# Elektrotehnika (ASUV)

## Vežbe 4

## Kondenzatori

Visoka škola elektrotehnike i računarstva
strukovnih studija, Beograd 2019

# Elektrostatika-Kondenzatori

**I.7.12.** Veza kondenzatora prikazana na slici priključena je na napon $U_{AB}$ = 50 V. Kapacitivnosti kondenzatora su:

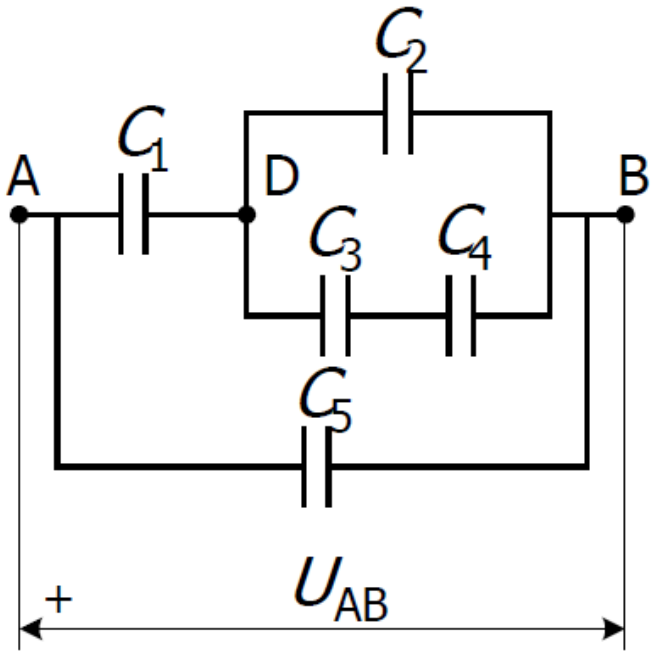
$C_1$ = 20 pF, $C_2$ = 12 pF, $C_3$ = 40 pF,

$C_4$ = 18 pF, $C_5$ = 15 pF.

a) Izračunati ekvivalentnu kapacitivnost veze $C_e$.

b) Izračunati napon $U_1$ na kondenzatoru $C_1$.

c) Izračunati energiju $W_{e4}$ kondenzatora $C_4$.



# Elektrostatika-Kondenzatori

a)

$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4} \quad \Rightarrow \quad C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{40\ \text{pF} \cdot 18\ \text{pF}}{40\ \text{pF} + 18\ \text{pF}} = 12{,}41\ \text{pF}$$

$$C_{234} = C_2 + C_{34} = 12\ \text{pF} + 12{,}41\ \text{pF} = 24{,}41\ \text{pF}$$

$$\frac{1}{C_{1234}} = \frac{1}{C_1} + \frac{1}{C_{234}} \quad \Rightarrow \quad C_{1234} = \frac{C_1 C_{234}}{C_1 + C_{234}} = \frac{20\ \text{pF} \cdot 24{,}42\ \text{pF}}{20\ \text{pF} + 24{,}41\ \text{pF}} = 11\ \text{pF}$$

$$C_e = C_{12345} = C_{1234} + C_5 = 11\ \text{pF} + 15\ \text{pF} = 26\ \text{pF}$$

# Elektrostatika-Kondenzatori

b)

$$Q_{1234} = Q_1 = Q_{234} = U_{AB} \cdot C_{1234} = 50\ \text{V} \cdot 11 \cdot 10^{-12}\text{F} = 550\ \text{pC}$$

$$U_1 = U_{AD} = \frac{Q_1}{C_1} = \frac{Q_{1234}}{C_1} = \frac{550 \cdot 10^{-12}\text{C}}{20 \cdot 10^{-12}\text{F}} = 27{,}5\ \text{V}$$

# Elektrostatika-Kondenzatori

c)

$$U_{DB} = U_{AB} - U_{AD} = 22{,}5\ \text{V}$$

$$Q_{34} = Q_3 = Q_4 = U_{DB} \cdot C_{34} = 22{,}5\ \text{V} \cdot 12{,}41\ \text{pF} = 279{,}22\ \text{pC}$$

$$W_{e4} = \frac{Q_4^{\,2}}{2C_4} = \frac{Q_{34}^{\,2}}{2C_4} = \frac{\left(279{,}22 \cdot 10^{-12}\,\text{C}\right)^2}{2 \cdot 18 \cdot 10^{-12}\,\text{F}} = 2{,}16\ \text{nJ}$$

# Elektrostatika-Kondenzatori

**I.7.13.** Za kolo na slici poznato je:

$C_1 = 60\,\mu\text{F}$, $C_2 = 15\,\mu\text{F}$,

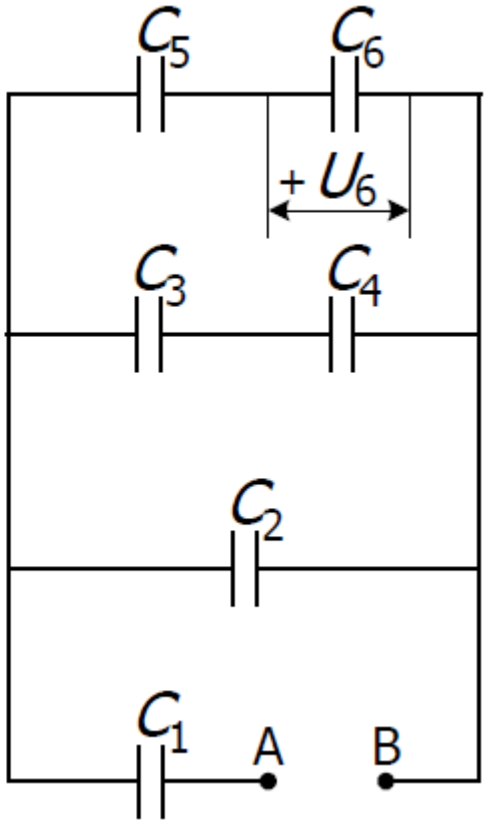
$C_4 = 10\,\mu\text{F}$, $C_5 = 30\,\mu\text{F}$, $C_6 = 15\,\mu\text{F}$.

Napon na kondenzatoru $C_6$ je $U_6 = 5$ V,
dok je elektrostatička energija kondenzatora $C_4$
$W_{e4} = 45 \cdot 10^{-4}\,\text{J}$.

a) Odrediti kapacitivnost kondenzatora $C_3$.

b) Izračunati ekvivalentnu kapacitivnost veze $C_e$.

c) Izračunati napon $U_{AB}$.

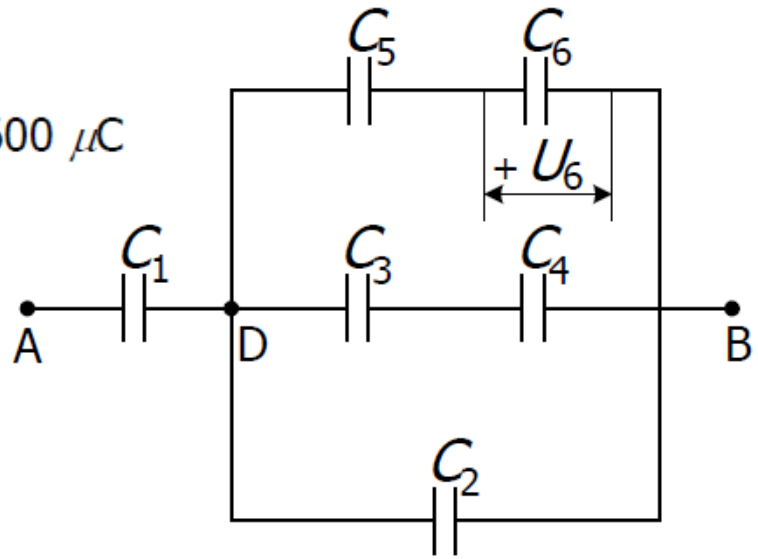


# Elektrostatika-Kondenzatori

$$Q_6 = Q_5 = Q_{56} = U_6 \cdot C_6 = 40\ \text{V} \cdot 15 \cdot 10^{-6}\text{F} = 600\ \mu\text{C}$$

$$U_5 = \frac{Q_5}{C_5} = \frac{Q_{56}}{C_5} = \frac{600 \cdot 10^{-6}\text{C}}{30 \cdot 10^{-6}\text{F}} = 20\ \text{V}$$

$$U_{DB} = U_5 + U_6 = 60\ \text{V}$$




$$W_{e4} = \frac{1}{2} C_4 \cdot U_4^{\,2} \quad \Rightarrow \quad U_4 = \sqrt{\frac{2W_{e4}}{C_4}} = \sqrt{\frac{2 \cdot 45 \cdot 10^{-4}\text{J}}{10 \cdot 10^{-6}\text{F}}} = 30\ \text{V}$$

$$Q_4 = C_4 \cdot U_4 = 10 \cdot 10^{-6}\text{F} \cdot 30\ \text{V} = 300\ \mu\text{C}.$$

# Elektrostatika-Kondenzatori

$$Q_3 = Q_4 = Q_{34} = 300\ \mu C$$

$$U_3 = U_{AD} - U_4 = 30\ \text{V}$$

$$C_3 = \frac{Q_3}{U_3} = \frac{300 \cdot 10^{-6}\,\text{C}}{30\ \text{V}} = 10\ \mu C$$

# Elektrostatika-Kondenzatori

b)

$$\frac{1}{C_{56}} = \frac{1}{C_5} + \frac{1}{C_6} \quad \Rightarrow \quad C_{56} = \frac{C_5 C_6}{C_5 + C_6} = \frac{30\ \mu\text{F} \cdot 15\ \mu\text{F}}{30\ \mu\text{F} + 15\ \mu\text{F}} = 10\ \mu\text{F}$$

$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4} \quad \Rightarrow \quad C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{10\ \mu\text{F} \cdot 15\ \mu\text{F}}{10\ \mu\text{F} + 15\ \mu\text{F}} = 5\ \mu\text{F}$$

$$C_{23456} = C_2 + C_{34} + C_{56} = 15\ \mu\text{F} + 5\ \mu\text{F} + 10\ \mu\text{F} = 30\ \mu\text{F}$$

$$\frac{1}{C_e} = \frac{1}{C_{123456}} = \frac{1}{C_1} + \frac{1}{C_{23456}} \quad \Rightarrow \quad C_e = C_{123456} = \frac{C_1 C_{23456}}{C_1 + C_{23456}} = \frac{60\ \mu\text{F} \cdot 30\ \mu\text{F}}{60\ \mu\text{F} + 30\ \mu\text{F}} = 20\ \mu\text{F}$$

# Elektrostatika-Kondenzatori

c) $Q_{23456} = C_{23456} \cdot U_{DB} = 30 \cdot 10^{-6}\,\mathrm{F} \cdot 60\ \mathrm{V} = 1800\ \mu\mathrm{C}$

$Q_1 = Q_{23456} = 1800\ \mu\mathrm{C}$

$$U_{AD} = \frac{Q_1}{C_1} = \frac{Q_{23456}}{C_1} = \frac{1800 \cdot 10^{-6}\,\mathrm{C}}{60 \cdot 10^{-6}\,\mathrm{F}} = 30\ \mathrm{V}$$

$U_{AB} = U_{AD} + U_{DB} = 30\ \mathrm{V} + 60\ \mathrm{V} = 90\ \mathrm{V}$

# Elektrostatika-Kondenzatori

**I.7.14.** Kapacitivnosti kondenzatora vezanih kao na slici su:

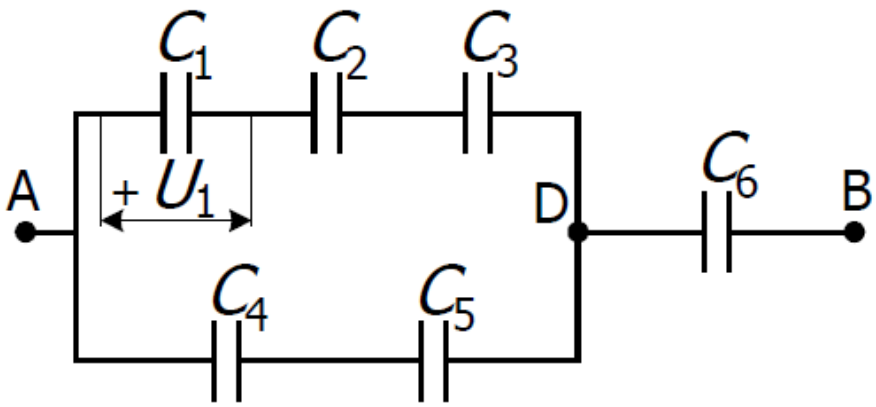
$C_1 = 20\,\mu F$, $C_2 = 20\,\mu F$, $C_3 = 10\,\mu F$,

$C_4 = 15\,\mu F$, $C_5 = 30\,\mu F$, $C_6 = 10\,\mu F$.

Napon na kondenzatoru $C_1$ je $U_1 = 5$ V.

a) Izračunati ekvivalentnu kapacitivnost veze $C_e$.

b) Izračunati napon $U_{AB}$, kao i napone i količine naelektrisanja svih kondenzatora.



# Elektrostatika-Kondenzatori

a) $$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = 0{,}05\,\frac{1}{\mu\text{F}} + 0{,}05\,\frac{1}{\mu\text{F}} + 0{,}1\,\frac{1}{\mu\text{F}} = 0{,}2\,\frac{1}{\mu\text{F}}$$

$$C_{123} = \frac{1}{0{,}2\,\frac{1}{\mu\text{F}}} = 5\ \mu\text{F}$$

$$\frac{1}{C_{45}} = \frac{1}{C_4} + \frac{1}{C_5} \quad \Rightarrow \quad C_{45} = \frac{C_4 C_5}{C_4 + C_5} = \frac{15\ \mu\text{F} \cdot 30\ \mu\text{F}}{15\ \mu\text{F} + 30\ \mu\text{F}} = 10\ \mu\text{F}$$

$$C_{12345} = C_{123} + C_{45} = 5\ \mu\text{F} + 10\ \mu\text{F} = 15\ \mu\text{F}$$

$$\frac{1}{C_e} = \frac{1}{C_{123456}} = \frac{1}{C_{12345}} + \frac{1}{C_6} \quad \Rightarrow \quad C_e = C_{123456} = \frac{C_{12345} C_6}{C_{12345} + C_6} = \frac{15\ \mu\text{F} \cdot 10\ \mu\text{F}}{15\ \mu\text{F} + 10\ \mu\text{F}} = 6\ \mu\text{F}$$

# Elektrostatika-Kondenzatori

b) $$Q_1 = Q_2 = Q_3 = Q_{123} = U_1 \cdot C_1 = 5\ \text{V} \cdot 20 \cdot 10^{-6}\text{F} = 100\ \mu\text{C}$$

$$U_2 = \frac{Q_2}{C_2} = \frac{Q_{123}}{C_2} = \frac{100 \cdot 10^{-6}\text{C}}{20 \cdot 10^{-6}\text{F}} = 5\ \text{V}$$

$$U_3 = \frac{Q_3}{C_3} = \frac{Q_{123}}{C_3} = \frac{100 \cdot 10^{-6}\text{C}}{10 \cdot 10^{-6}\text{F}} = 10\ \text{V}$$

$$U_{AD} = U_1 + U_2 + U_3 = 20\ \text{V}$$

# Elektrostatika-Kondenzatori

$$Q_{45} = Q_4 = Q_5 = U_{AD} \cdot C_{45} = 20\ \text{V} \cdot 10 \cdot 10^{-6}\text{F} = 200\ \mu\text{C}$$

$$U_4 = \frac{Q_4}{C_4} = \frac{Q_{45}}{C_4} = \frac{200 \cdot 10^{-6}\text{C}}{15 \cdot 10^{-6}\text{F}} = 13{,}33\ \text{V}$$

$$U_5 = \frac{Q_5}{C_5} = \frac{Q_{45}}{C_5} = \frac{200 \cdot 10^{-6}\text{C}}{30 \cdot 10^{-6}\text{F}} = 6{,}67\ \text{V}.$$

$$Q_{12345} = Q_{123} + Q_{45} = 100\ \mu\text{C} + 200\ \mu\text{C} = 300\ \mu\text{C} = Q_6.$$

$$U_6 = U_{DB} = \frac{Q_6}{C_6} = \frac{300 \cdot 10^{-6}\text{C}}{10 \cdot 10^{-6}\text{F}} = 30\ \text{V}$$

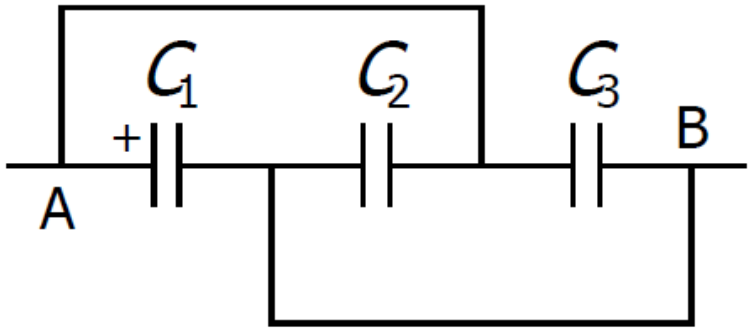
$$U_{AB} = U_{AD} + U_{DB} = 20\ \text{V} + 30\ \text{V} = 50\ \text{V}$$

# Elektrostatika-Kondenzatori

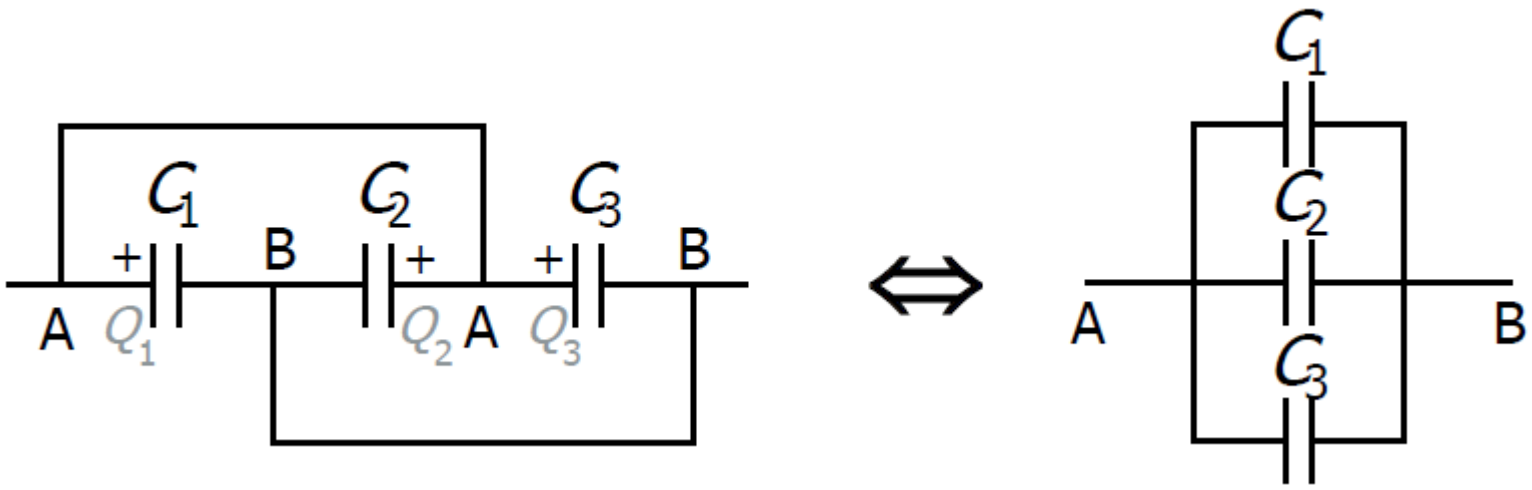
**I.7.15.** Kondenzatori kapacitivnosti $C_1 = 10$ pF, $C_2 = 20$ pF i $C_3 = 50$ pF vezani su u grupu kao na slici. Odrediti:

a) ekvivalentnu kapacitivnost veze $C_e$ između tačaka A i B;

b) napon $U_{AB}$ ako je naelektrisanje na pozitivnoj elektrodi kondenzatora $C_1$ $Q_1 = 100$ pC. Takođe odrediti količine naelektrisanja na kondenzatorima $C_2$ i $C_3$.



# Elektrostatika-Kondenzatori




$$C_e = C_1 + C_2 + C_3 = 10\ \text{pF} + 20\ \text{pF} + 50\ \text{pF} = 80\ \text{pF}$$

# Elektrostatika-Kondenzatori

b)

$$U_1 = U_{AB} = \frac{Q_1}{C_1} = \frac{100 \cdot 10^{-12}\text{C}}{10 \cdot 10^{-12}\text{F}} = 10\ \text{V}$$

$$Q_2 = U_{AB} \cdot C_2 = 10\ \text{V} \cdot 20 \cdot 10^{-12}\text{F} = 200\ \text{pC}$$

$$Q_3 = U_{AB} \cdot C_3 = 10\ \text{V} \cdot 50 \cdot 10^{-12}\text{F} = 500\ \text{pC}$$