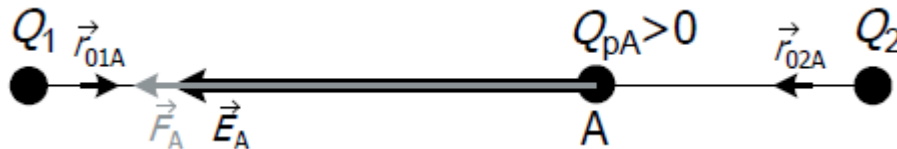
$$d) \quad U_{AB} = V_A - V_B = 0 \text{ V} - 0,09 \text{ V} = -0,09 \text{ V}$$

$$U_{BA} = V_B - V_A = -U_{AB} = 0,09 \text{ V}$$

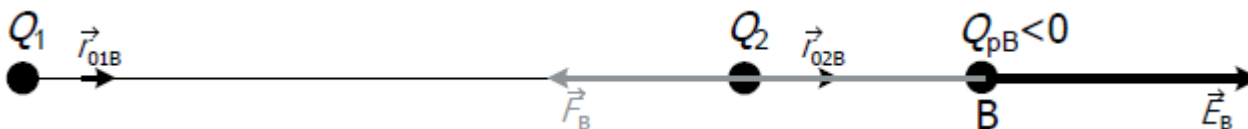


Elektrostatika

e) $\vec{F}_A = \vec{E}_A \cdot Q_{pA} = 2,7 \frac{\text{N}}{\text{C}} \vec{r}_{02A} \cdot 1 \cdot 10^{-12} \text{C} = 2,7 \cdot 10^{-12} \text{N} \vec{r}_{02A} = 2,7 \text{ pN} \vec{r}_{02A} = -2,7 \text{ pN} \vec{r}_{01A}$



f) $\vec{F}_B = \vec{E}_B \cdot Q_{pB} = 1,575 \frac{\text{N}}{\text{C}} \vec{r}_{01B} \cdot (-1 \cdot 10^{-12} \text{C}) = -1,575 \cdot 10^{-12} \text{N} \vec{r}_{01B} = -1,575 \text{ pN} \vec{r}_{01B} = -1,575 \text{ pN} \vec{r}_{02B}$





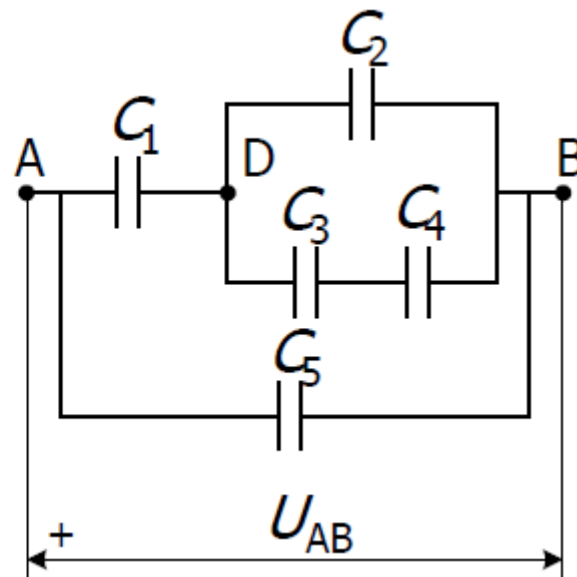
Elektrostatika

I.7.12. Veza kondenzatora prikazana na slici priključena je na napon $U_{AB} = 50 \text{ V}$. Kapacitivnosti kondenzatora su:

$$C_1 = 20 \text{ pF}, C_2 = 12 \text{ pF}, C_3 = 40 \text{ pF},$$

$$C_4 = 18 \text{ pF}, C_5 = 15 \text{ pF}.$$

- a) Izračunati ekvivalentnu kapacitivnost veze C_e .
- b) Izračunati napon U_1 na kondenzatoru C_1 .
- c) Izračunati energiju W_{e4} kondenzatora C_4 .





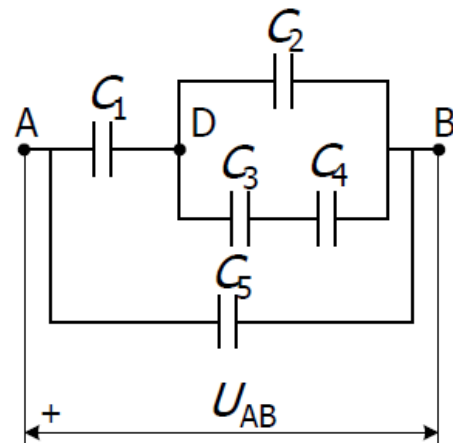
Elektrostatika

a)
$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4} \quad C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{40 \text{ pF} \cdot 18 \text{ pF}}{40 \text{ pF} + 18 \text{ pF}} = 12,41 \text{ pF}$$

$$C_{234} = C_2 + C_{34} = 12 \text{ pF} + 12,41 \text{ pF} = 24,41 \text{ pF}$$

$$\frac{1}{C_{1234}} = \frac{1}{C_1} + \frac{1}{C_{234}} \quad C_{1234} = \frac{C_1 C_{234}}{C_1 + C_{234}} = \frac{20 \text{ pF} \cdot 24,41 \text{ pF}}{20 \text{ pF} + 24,41 \text{ pF}} = 11 \text{ pF}$$

$$C_e = C_{12345} = C_{1234} + C_5 = 11 \text{ pF} + 15 \text{ pF} = 26 \text{ pF}$$





Elektrostatika

b) Izračunati napon U_1 na kondenzatoru C_1 .

$$U_1 = U_{AD} = \frac{Q_1}{C_1} = \frac{Q_{1234}}{C_1} = \frac{550 \cdot 10^{-12} \text{ C}}{20 \cdot 10^{-12} \text{ F}} = 27,5 \text{ V}$$

$$Q_{1234} = Q_1 = Q_{234} = U_{AB} \cdot C_{1234} = 50 \text{ V} \cdot 11 \cdot 10^{-12} \text{ F} = 550 \text{ pC}$$

c) Izračunati energiju W_{e4} kondenzatora C_4 .

$$W_{e4} = \frac{Q_4^2}{2C_4} = \frac{Q_{34}^2}{2C_4} = \frac{(279,22 \cdot 10^{-12} \text{ C})^2}{2 \cdot 18 \cdot 10^{-12} \text{ F}} = 2,16 \text{ nJ}$$

$$Q_{34} = Q_3 = Q_4 = U_{DB} \cdot C_{34} = 22,5 \text{ V} \cdot 12,41 \text{ pF} = 279,22 \text{ pC}$$

$$U_{DB} = U_{AB} - U_{AD} = 22,5 \text{ V}$$

