MERNI INFORMACIONI SISTEMI

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Odabiranje

- ***Odabiranje*** je postupak kojim se dobijaju odbirci signala (vrednosti amplitude) u određenim trenucima vremena
- ***Uniformno odabiranje*** je odabiranje svakih T jedinica vremena

$$x_k = x(kT) = x(t)\Big|_{t=0,\pm T,\pm 2T,\pm 3T,\ldots}$$

Učestanost odabiranja

$$F_0 = \frac{1}{T}$$

Perioda ili vremenski razmak između dva odabiranja

Definicija slučajnog signala

- **Deterministički signal** je signal koji je na jedinstven način određen dobro definisanim procesom (može se opisati matematičkim relacijama, tabelama, pravilima)
- **Slučajni signal** je signal čije se vrednosti ne mogu unapred predvideti
- Slučajni signal $\{\xi[n]\}$ se može posmatrati kao jedna moguća realizacija slučajnog procesa opisuje se korišćenjem **statističkih principa**
- **Primena**: govor, muzika, slika, šum, vremenski promenljivi telekomunikacioni kanali, bilo koja informacija koja je funkcija vremena

Osobine slučajnog signala

- **Kumulativna funkcija raspodele** signala $\{\xi[n]\}$ pokazuje sa kojom je verovatnoćom vrednost signala $\xi[n]$ u funkciji indeksa n manja ili jednaka nekoj vrednosti x
- $P(x,n)$= **Probability** $(\xi[n] \leq x)$
- **Funkcija gustine verovatnoće** definiše se kao izvod kumulativne funkcije raspodele

$$p(x,n) = \frac{\mathrm{d}P(x,n)}{\mathrm{d}x}$$

$$P(x,n) = \int_{-\infty}^{x} p(u,n)\,\mathrm{d}u$$

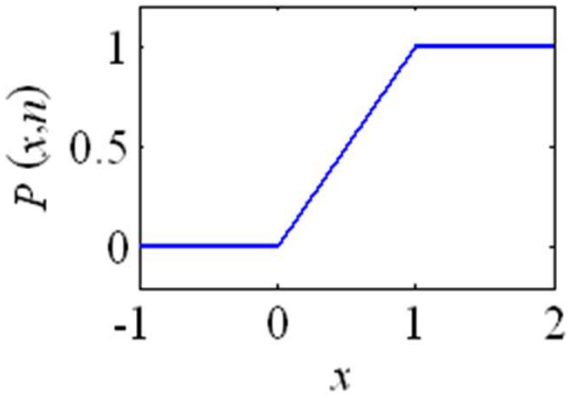
x i $\xi[n]$ mogu da imaju bilo koju vrednost iz opsega

$$-\infty < x < +\infty$$

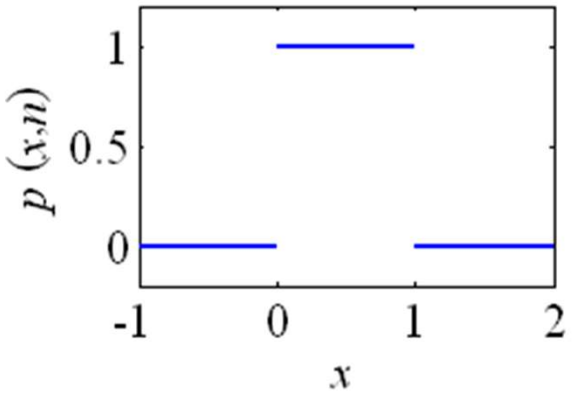
$$-\infty < \xi[n] < +\infty$$

Primer slučajnog signala

$$P(x,n)=\begin{cases}0, & x<0\\ x, & 0\le x\le 1\\ 1, & x>1\end{cases}$$



$$p(x,n)=\begin{cases}0, & x<0\\ 1, & 0\le x\le 1\\ 0, & x>1\end{cases}$$



- Kumulativna funkcija raspodele je linearno rastuća
- Funkcija gustine verovatnoće je konstantna u opsegu $0 \le x \le 1$

Sinusoidalni signal

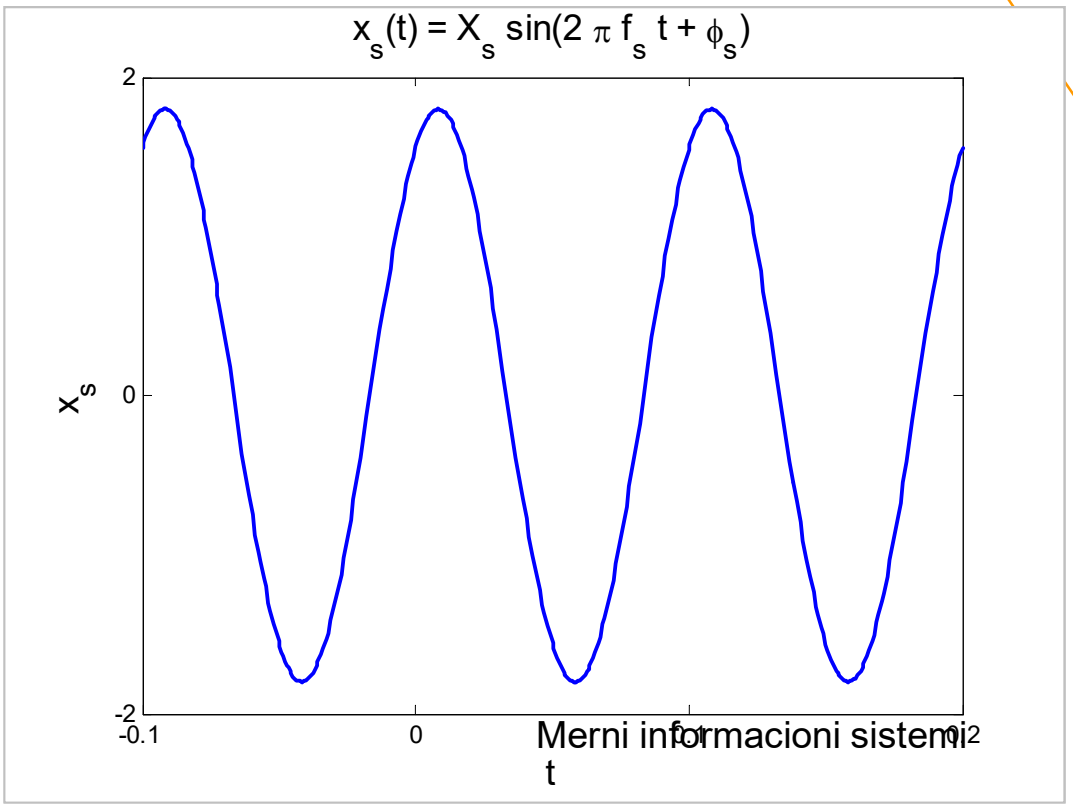
$$x_s(t) = X_s \sin(2\pi f_s t + \phi_s)$$

Amplituda

Faza u radijanima

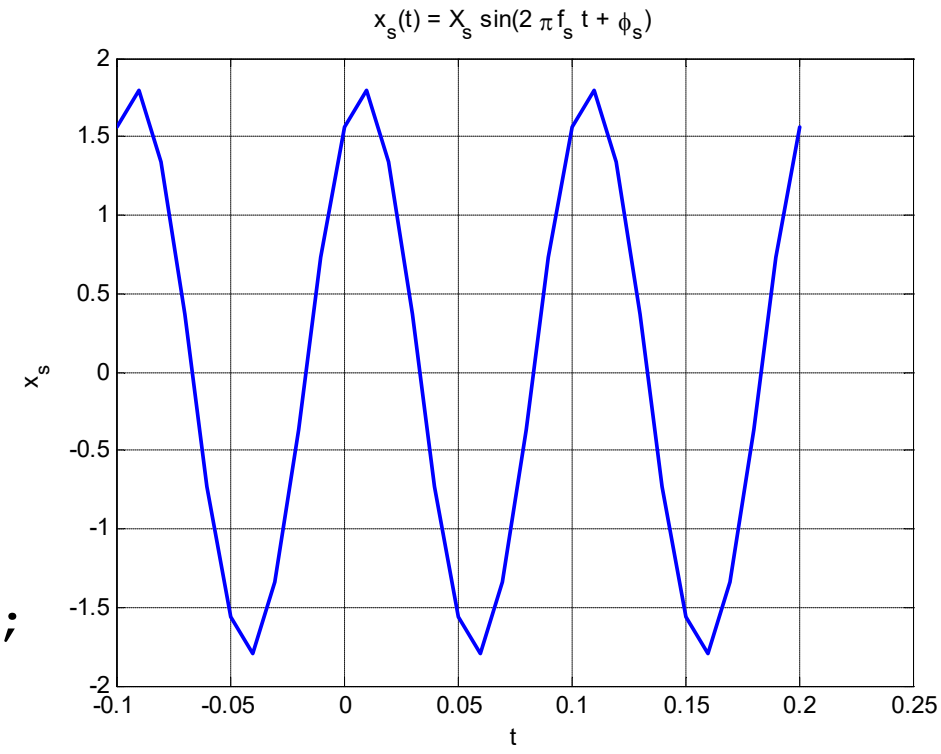
Vreme u sekundama (s)

Frekvencija u Hertz (Hz)



MATLAB kod za sin signal

```
Xs  = 1.8;
fs = 10;
fi = pi/3;
t1 = -0.1;
tstep = 0.01;
t2 = 0.2;
t = t1:tstep:t2;
x = Xs*sin(2*pi*fs*t+fi);
plot(t, x)
xlabel('t')
ylabel('x_s')
title('x_s(t) = X_s sin(2 \pi f_s t + \phi_s)')
grid on
```



Diskretni signal – sekvenca

- Diskretni signal se predstavlja kao niz brojeva (***sequence***) koji je uređen po vremenu nastanka
- U praksi se radi sa sekvencama konačnog trajanja (**finite-length** sequences)
- Sekvence se najčešće prave odabiranjem vrednosti kontinualnog signala
- Predstavlja se sekvencom-nizom brojeva

$$\{x[n]\}$$

Sinusoidalna sekvenca

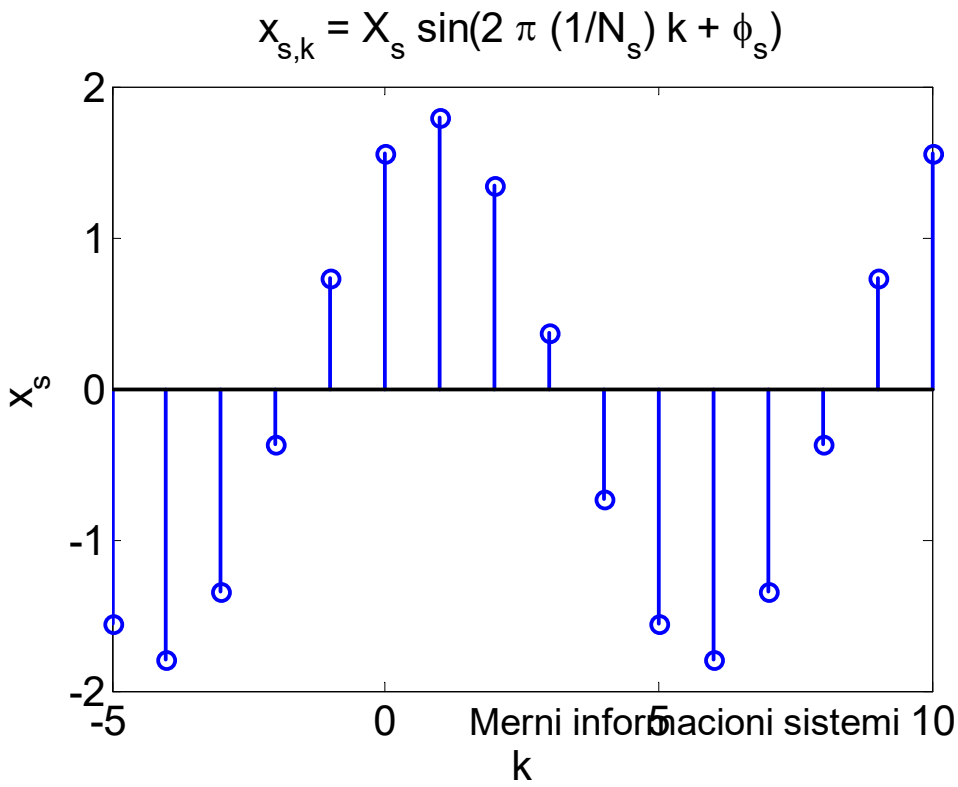
$$x_{s,k} = X_s \sin(2\pi \frac{1}{N_s} k + \phi_s)$$

Amplituda

Faza

indeks

Perioda

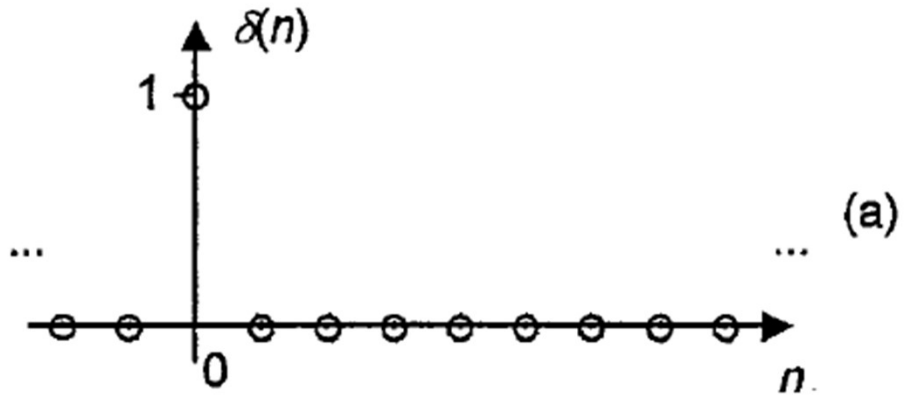


Kauzalna sekvenca

- Sekvenca koja ima vrednosti 0 za sve negativne indekse je **kauzalna**
- Sekvenca koja ima vrednosti različite od 0 za sve indekse iz određenog opsega je **sekvenca konačne dužine**

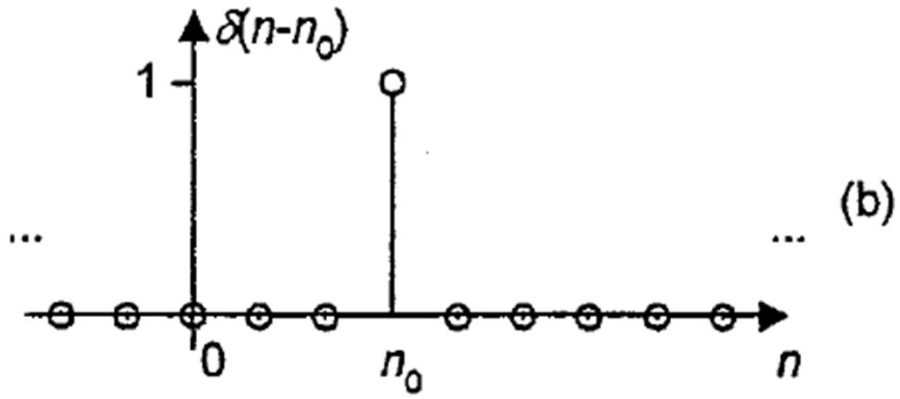
Jedinični impuls
Dirakova delta funkcija

$$\delta[n] = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases}$$



Zakašnjen jedinični impuls

$$\delta[n-n_0]=\begin{cases}1, n=n_0\\ 0, n\neq n_0\end{cases}$$



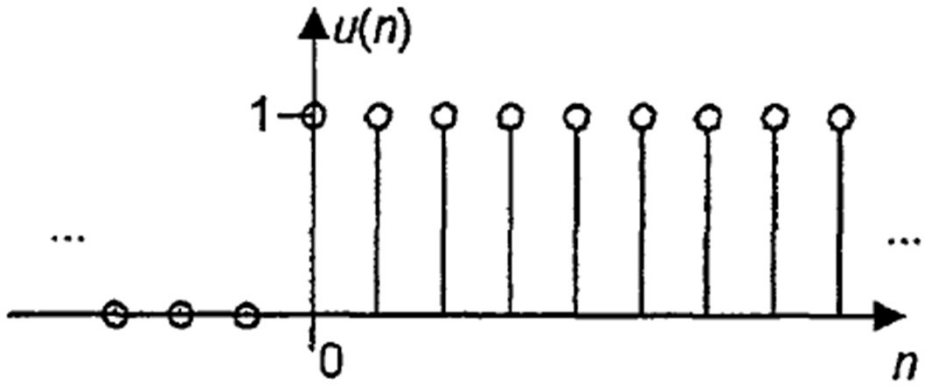
Osobina selektivnosti

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

Izdvajanje člana niza za *n=k*

Jedinični odskočni niz

$$u[n] = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$$



Jedinični odskočni niz
Hevisajdova (Heaviside) funkcija

$$u[n] = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$$

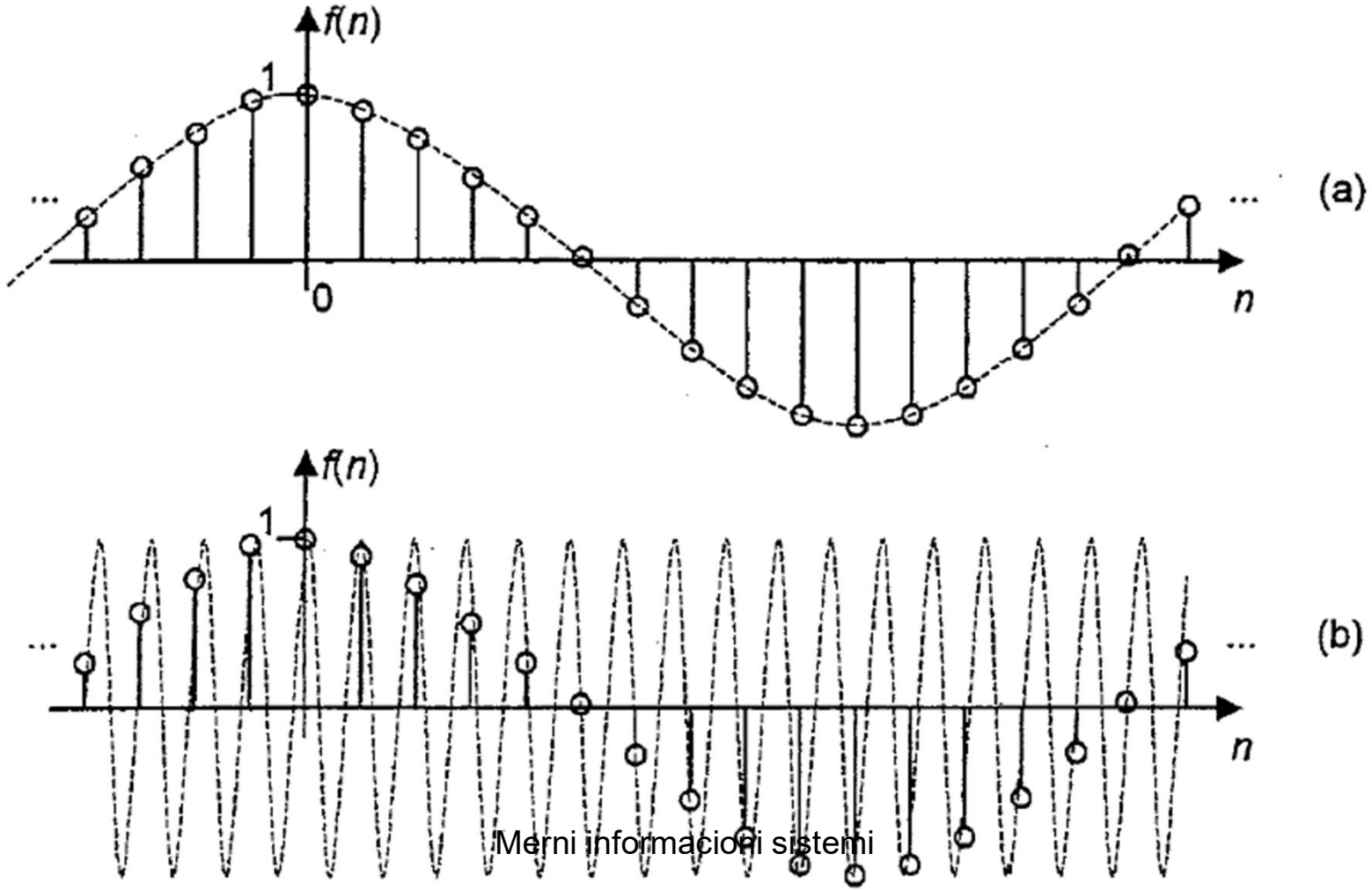
$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

Periodična funkcija

$$x[n] = x[n+N]$$

***N* je perioda ponavljanja**

Različiti kontinulani signali – isti diskretni signali



Kompleksni eksponencijalni niz

$$x[n] = e^{j\omega n}$$

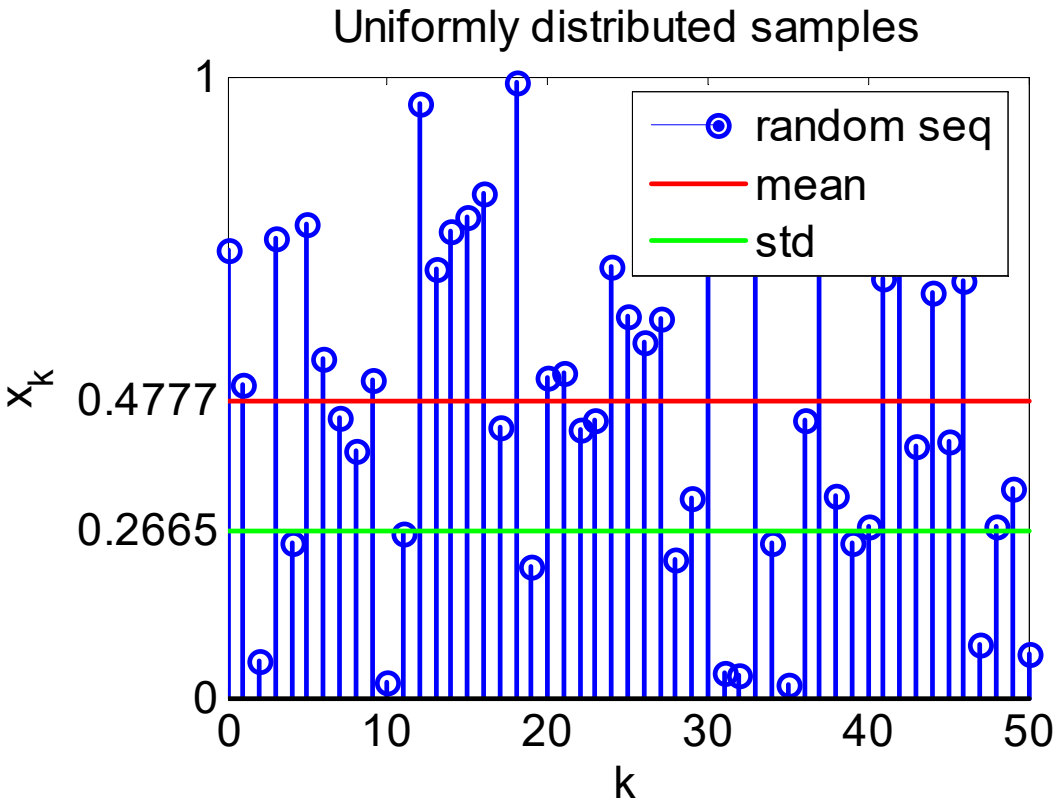
$$x[n] = \cos(\omega n) + j\sin(\omega n)$$

Periodični signali

- Deterministički signal može biti periodičan – posmatra se samo opseg vremena koji se kasnije ponavlja beskonačno puta
- Aperiodični signal je onaj koji nema osobinu periodičnosti

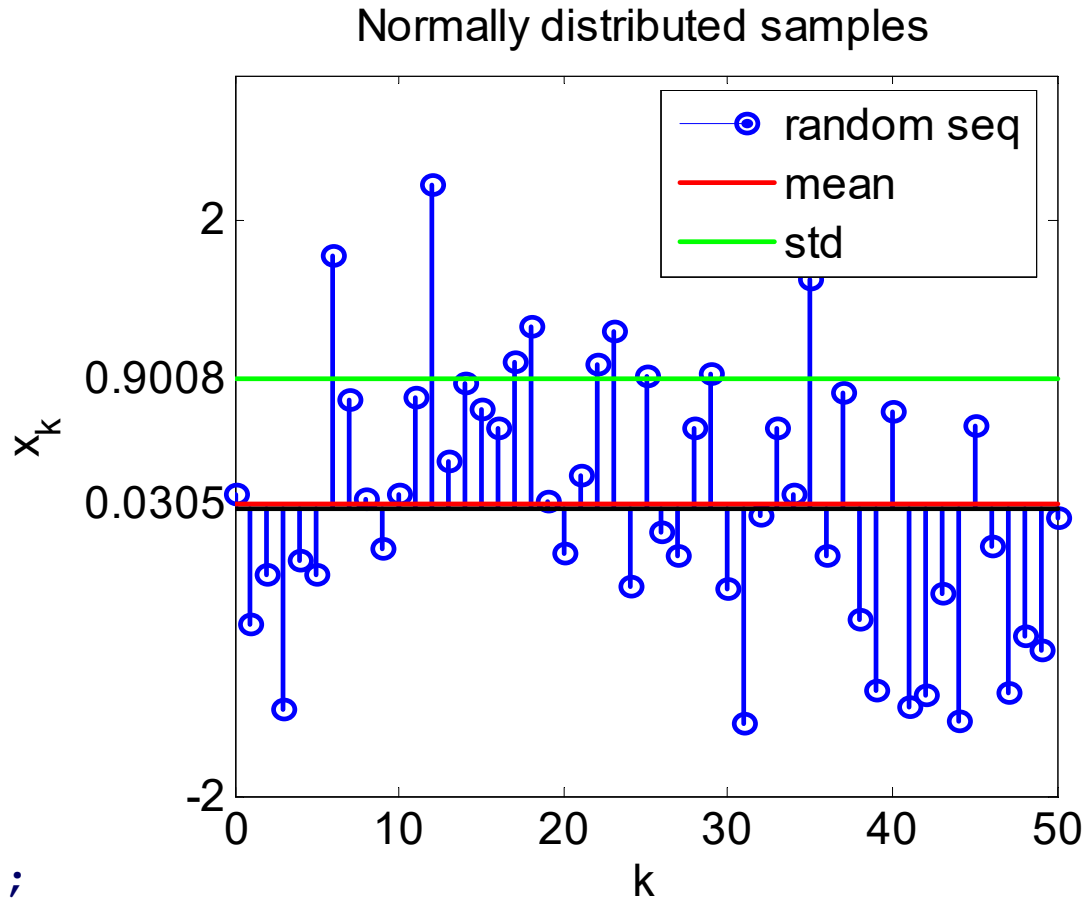
Random signal - MATLAB

```
xk = 0:1:50;
x = rand(size(k));
m = mean(x);
s = std(x);
stem(k,x)
hold on
plot([k(1) k(end)],…
     [m m],'r',...
     [k(1) k(end)],…
     [s s],'g')
hold off
xlabel('k')
ylabel('x_k')
ytick = [0 s m 1];
set(gca,'YTick',ytick)
legend('random seq',…
       'mean','std')
title('Uniformly distributed samples')
```



Random signal - MATLAB

```
k = 0:1:50;
x = randn(size(k));
m = mean(x);
s = std(x);
stem(k,x)
hold on
plot([k(1) k(end)],…
     [m m],'r',…
     [k(1) k(end)],…
     [s s],'g')
hold off
xlabel('k')
ylabel('x_k')
legend('random seq',…
       'mean','std')
ytick = sort([-2 s m 2]);
set(gca,'YTick',ytick)
title('Normally distributed samples')
```



Diskretna Furijeova transformacija

Sekvenca u vremenskom domenu $x[n]$ dužine N preslikava se u sekvencu $X[k]$, iste dužine gde se članovi računaju po formuli

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}, \quad 0 \le k \le N-1$$

Matlab `fft` realizuje DFT

`X=fft[x]`

- Iz zadate sekvence u vremenskom domenu $x[n]$ dobija se nova sekvenca u $X[k]$

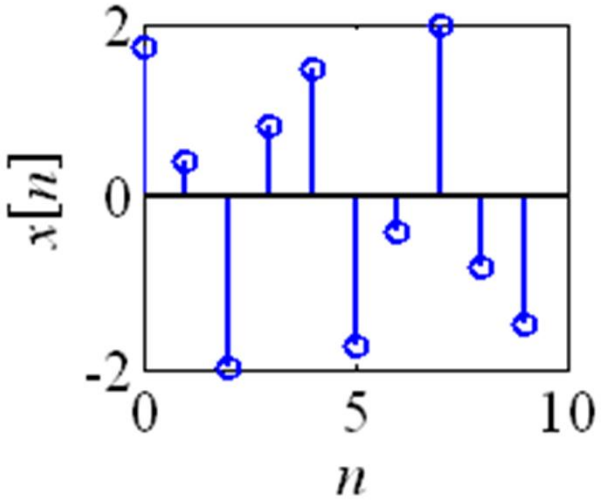
```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128]
X = fft(x)
```

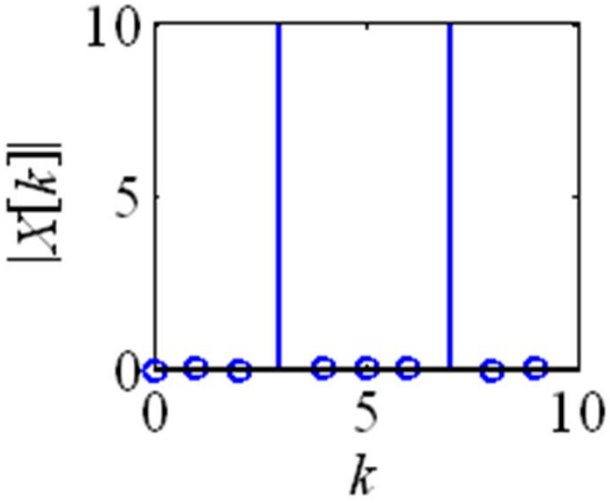
Kompleksni brojevi

```
x =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

```
X =
  Columns 1 through 4
   1.9922  1.1861-0.6487i 0.7969-0.3984i 0.6889-0.1799i
  Columns 5 through 8
   0.6641  0.6889+0.1799i 0.7969+0.3984i 1.1861+0.6487i
```

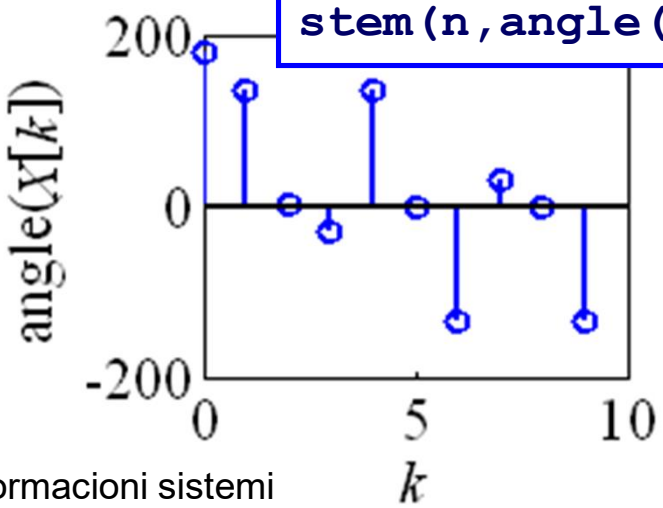
FFT



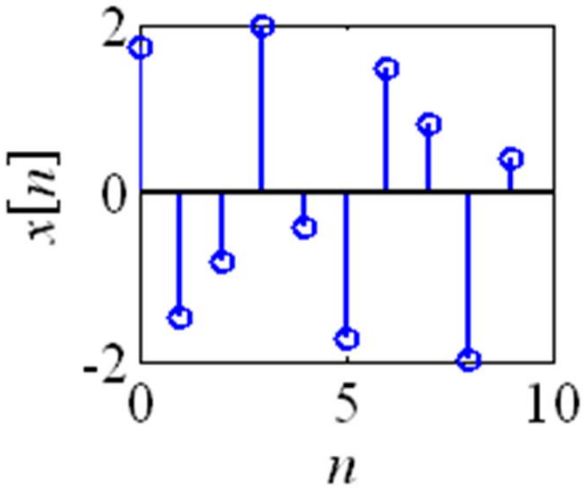


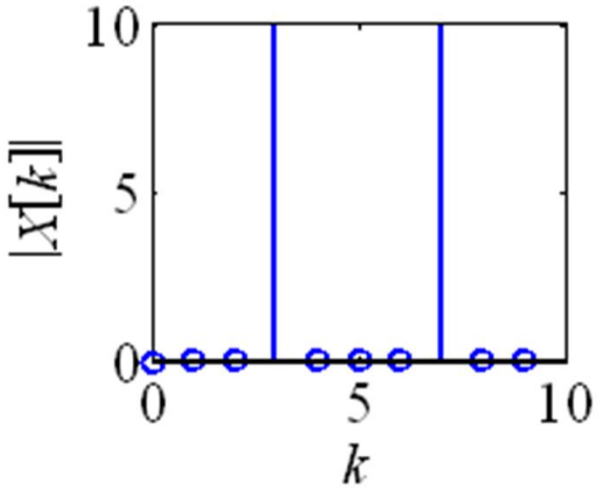

```
N = 10;
n = 0:N-1;
k = 3;
f = k/N;
fi = -pi/6;
x = 2*cos(2*pi*f*n+fi);
stem(n,x)
X = fft(x)
stem(n,abs(X))
```

```
stem(n,angle(X)*180/pi)
```

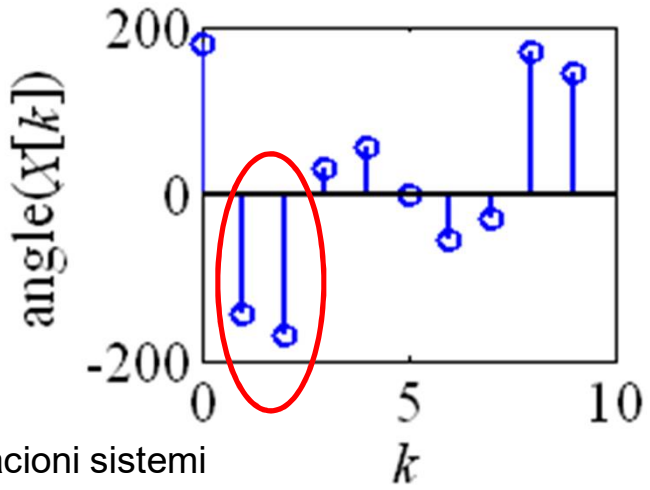


FFT - promena faze

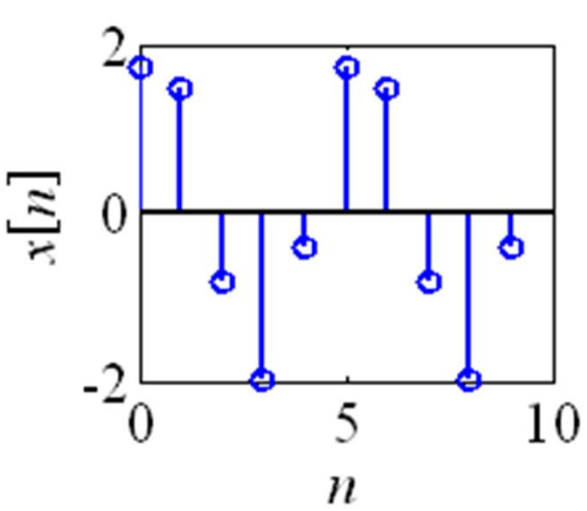


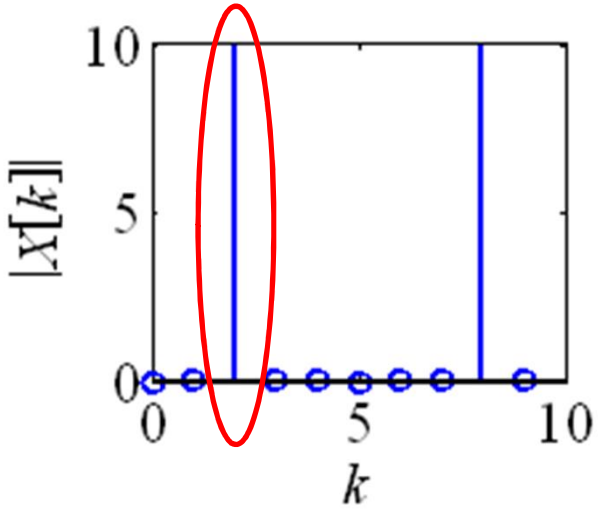



```
N = 10;
n = 0:N-1;
k = 3;
f = k/N;
fi = pi/6;
x = 2*cos(2*pi*f*n+fi);
stem(n,x)
X = fft(x)
stem(n,abs(X))
```

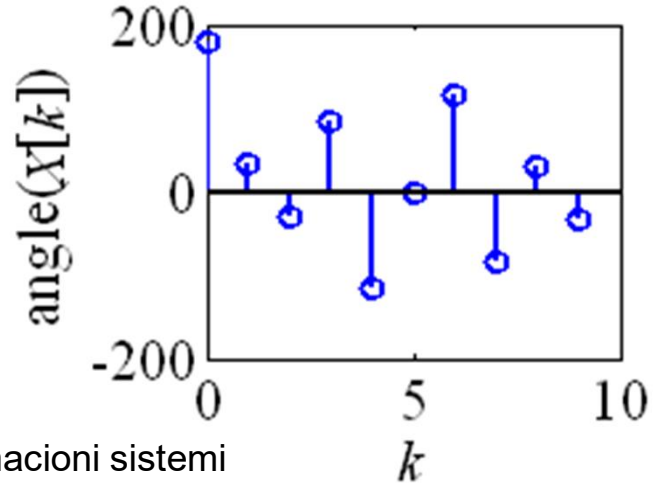


FFT – promena *k*

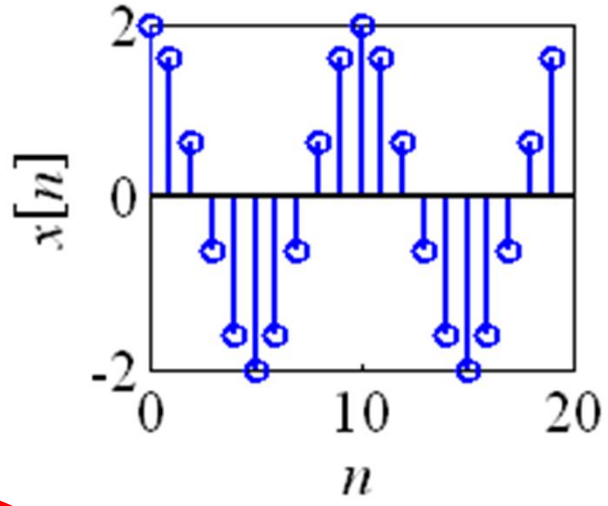


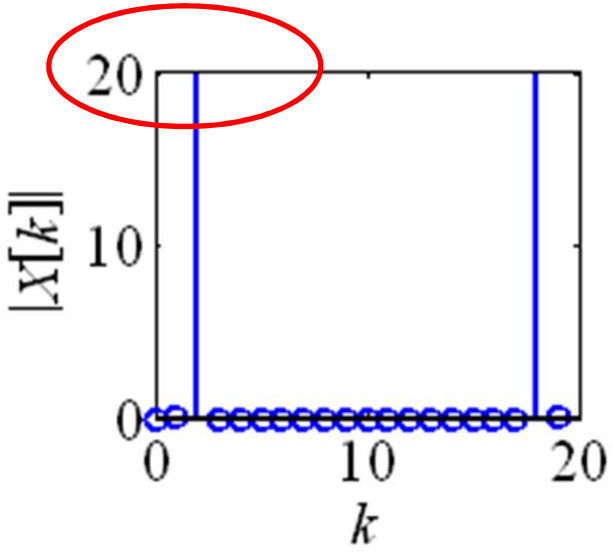



```
N = 10;
n = 0:N-1;
k = 2;
f = k/N;
fi = -pi/6;
x = 2*cos(2*pi*f*n+fi);
stem(n,x)
X = fft(x)
stem(n,abs(X))
```

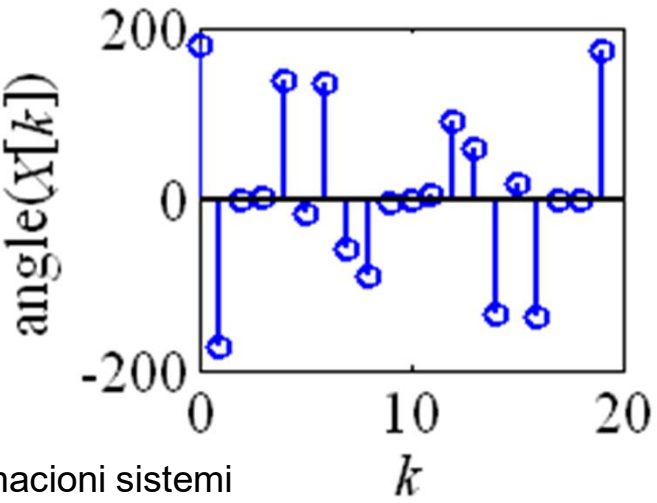


FFT – promena *N*

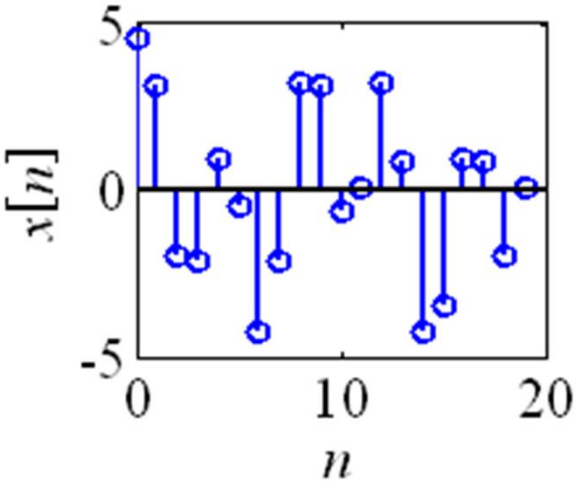


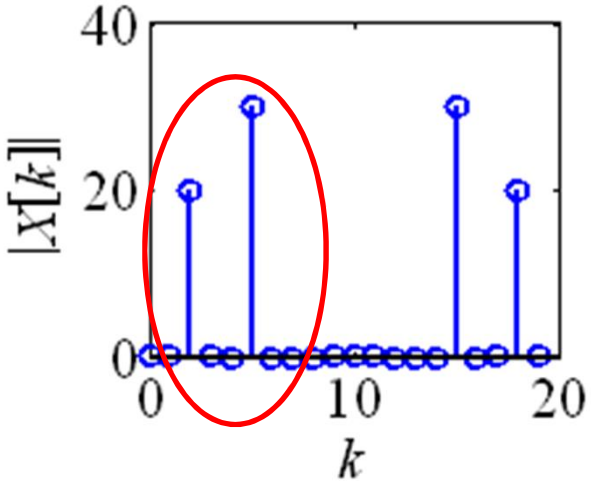



```
N = 20;
n = 0:N-1;
k = 3;
f = k/N;
fi = -pi/6;
x = 2*cos(2*pi*f*n+fi);
stem(n,x)
X = fft(x)
stem(n,abs(X))
```

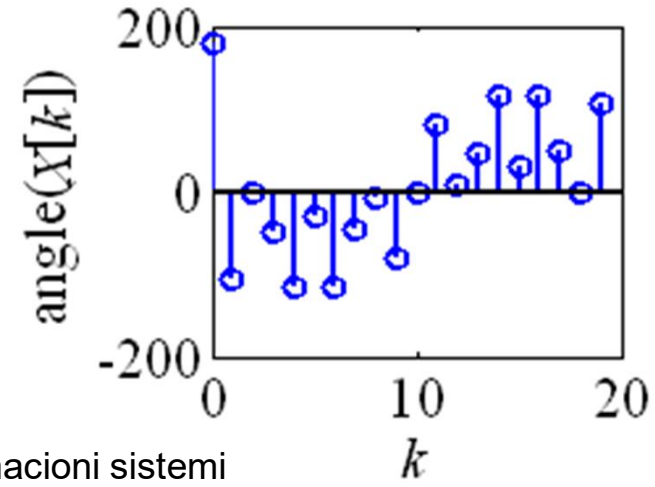


FFT – dve sinusoide



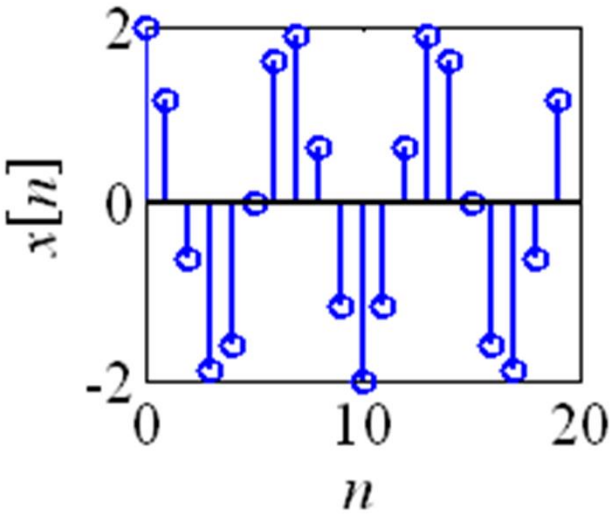


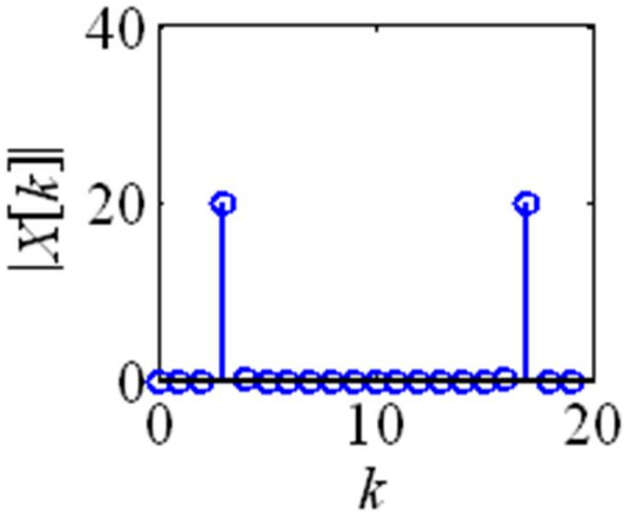

```
N=20;
n=0:N-1;
k1=2;     k2=5;
f1=k1/N;  f2=k2/N;
fi2=-pi/6;
x = 2*cos(2*pi*f1*n+0)+...
    3*cos(2*pi*f2*n+fi2);
X = fft(x)
```



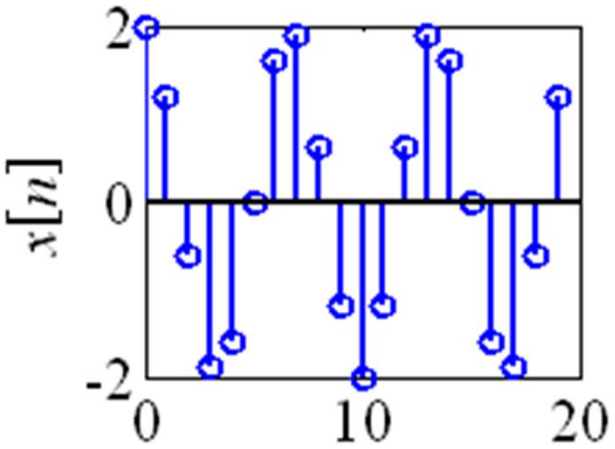
FFT - aliasing

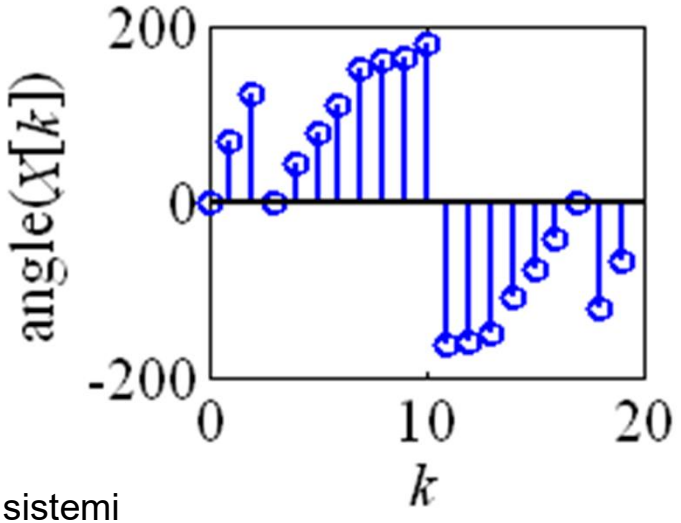
```
k=3;
f=k/N;
```






```
k=N+3;
f=k/N;
```

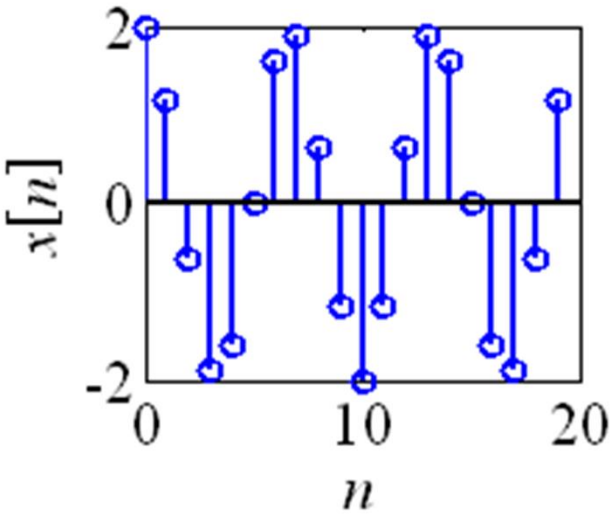


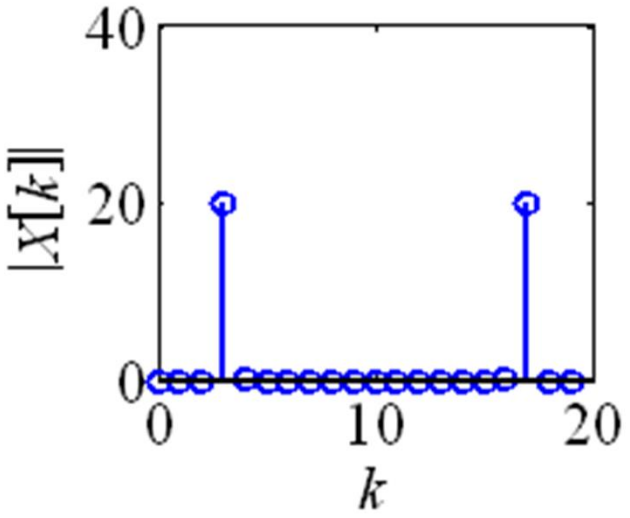



```
x = 2*cos(2*pi*f*n);
```

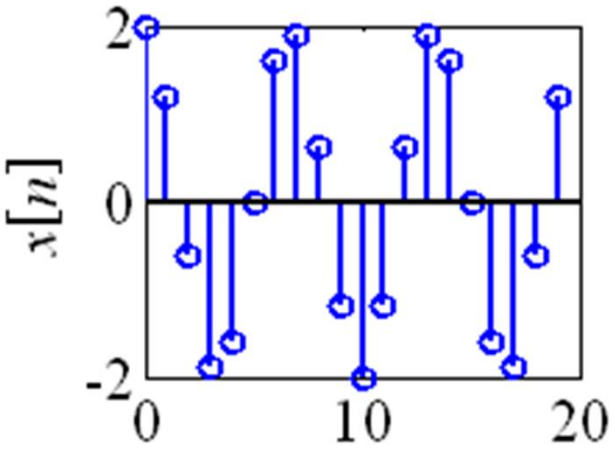
FFT - folding

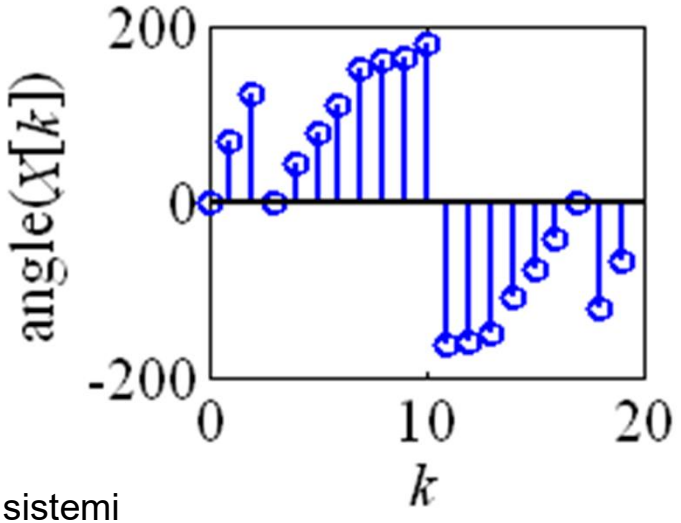
```
k=3;
f=k/N;
```






```
k=N-3;
f=k/N;
```






```
x = 2*cos(2*pi*f*n);
```

Jednoznačnost FFT

```
k=3;
f=k/N;
```

$$k < N/2$$

```
x = 2*cos(2*pi*f*n);
```

$$f < 1/2$$

Jednoznačnost FFT

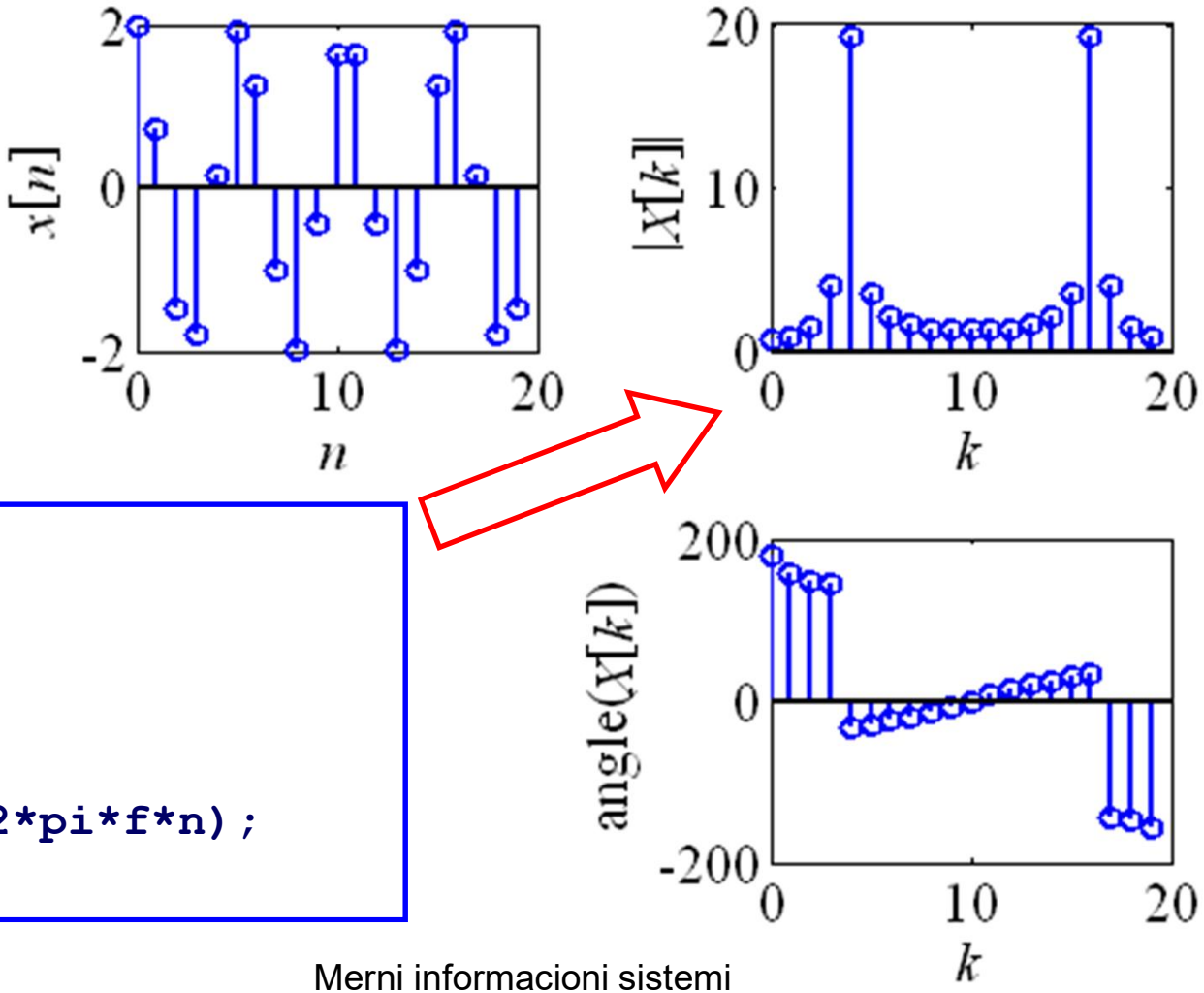
```
x = 2*cos(2*pi*f*n);
```

```
f < 1/2
f = 1-f
f = 1+f
f = 2-f
f = 2+f
```

FFT – curenje spektra

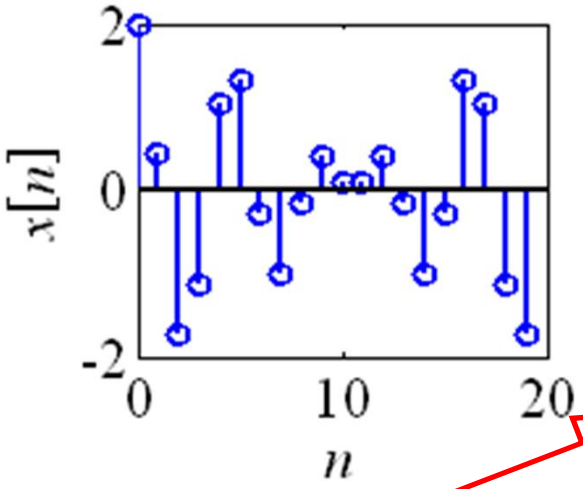
frequency leakage

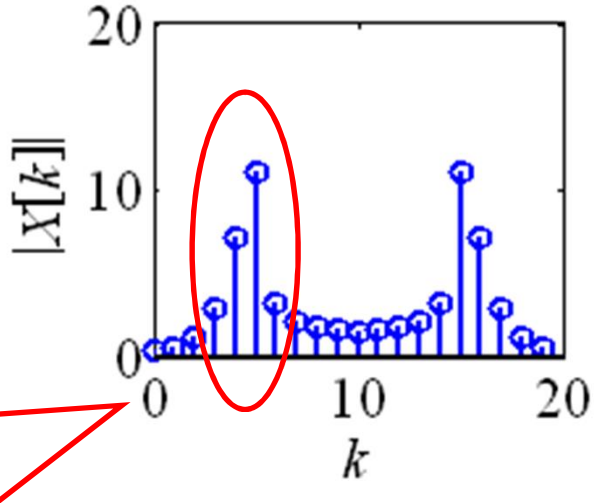
```
N = 20;
M = 21;
n = 0:N-1;
k = 4;
f = k/M;
x = 2*cos(2*pi*f*n);
X = fft(x)
```



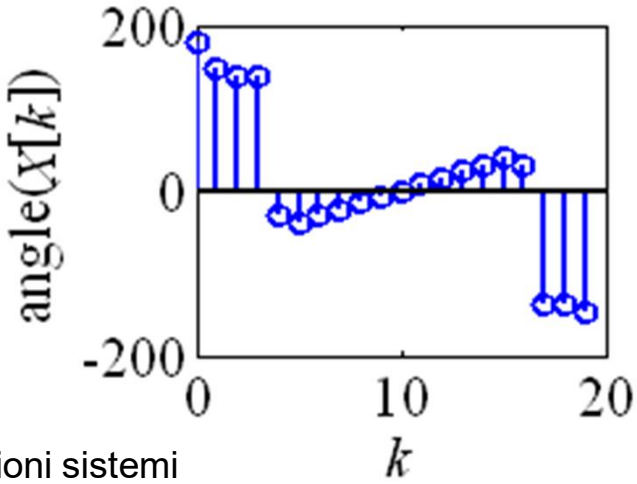
FFT – frekvencijska rezolucija

frequency resolution



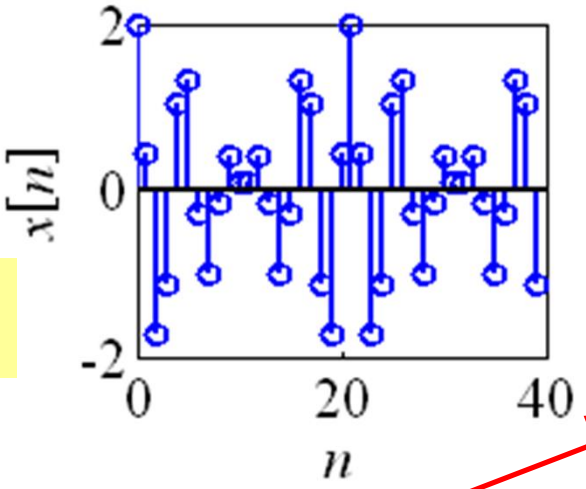


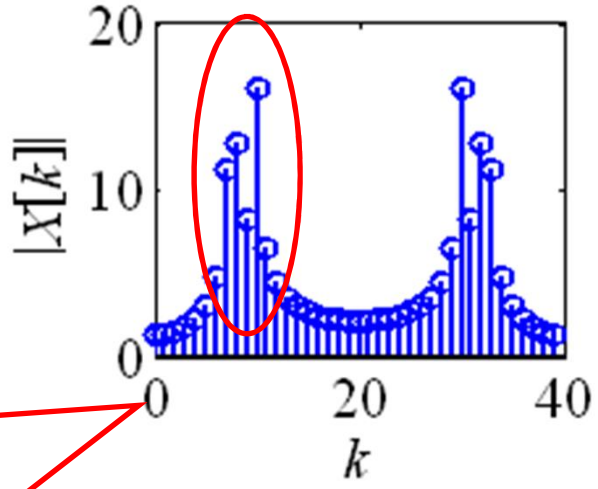

```
N = 20;
M = 21;
n = 0:N-1;
k1=4; f1=k1/M;
k2=5; f2=k2/M;
x=cos(2*pi*f1*n)+...
  cos(2*pi*f2*n);
X = fft(x)
```

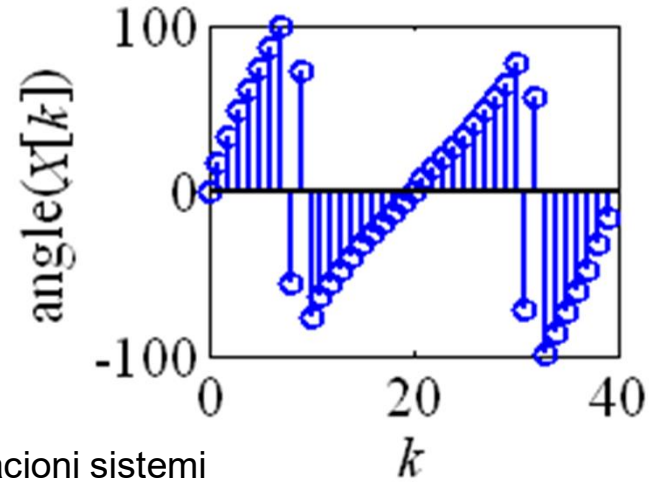


FFT – frekvencijska rezolucija

$$\Delta f = 1/N$$

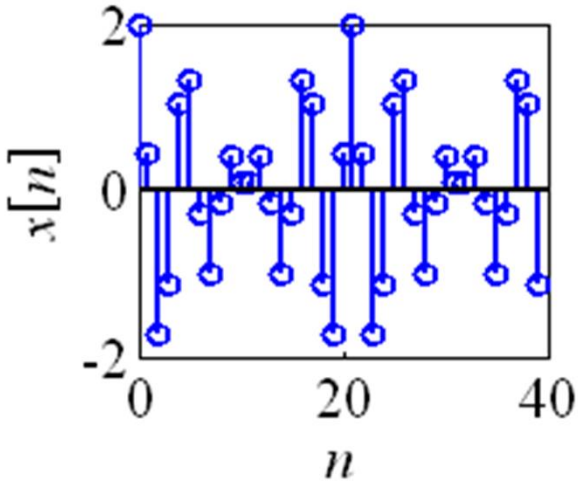


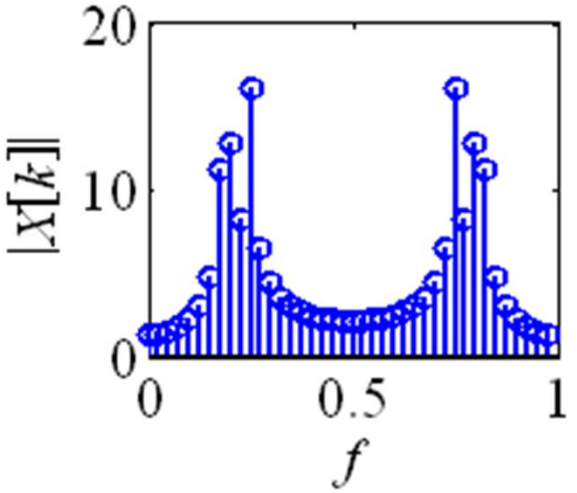


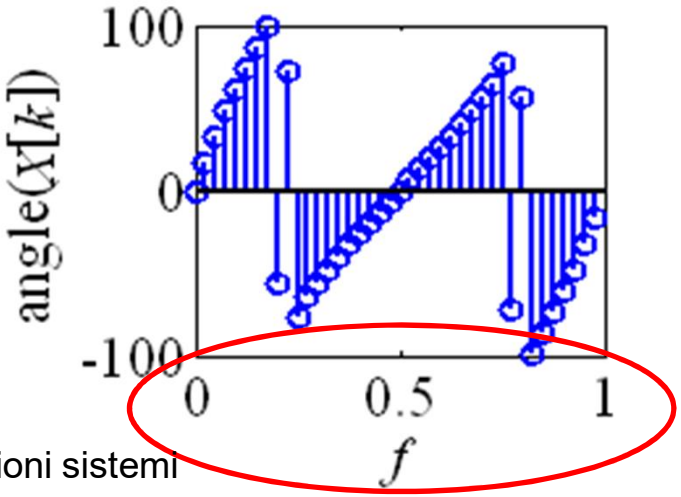



```
N = 40;
M = 21;
n = 0:N-1;
k1=4; f1=k1/M;
k2=5; f2=k2/M;
x=cos(2*pi*f1*n)+...
  cos(2*pi*f2*n);
X = fft(x)
```

FFT – digitalna frekvencija

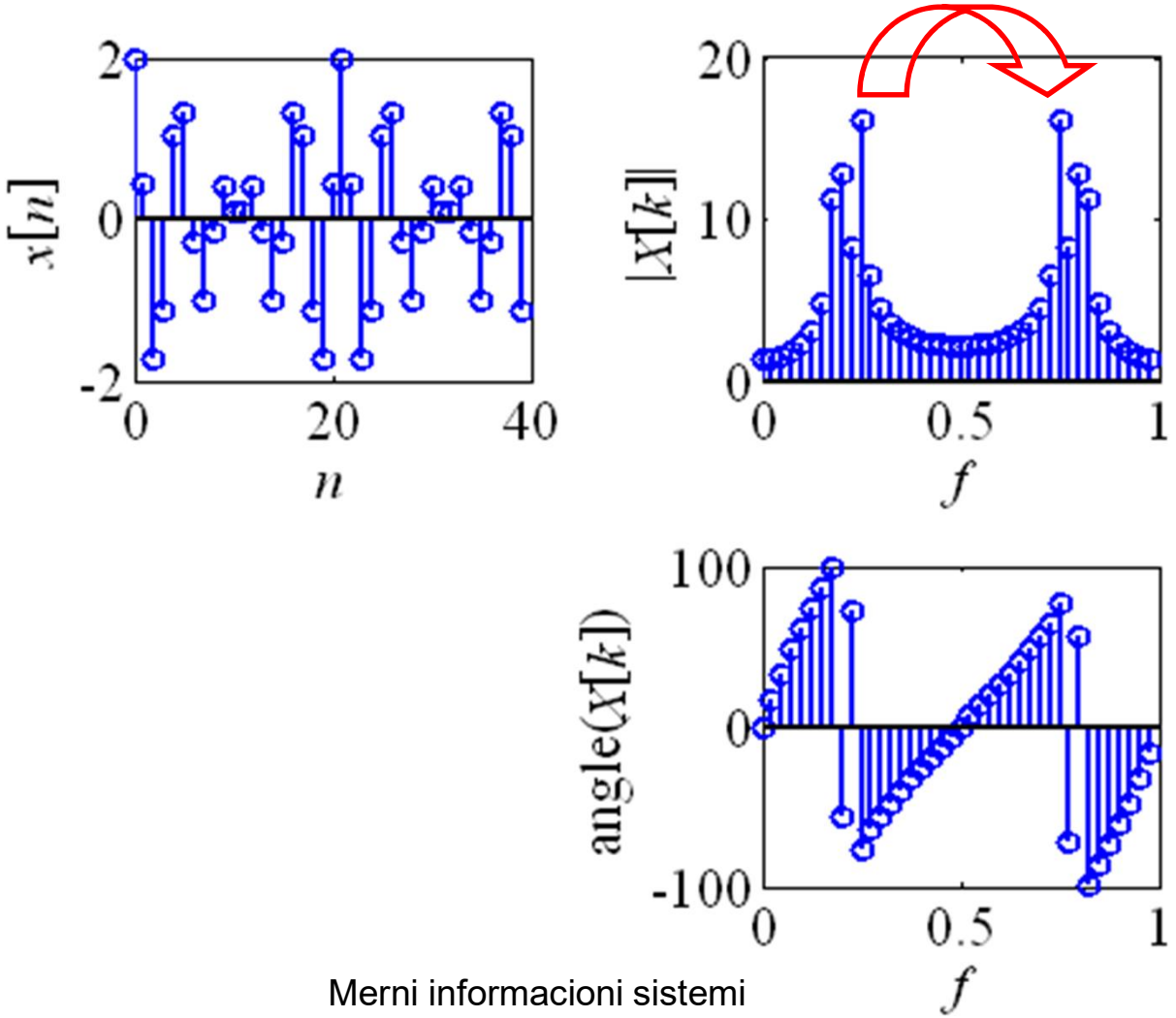




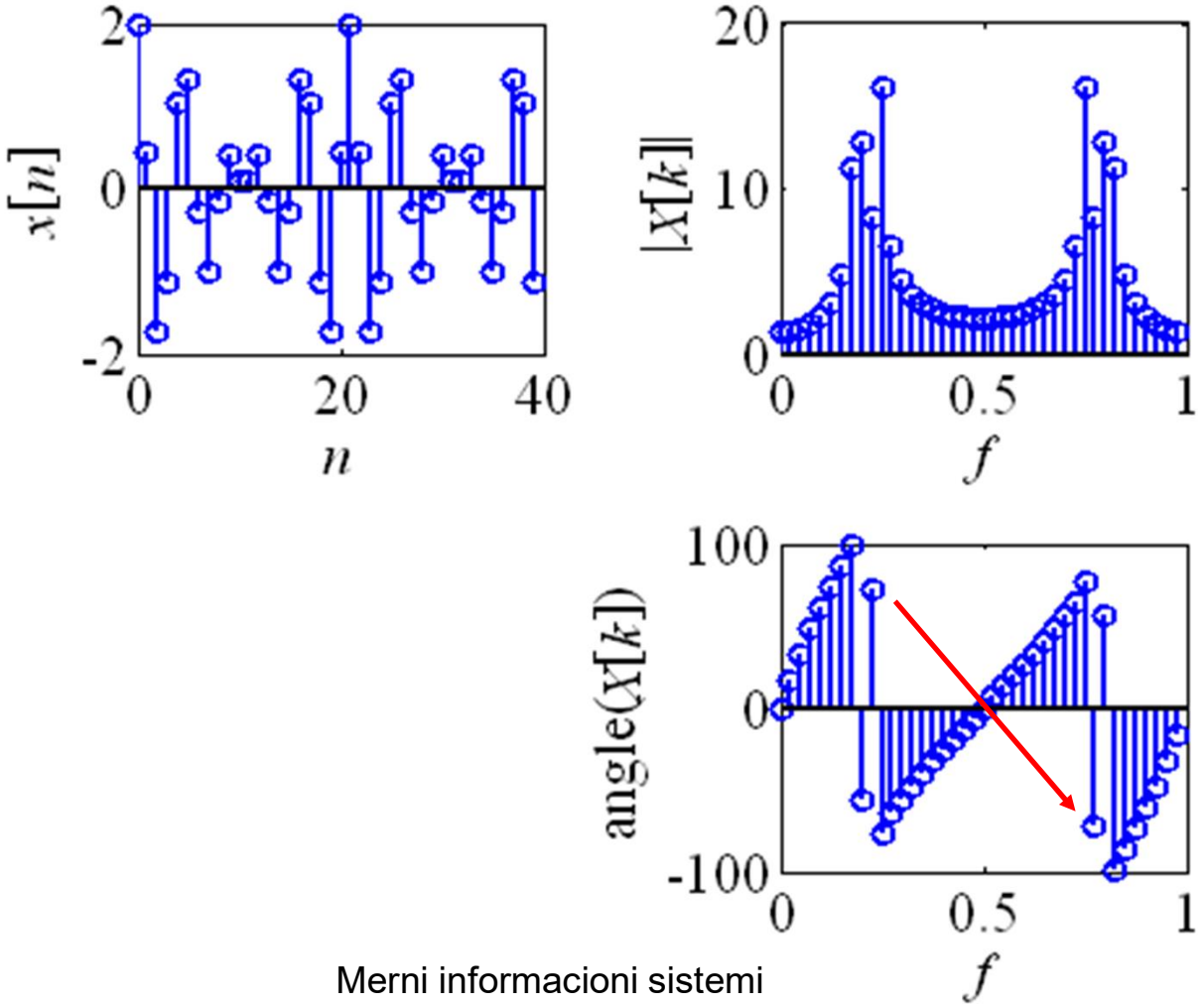



```
stem(n/N,abs(y))
xlabel('f')
```

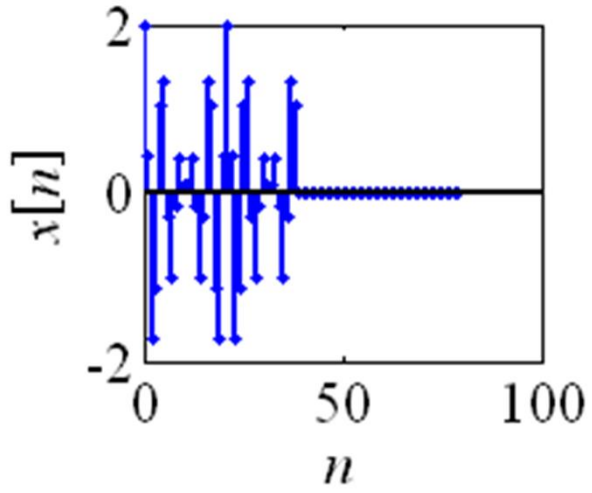
FFT – simetrija

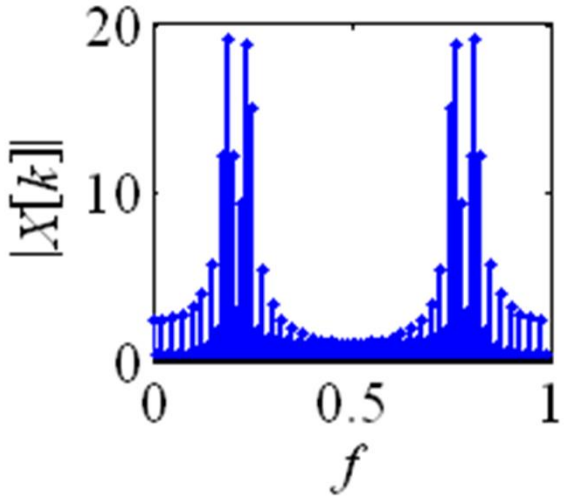


FFT – antisimetrija

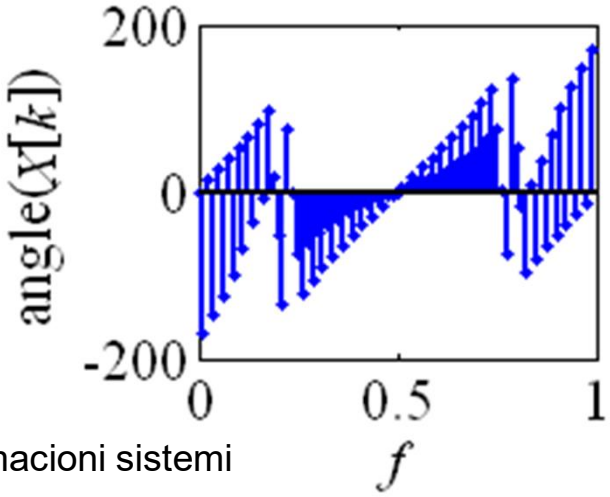


FFT – dopuna nulama

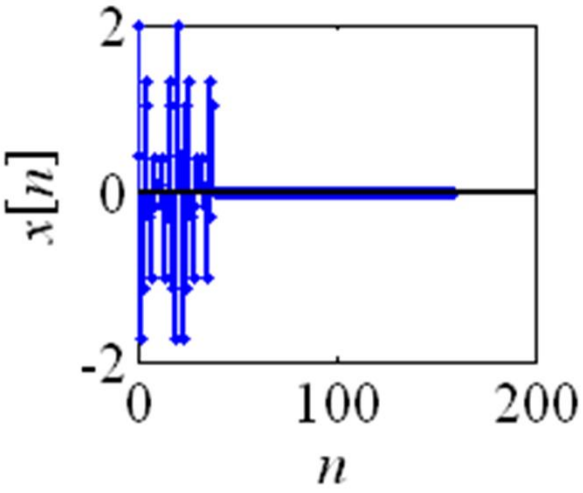


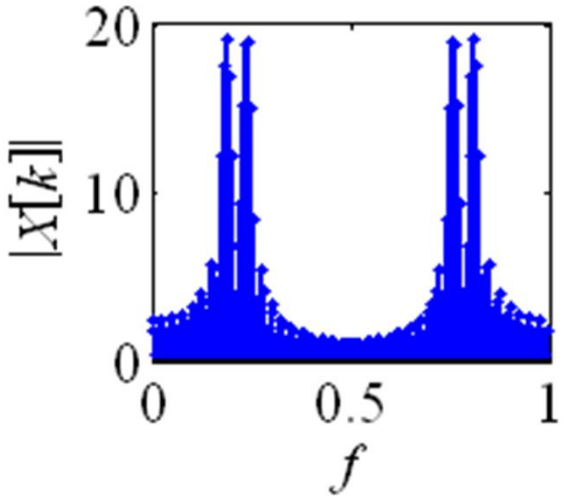



```
N = 80;
. . .
x = x.*(n<(N/2-1));
```

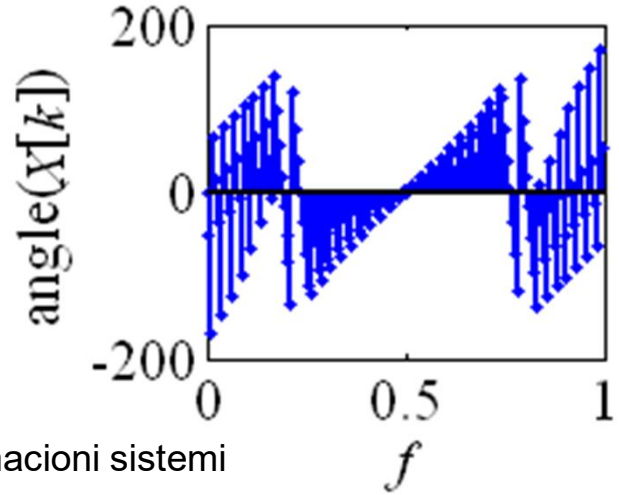


FFT – dopuna nulama (2)

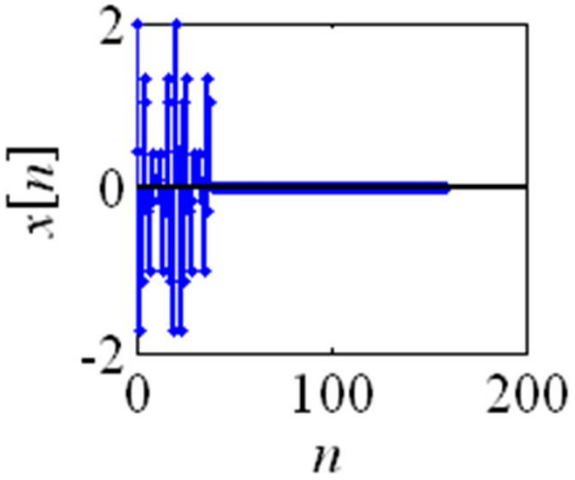


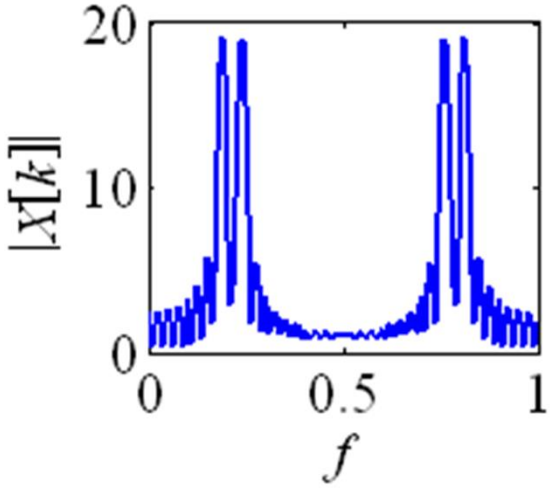


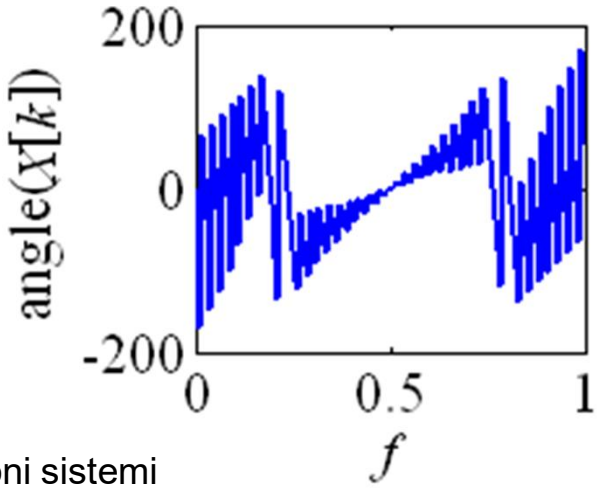

```
N = 160;
. . .
x = x.*(n<39);
```



FFT – dopuna nulama (2)

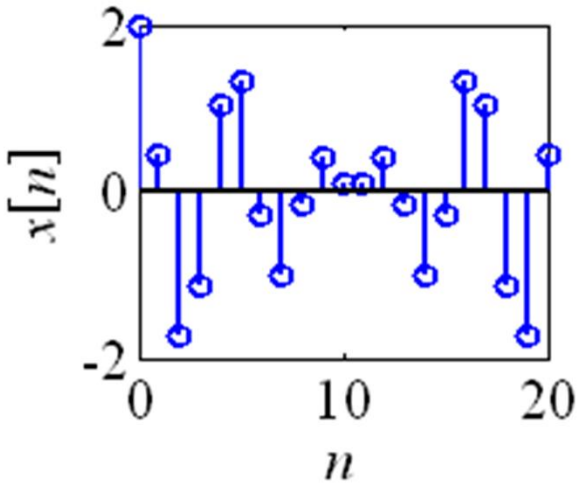


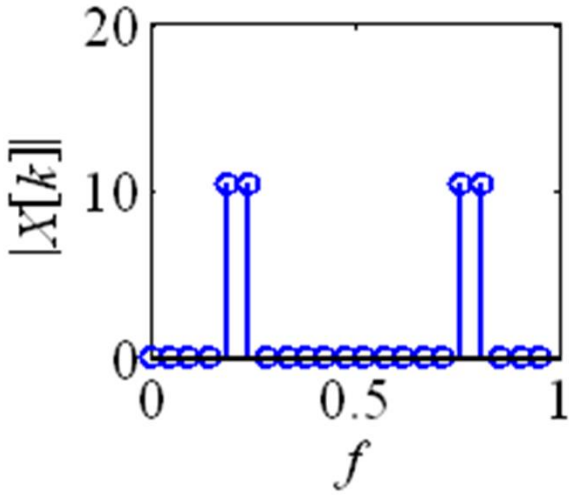


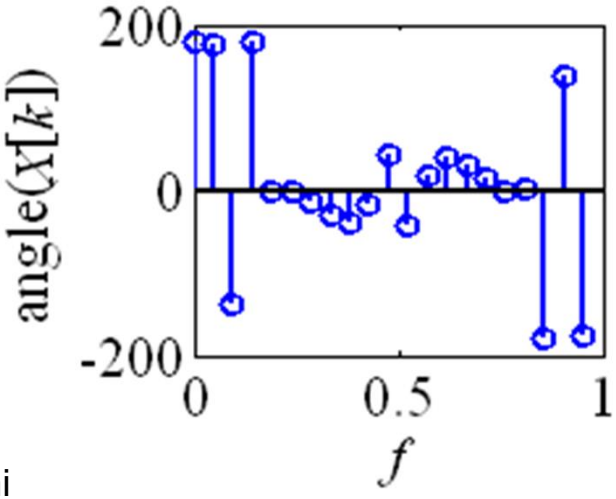



```
plot(n/N,abs(y),'-')
. . .
plot(n/N,angle(y)*180/pi,'-')
```

FFT – perfektna dopuna








```
N = 21;
. . .
k1 = 4;
f1 = k1/M;
k2 = 5;
f2 = k2/M;
x = cos(2*pi*f1*n)+cos(2*pi*f2*n);
x = x.*(n<21);
```

Osobine DFT

Uređeni par

sekvenca u vremenskom domenu $x[n]$

sekvenca transformacije $X[k]$

$$\{x[n] \leftrightarrow \{X[k]\}\}$$

$$\{y[n] \leftrightarrow \{Y[k]\}\}$$

Jednoznačnost DFT

Ako su dve sekvence iste
tada su iste i njihove transformacije

$$\{x[n]\} = \{y[n]\} \Leftrightarrow \{X[k]\} = \{Y[k]$$

Linearnost DFT

Linearna kombinacija dve sekvence jednaka je linearnoj kombinaciji transformacija gde su a i b konstante

$$\mathrm{DFT}(a\{x[n]\}+b\{y[n]\}) = a\,\mathrm{DFT}(\{x[n]\})+b\,\mathrm{DFT}(\{y[n]\})$$

Simetrija DFT

Transformacija je simetrična
pri čemu * označava
konjugovano kompleksnu vrednost

$$X[N-k] = X[k]^*$$

Moduo je simetričan a faza je antisimetrična

Realan signal – kompleksna transformacija

Ako je sekvenca $x[n]$
u vremenskom domenu REALNA
sekvenca transformacije $X[k]$ je
KOMPLEKSNA sekvenca

Parseval-ova teorema

Transformacija je simetrična

pri čemu * označava konjugovano kompleksnu vrednost

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

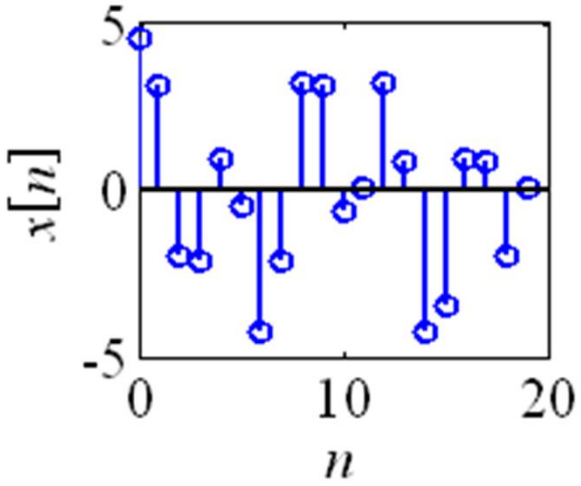
Pojam snage

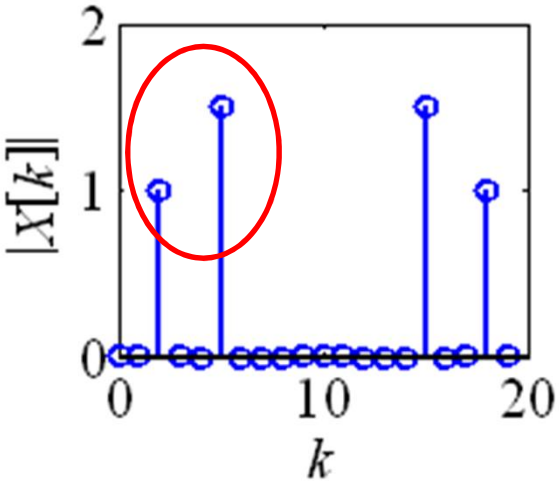
Normalizovana transformacija

Normalizovana transformacija da bi moduo u transformaciji odgovarao amplitudi sinusoide

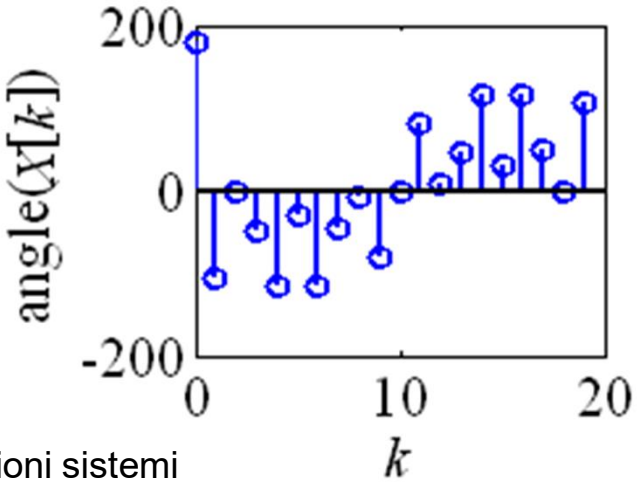
$$\{X_{\text{norm}}[k]\} = \{\frac{1}{N} X[k]\}$$

FFT – sa normalizacijom

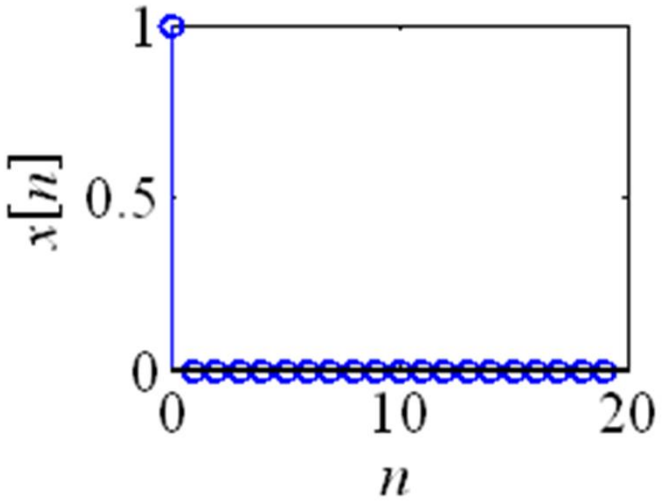


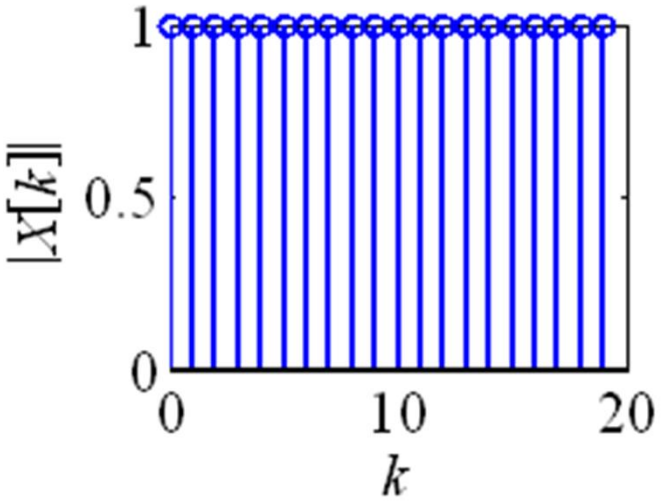



```
N=20; n=0:N-1;
k1=2;     k2=5;
f1=k1/N; f2=k2/N;
fi2=-pi/6;
x = 2*cos(2*pi*f1*n+0)+...
    3*cos(2*pi*f2*n+fi2);
X = fft(x)
. . .
stem(n,abs(X)/N)
```

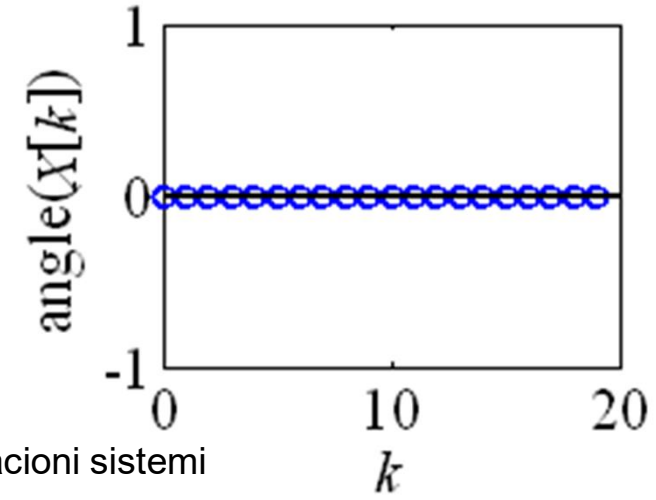


DFT jedinične funkcije

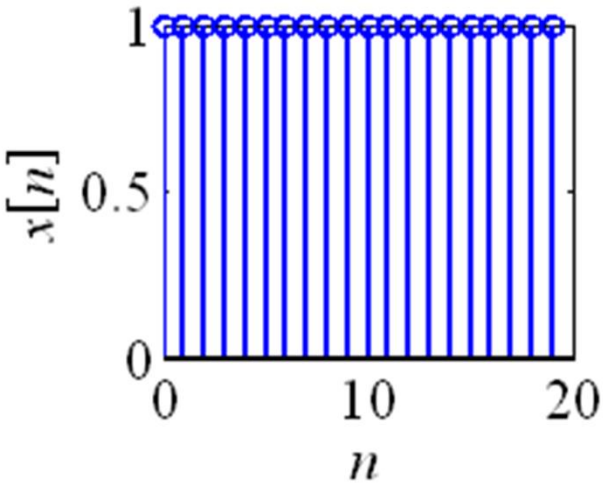


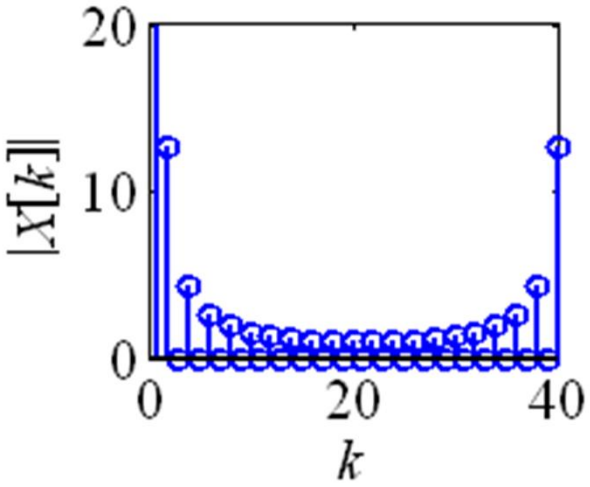



```
N=20; n=0:N-1;
x = (n==0);
X = fft(x)
. . .
stem(n,abs(X)/N)
```

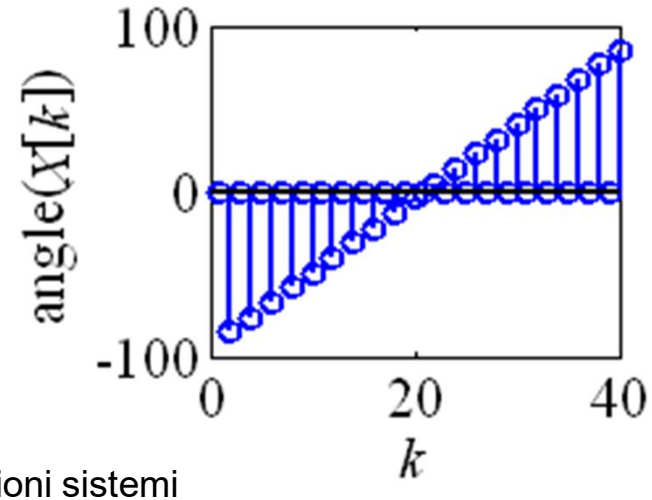


DFT odskočne funkcije

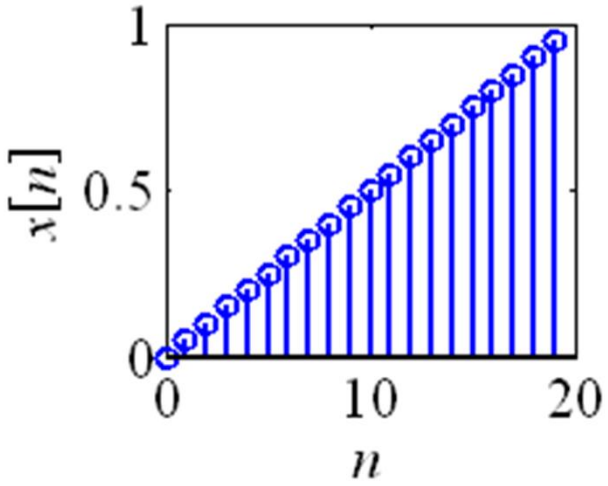


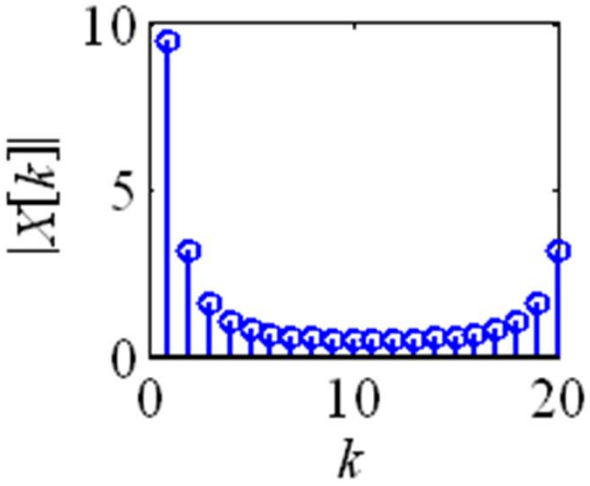



```
N=20; n=0:N-1;
x = (n>=0);
X = fft(x,2*N)
. . .
stem(n,abs(X)/N)
```

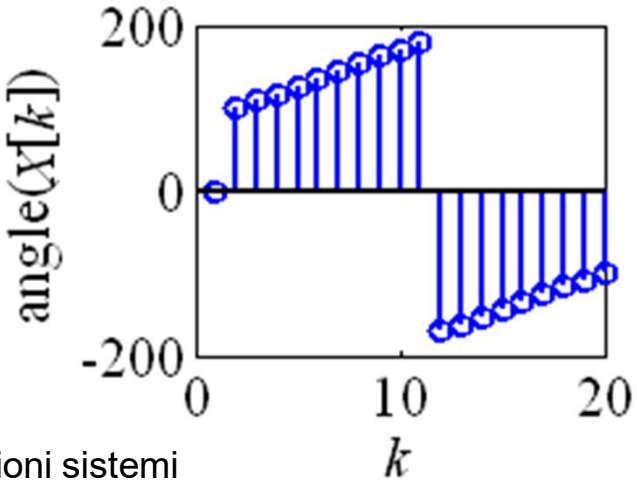


DFT rastuće funkcije

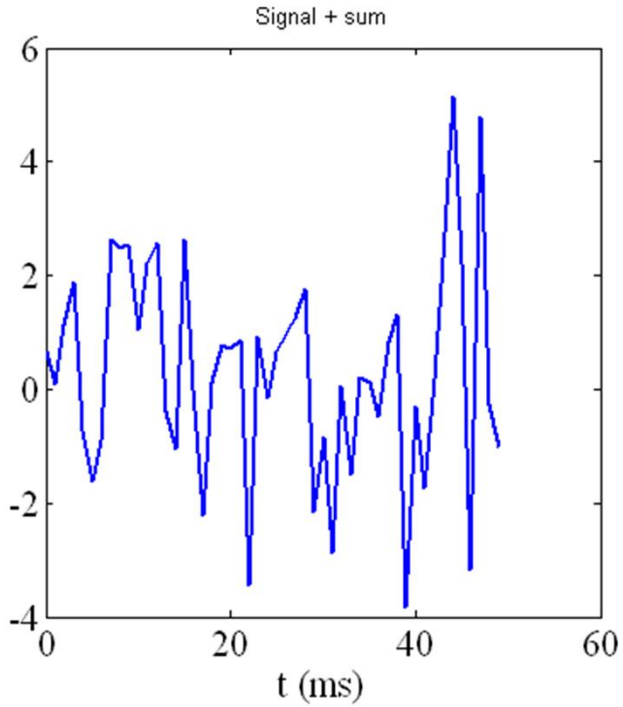


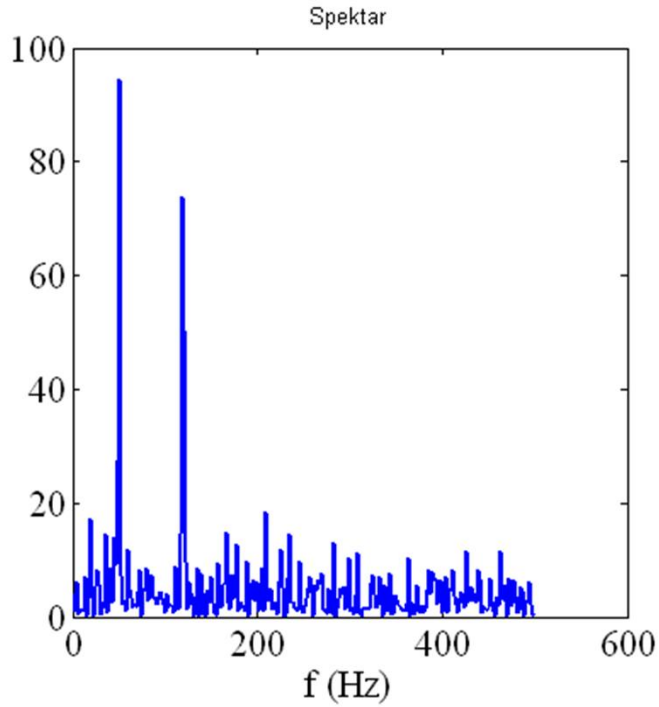



```
N=20; n=0:N-1;
x = n/N;
X = fft(x,N)
. . .
stem(n,abs(X)/N)
```



DFT kontinualne funkcije






```
t = 0:0.001:0.6;
x = sin(2*pi*50*t)+sin(2*pi*120*t);
y = x + 2*randn(size(t));
Y = fft(y,512);
Pyy = Y.* conj(Y) / 512;
f = 1000*(0:256)/512;
```

Inverzna DFT

Sekvenca u vremenskom domenu $x[n]$ dužine N preslikava se u sekvencu $X[k]$, iste dužine gde se članovi računaju po formuli

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn}$$

Matlab `ifft` realizuje IDFT

`x=ifft[X]`

```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128]
X = fft(x)
y = ifft(X)
```

```
x =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

```
X =
  Columns 1 through 4
   1.9922  1.1861-0.6487i 0.7969-0.3984i 0.6889-0.1799i
  Columns 5 through 8
   0.6641  0.6889+0.1799i 0.7969+0.3984i 1.1861+0.6487i
```

```
y =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

DFT u više tačaka

- Y=fft(X,n) returns the n-point DFT
- If the length of X is less than n,
  X is padded with trailing zeros to length n
- If the length of X is greater than n,
  the sequence X is truncated
- When X is a matrix,
  the length of the columns
  are adjusted in the same manner

Matlab `ifft` realizuje IDFT

```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128]
X = fft(x,10)
y = ifft(X)
```

```
x =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

```
X =
  Columns 1 through 4
   1.9922  1.1861-0.6487i 0.7969-0.3984i 0.6889-0.1799i
  Columns 5 through 8
   0.6641  0.6889+0.1799i 0.7969+0.3984i 1.1861+0.6487i
```

```
y =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078 0 0
```

Matlab `fft` i `ifft`

```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128]
X = fft(x,10)
y = ifft(X)
```

```
x =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

```
X =
  Columns 1 through 4
   1.9922  1.1861-0.6487i 0.7969-0.3984i 0.6889-0.1799i
  Columns 5 through 8
   0.6641  0.6889+0.1799i 0.7969+0.3984i 1.1861+0.6487i
```

```
y =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078 0 0
```

Matlab `fft` i `ifft`

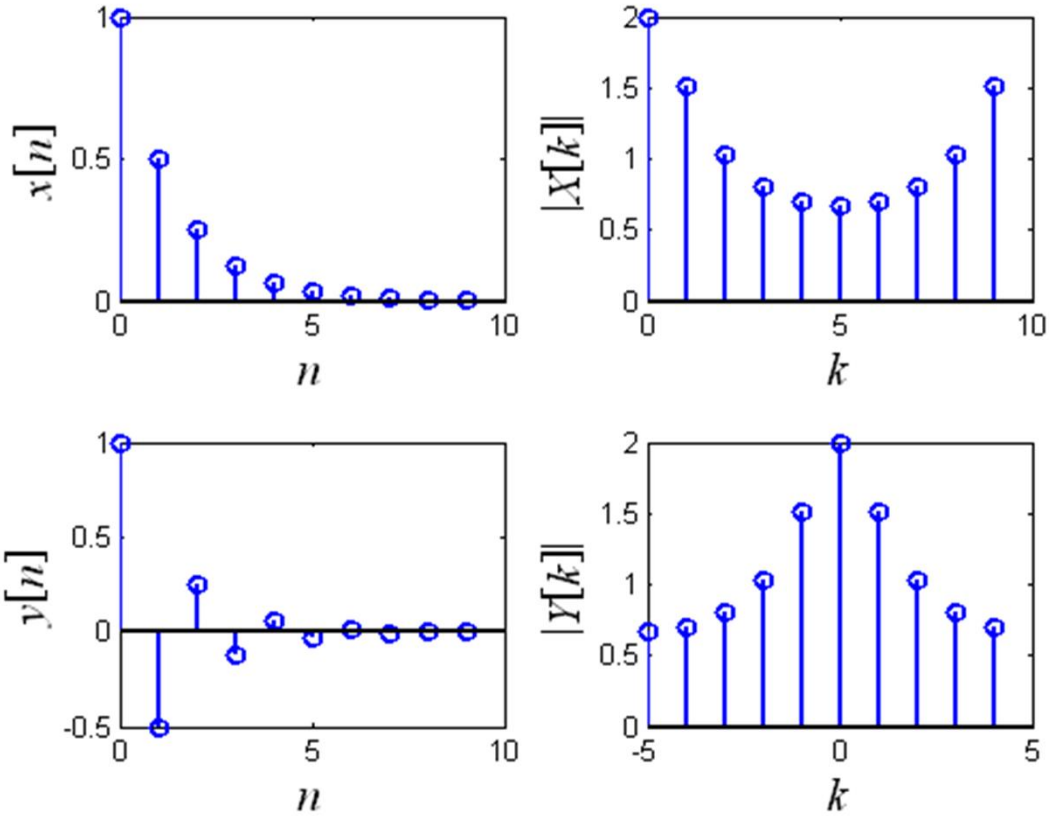
```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128]
X = fft(x,6)
y = ifft(X)
```

```
x =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

```
X =
  Columns 1 through 4
   1.9922  1.1861-0.6487i 0.7969-0.3984i 0.6889-0.1799i
  Columns 5 through 8
   0.6641  0.6889+0.1799i 0.7969+0.3984i 1.1861+0.6487i
```

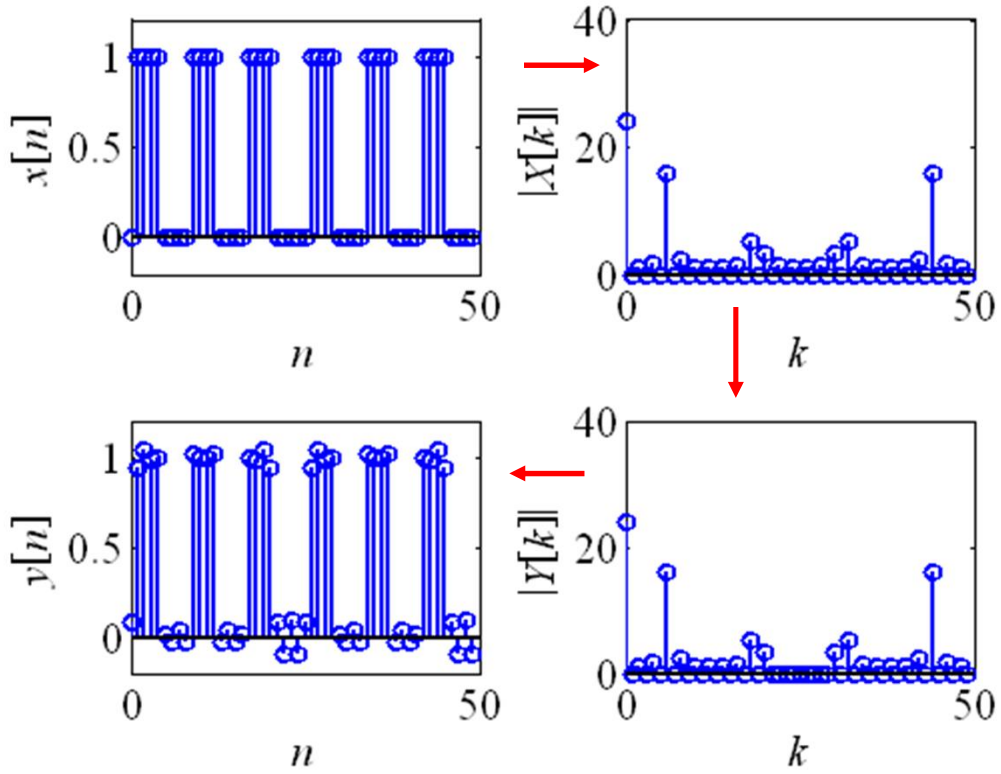
```
y =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313
```

Šiftovana DFT



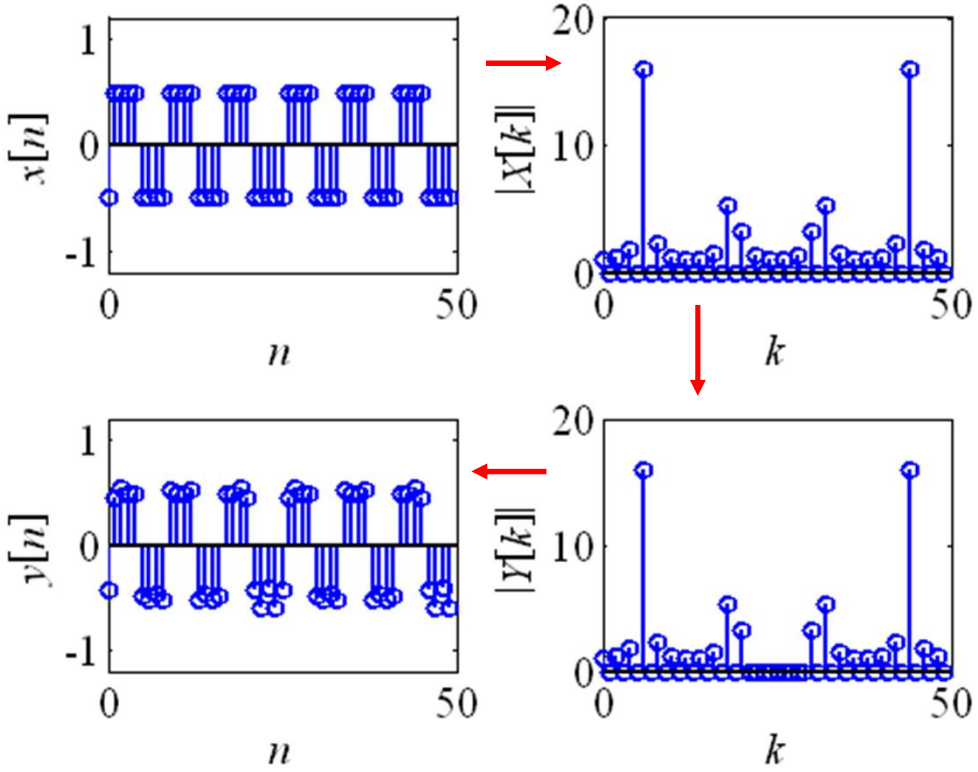

```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128 1/256 1/512]
X = fft(x)
Y = fftshift(X)
y = ifft(Y)
n = 0:length(x)-1;
```

Uticaj promene spektra



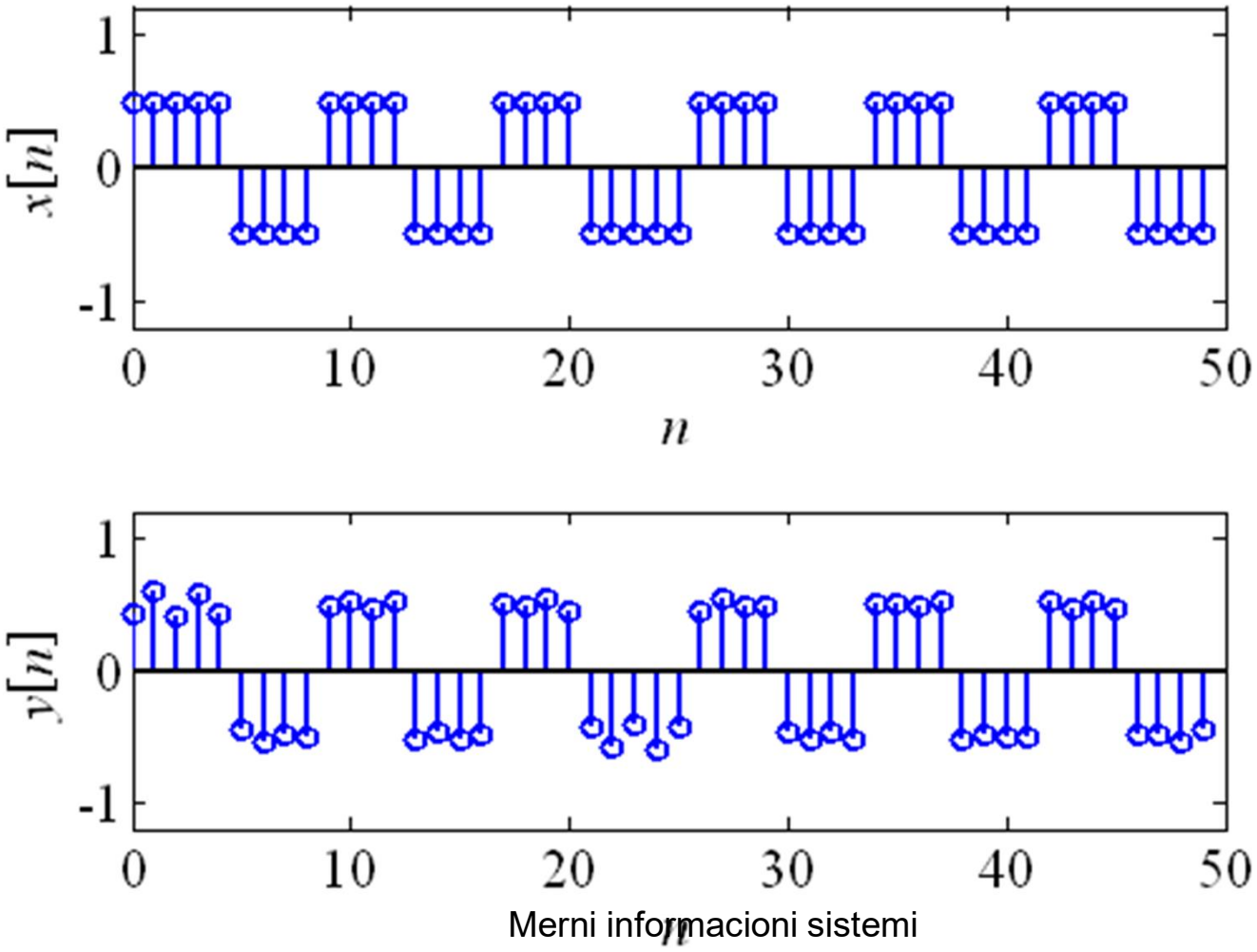

```
N=50; n=0:N-1; k=6; f=k/N;
x=2*sin(2*pi*f*n); x=x>0.25;
X=fft(x); Y=X; n2=3;
Y(N/2-n2:N/2+n2+1)=0*Y(N/2-n2:N/2+n2+1);
```

Uticaj promene spektra, DC=0




```
N=50; n=0:N-1; k=6; f=k/N;
x=2*sin(2*pi*f*n); x=(x>0)-0.5;
X=fft(x); Y=X; n2=3;
Y(N/2-n2:N/2+n2+1)=0*Y(N/2-n2:N/2+n2+1);
```

Uticaj ograničenja spektra



Odziv IIR sistema u vremenskom domenu

$$y[n] = \sum_{k=0}^{N} b_k x[n-k] - \sum_{k=1}^{M} a_k y[n-k]$$

Izlazni signal

Nerekurzivni deo

Rekurzivni deo

Diskretna Furijeova Transformacija DFT

Sekvenca u vremenskom domenu $x[n]$
preslikava se u
sekvencu u frekvencijskom domenu $X[k]$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}, \quad 0 \le k \le N-1$$

Inverzna DFT

Sekvenca u frekvencijskom domenu $X[k]$
preslikava se u
sekvencu u vremenskom domenu $x[n]$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn}$$

DFT diskretnih signala

Uređeni par

sekvenca u vremenskom domenu $x[n]$

sekvenca transformacije $X[k]$

$$\{x[n] \leftrightarrow \{X[k]\}\}$$

$$\{y[n] \leftrightarrow \{Y[k]\}\}$$

Linearnost DFT

Linearna kombinacija dve sekvence jednaka je linearnoj kombinaciji transformacija gde su a i b konstante

$$\mathrm{DFT}(a\{x[n]\}+b\{y[n]\}) = a\,\mathrm{DFT}(\{x[n]\})+b\,\mathrm{DFT}(\{y[n]\})$$

Primena DFT na sekvence u vremenskom domenu

$$y[n] = \sum_{l=0}^{N} b_l x[n-l] - \sum_{k=m}^{M} a_m y[n-m]$$

$$w = e^{j\frac{2\pi}{N}}$$

$$Y[k] = \left(\sum_{l=0}^{L} b_l w^{-lk} \right) X[k] - \left(\sum_{m=1}^{M} a_m w^{-mk} \right) Y[k]$$

$$Y[k] \left(1 + \sum_{m=1}^{M} a_m w^{-mk} \right) = \left(\sum_{l=0}^{L} b_l w^{-lk} \right) X[k]$$

Funkcija prenosa preko DFT

$$H[k] = \frac{Y[k]}{X[k]} = \frac{\sum_{l=0}^{L} b_l w^{-lk}}{1 + \sum_{m=1}^{M} a_m w^{-mk}}$$

$$w = e^{j\frac{2\pi}{N}}$$

Funkcija prenosa preko DFT

$$H[k] = \frac{Y[k]}{X[k]} = \frac{\sum_{l=0}^{L} b_l e^{-j\frac{2\pi k}{N} l}}{1 + \sum_{m=1}^{M} a_m e^{-j\frac{2\pi k}{N} m}}$$

$$\Delta f = \frac{1}{N}$$

$$H[k] = \frac{Y[k]}{X[k]} = \frac{\sum_{l=0}^{L} b_l e^{-j 2\pi k \Delta f l}}{1 + \sum_{m=1}^{M} a_m e^{-j 2\pi k \Delta f m}}$$

Funkcija prenosa preko DFT

$$H[k] = \frac{Y[k]}{X[k]} = \frac{\sum_{l=0}^{L} b_l e^{-j2\pi k\Delta f l}}{1 + \sum_{m=1}^{M} a_m e^{-j2\pi k\Delta f m}}$$

Digitalna frekvencija

$$f[k] = k\Delta f$$

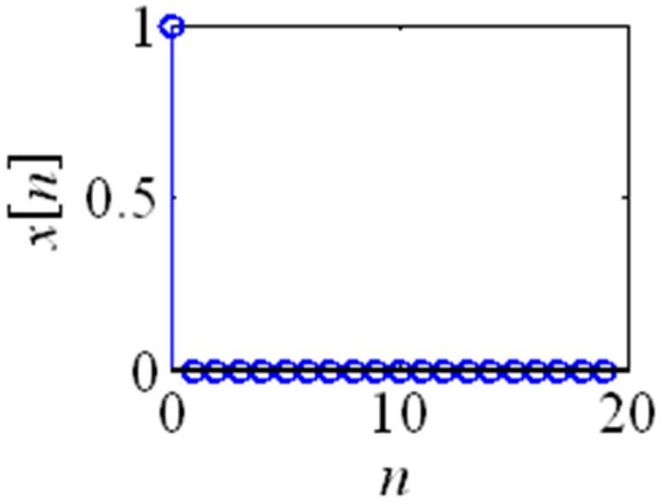
$$H[k] = \frac{Y[k]}{X[k]} = \frac{\sum_{l=0}^{L} b_l e^{-j2\pi f[k] l}}{1 + \sum_{m=1}^{M} a_m e^{-j2\pi f[k] m}}$$

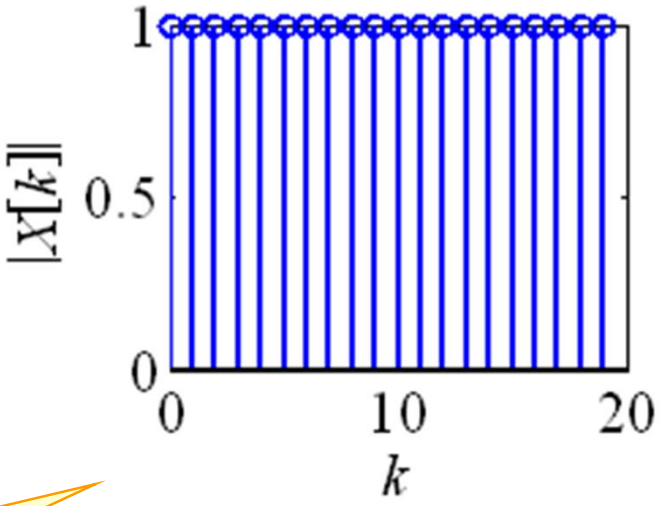
DFT - fft

- X = fft(x) returns
  discrete Fourier transform (DFT)
  of vector x
  computed with a
  fast Fourier transform (FFT) algorithm

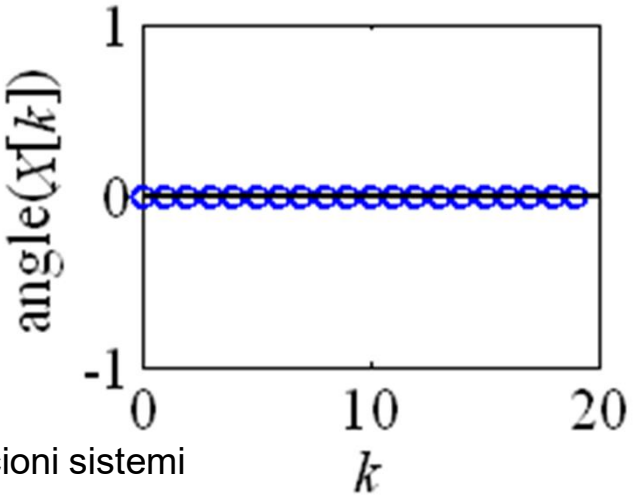
MATLAB: **doc fft**

DFT jedinične funkcije



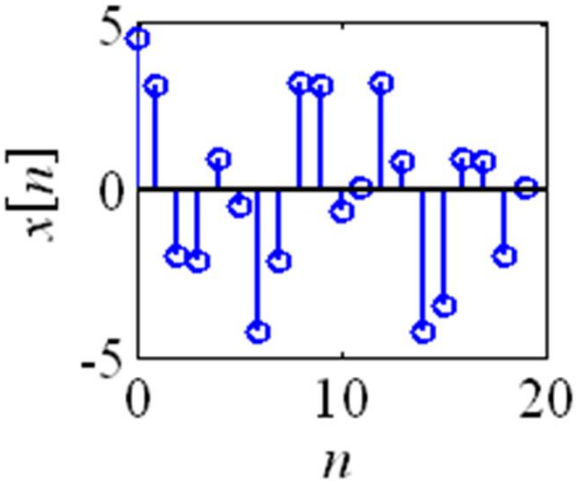


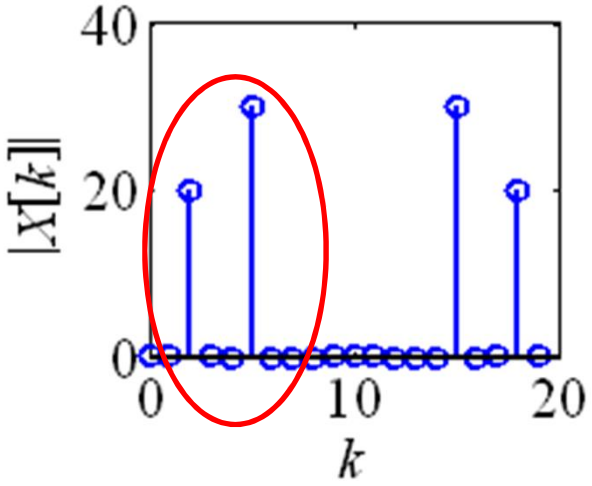
$$x[n]=\delta[n]$$
$$|X[k]|=1,\ \arg(X[k])=0$$



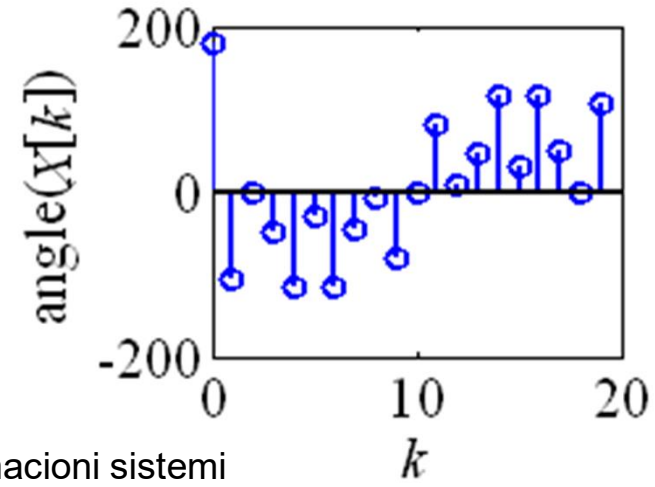

```
N=20; n=0:N-1;
x = (n==0);
X = fft(x)
```

FFT – dve sinusoide



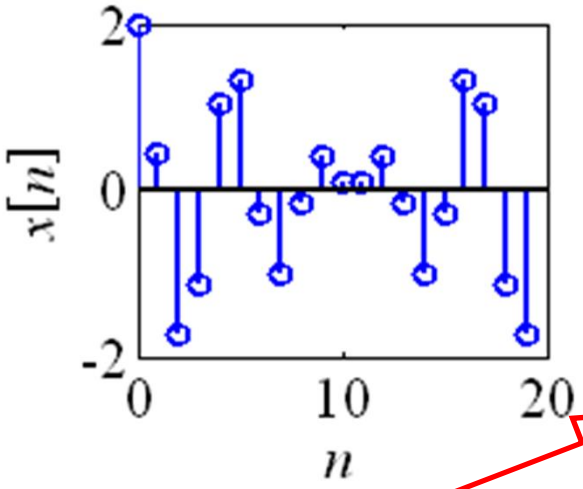


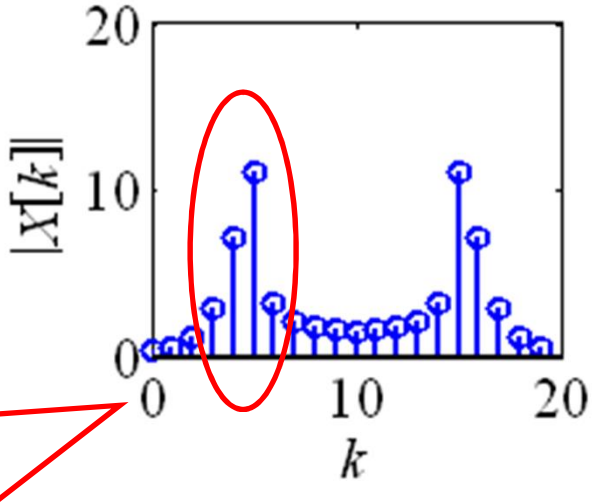

```
N=20;
n=0:N-1;
k1=2;     k2=5;
f1=k1/N;  f2=k2/N;
fi2=-pi/6;
x = 2*cos(2*pi*f1*n+0)+...
    3*cos(2*pi*f2*n+fi2);
X = fft(x)
```



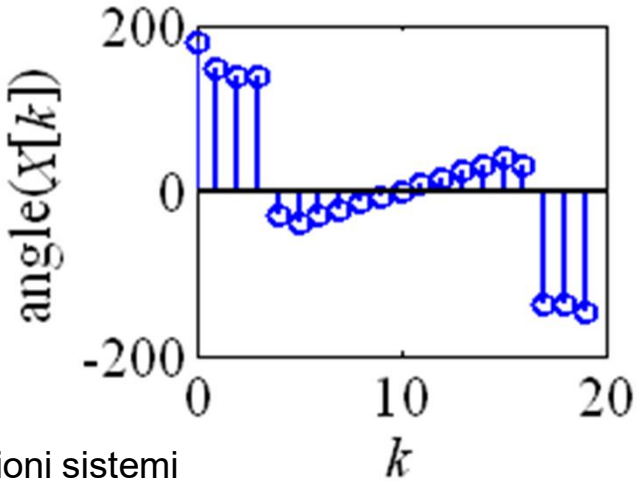
FFT – frekvencijska rezolucija

frequency resolution

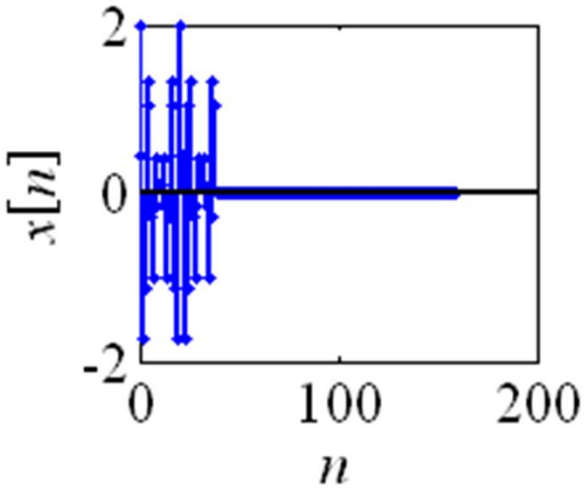


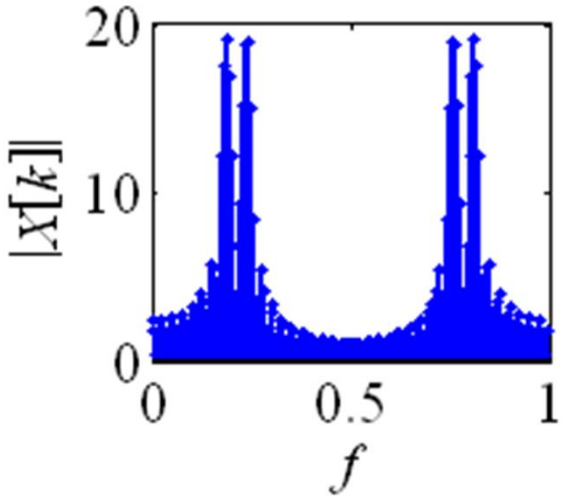


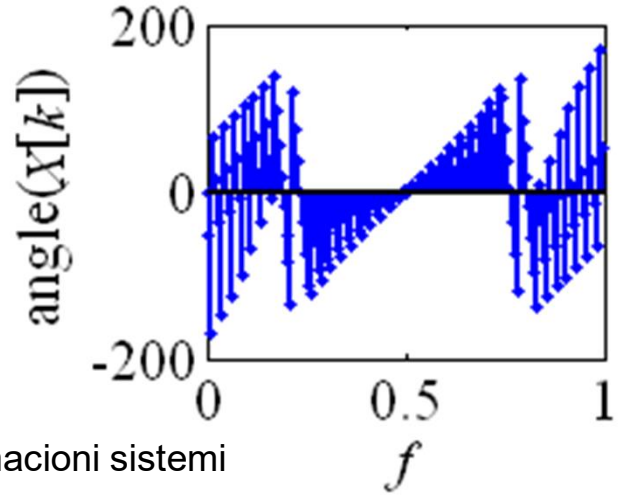

```
N = 20;
M = 21;
n = 0:N-1;
k1=4; f1=k1/M;
k2=5; f2=k2/M;
x=cos(2*pi*f1*n)+...
  cos(2*pi*f2*n);
X = fft(x)
```



FFT – dopuna nulama

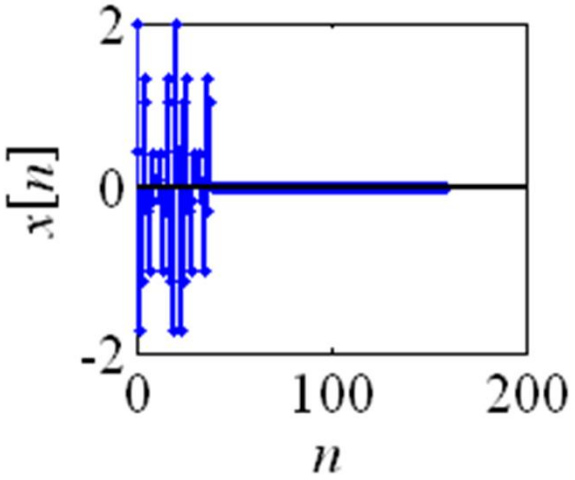


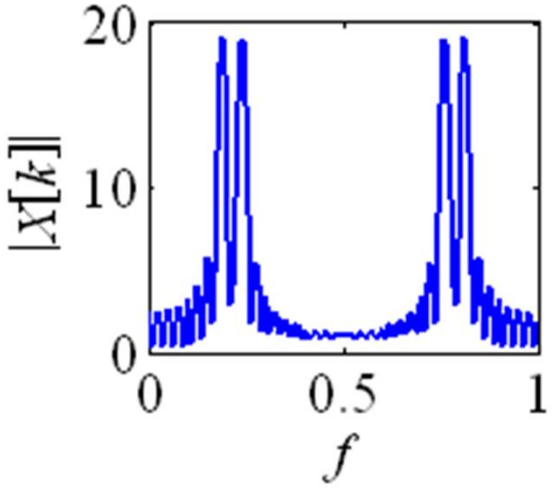


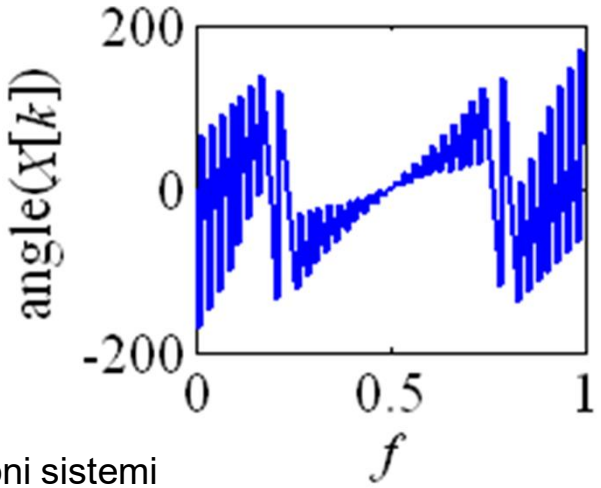



```
N = 160;
. . .
x = x.*(n<39);
```

FFT – dopuna nulama anvelopa DFT








```
plot(n/N,abs(y),'-')
. . .
plot(n/N,angle(y)*180/pi,'-')
```

Matlab `fft, ifft`

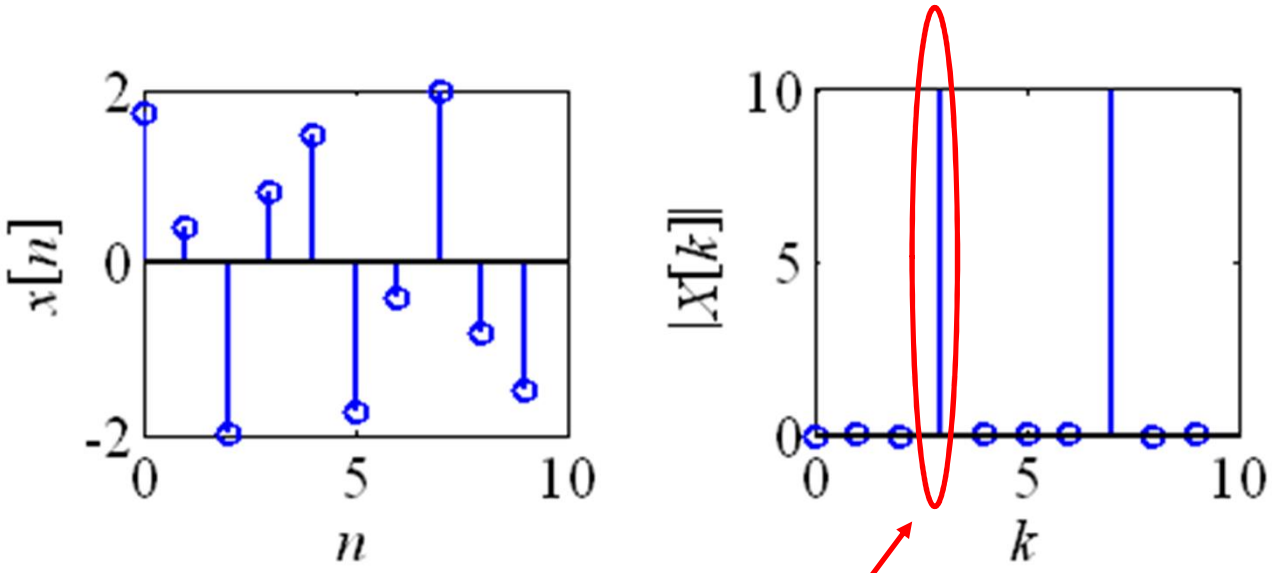
```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128]
X = fft(x,8+2)
y = ifft(X)
```

```
x =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

```
X =
  Columns 1 through 4
   1.9922  1.1861-0.6487i 0.7969-0.3984i 0.6889-0.1799i
  Columns 5 through 8
   0.6641  0.6889+0.1799i 0.7969+0.3984i 1.1861+0.6487i
```

```
y =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078 0 0
```

FFT




```
N = 10;
n = 0:N-1;
k = 3;
f = k/N;
fi = -pi/6;
x = 2*cos(2*pi*f*n+fi);
stem(n,x)
X = fft(x)
stem(n,abs(X))
```

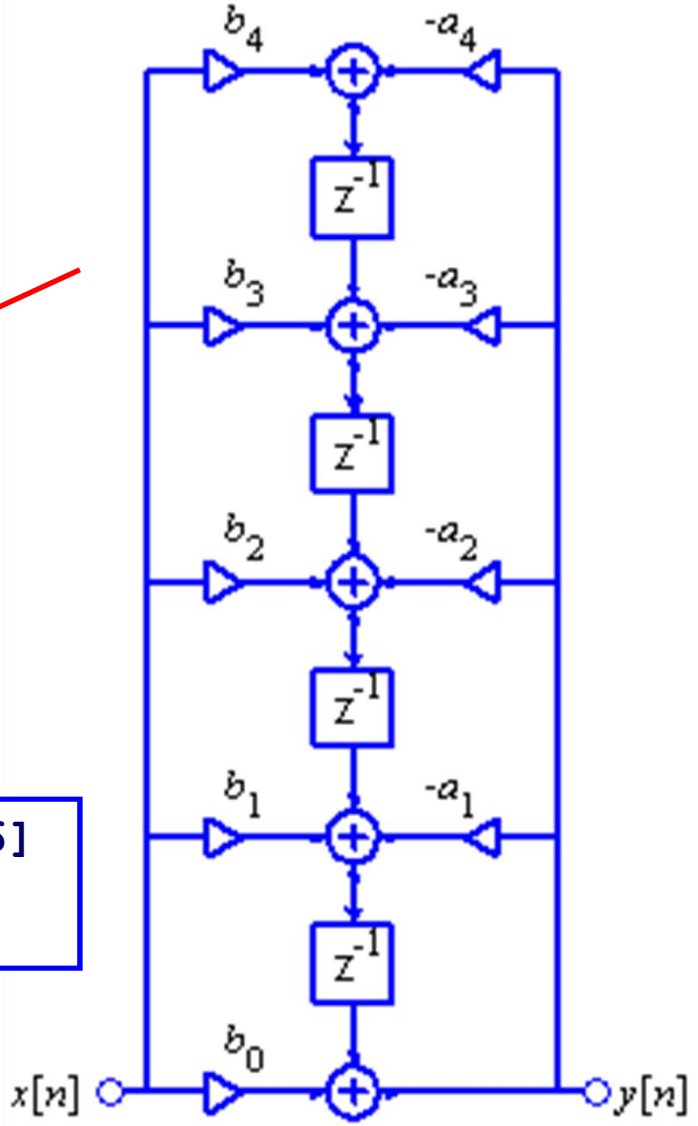
Projektovanje sistema

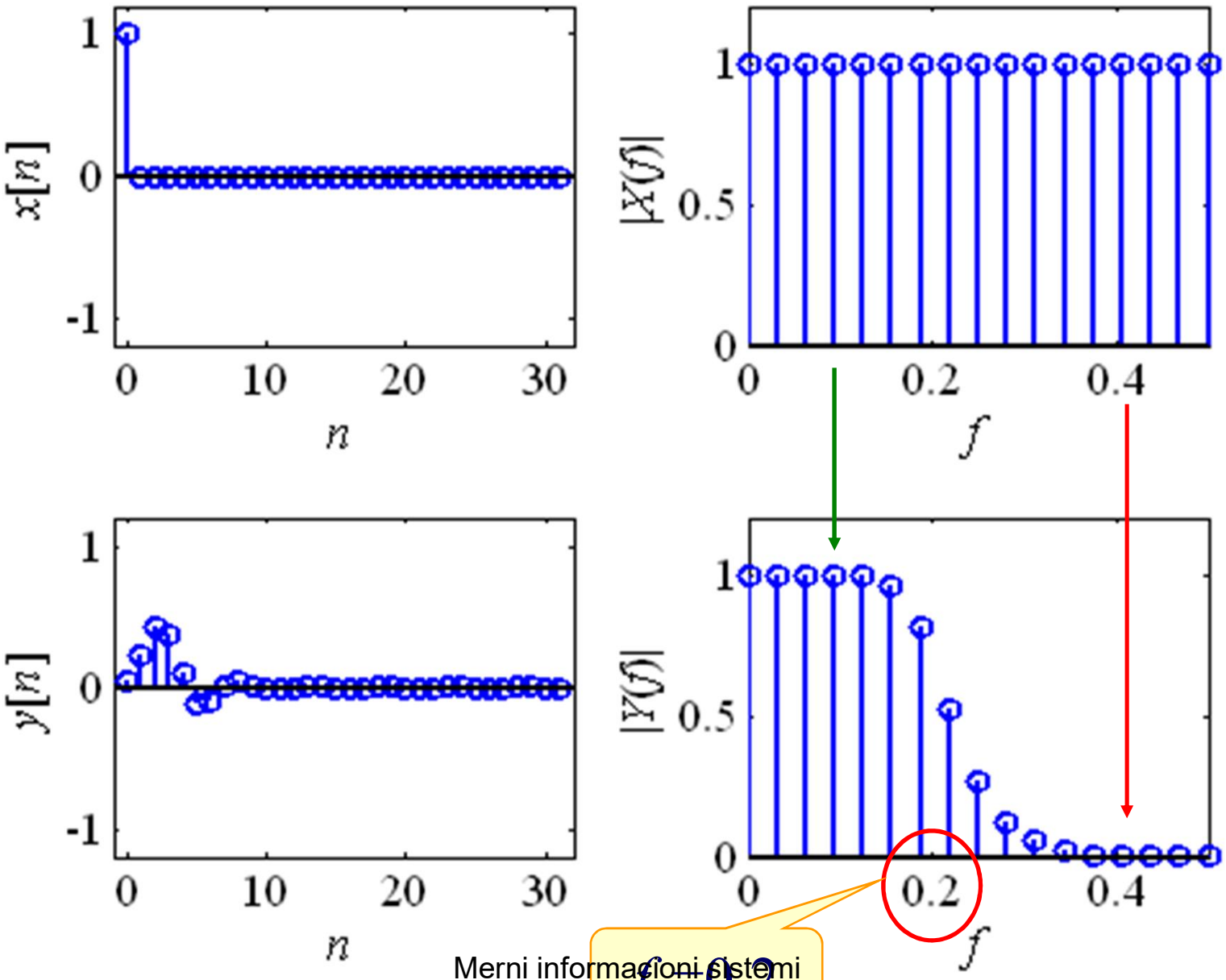
Izračunavanje koeficijenata sistema da zadovolje zadate specifikacije

```
b = [b0 b1 b2 b3 b4]

a = [1 a1 a2 a3 a4]
```

```
order = 4;
fn = 0.2;
[b,a] = butter(order,2*fn)
```

```
b = [0.0466 0.1863 0.2795 0.1863 0.0466]

a = [1 -0.7821 0.6800 -0.1827 0.0301]
```

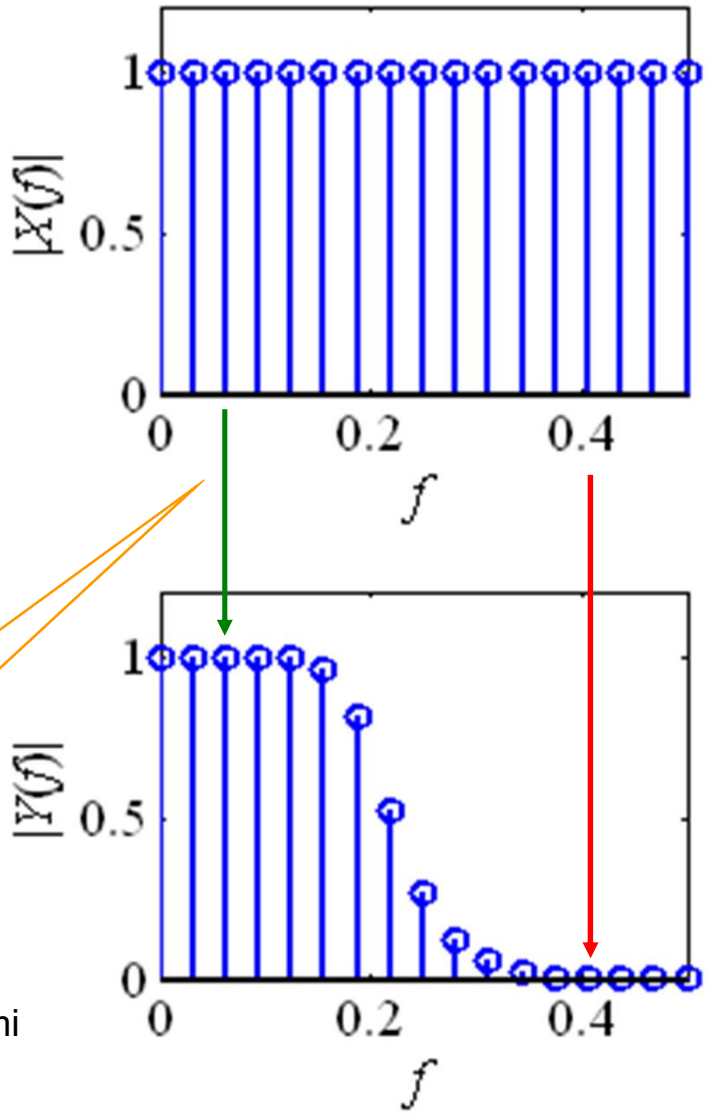



1
0
-1
x[n]
0
10
20
30
n
|X(f)|
1
0.5
0
0
0.2
0.4
f
y[n]
|Y(f)|
$f_n = 0.2$

Impulsna pobuda – frekvencijska karakteristika

Na osnovu odziva na impulsnu pobudu možemo da zaključimo da li će sinusoidalan signal da prođe kroz filtar

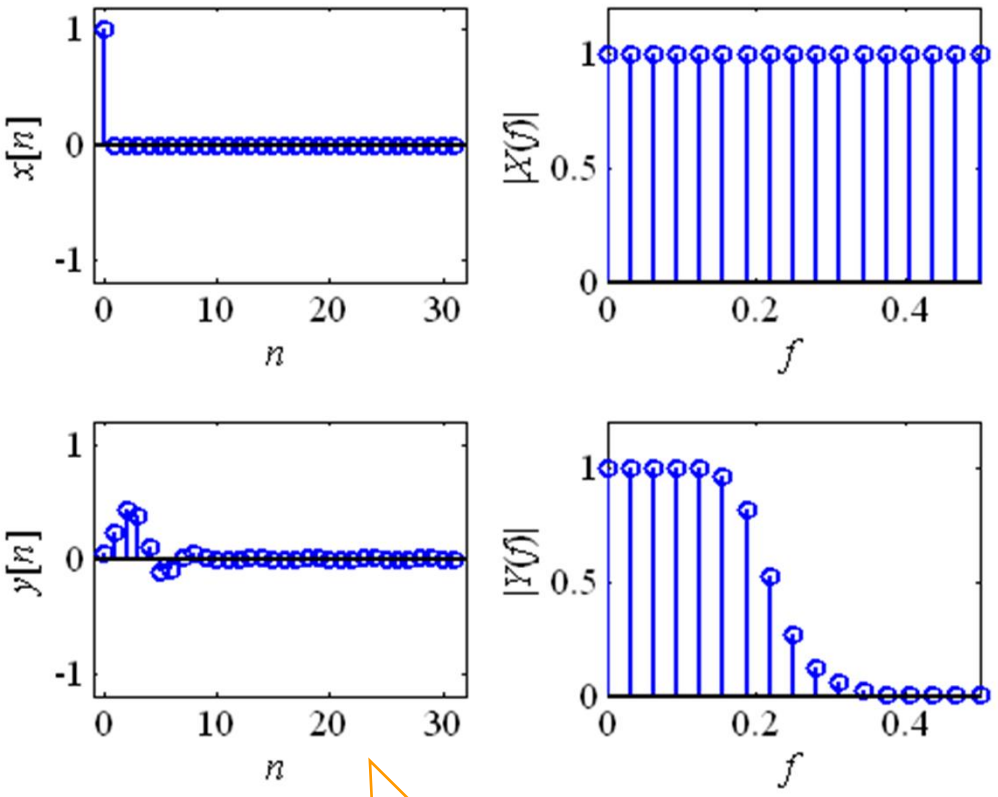
- bez slabljenje
- oslabljen
- pojačan



Bez slabljenja

Procesiranje sistemom

Procesiranje sistemom realizuje se funkcijom **`filter`**




```
N = 32;
n = 0:N-1;
x = (n==0);
y = filter(b,a,x);
X = fft(x);
Y = fft(y);
stem(n,y)
stem(n/N,abs(Y))
```

subplot(2,2,3)

DTFT

$$H[k] = \frac{Y[k]}{X[k]} = \frac{\sum_{l=0}^{L} b_l e^{-j2\pi f[k] l}}{1 + \sum_{m=1}^{M} a_m e^{-j2\pi f[k] m}}$$

Kontinualna frekvencija

$$f[k] \rightarrow f$$

$$[.] \rightarrow (.)$$

$$H(f) = \frac{\sum_{l=0}^{L} b_l e^{-j2\pi f l}}{1 + \sum_{m=1}^{M} a_m e^{-j2\pi f m}}$$

DTFT

$$H(f) = \frac{\sum_{l=0}^{L} b_l e^{-j2\pi f l}}{1 + \sum_{m=1}^{M} a_m e^{-j2\pi f m}}$$

ugaona učestanost

$$2\pi f \rightarrow \omega$$

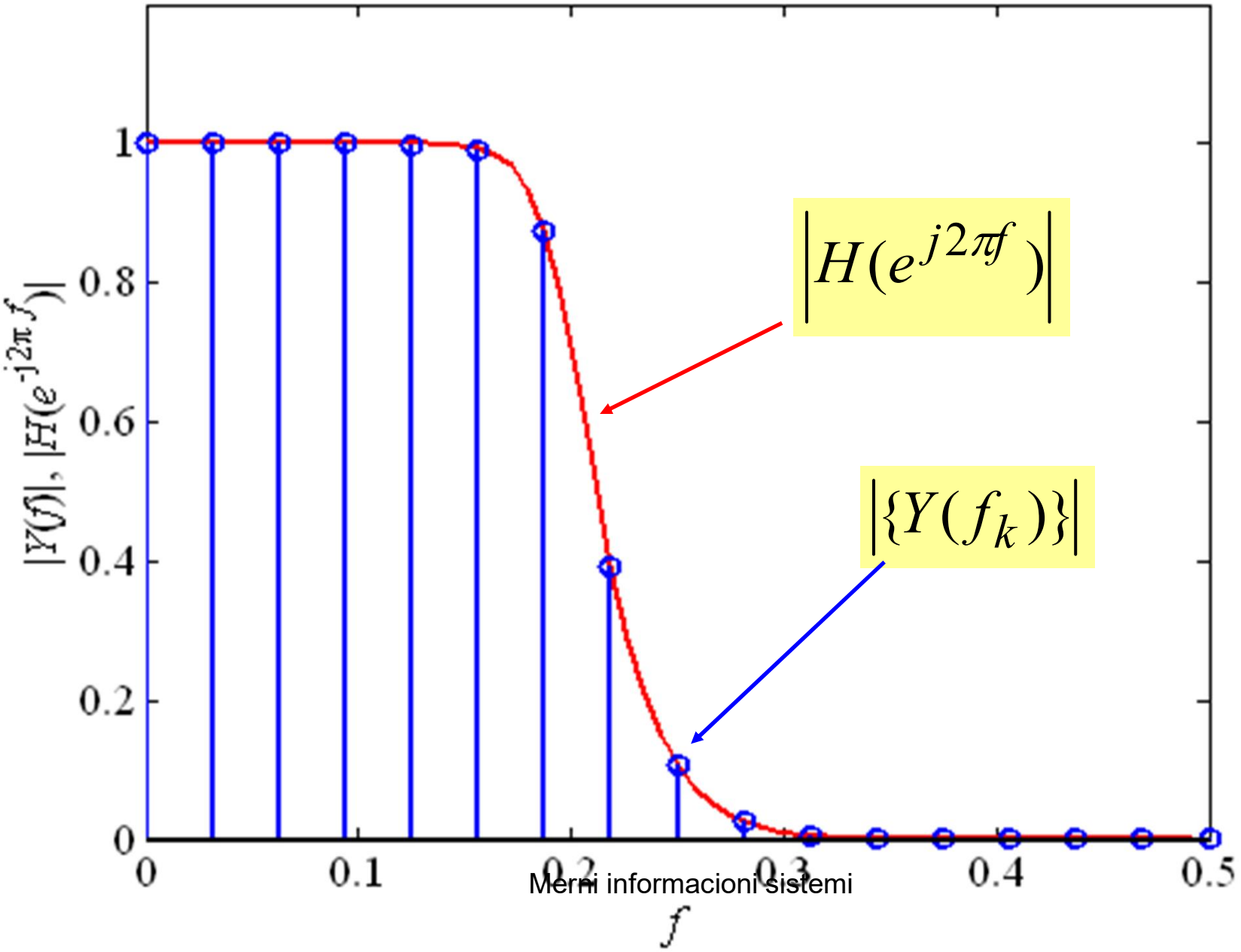
Frekvencijski odziv

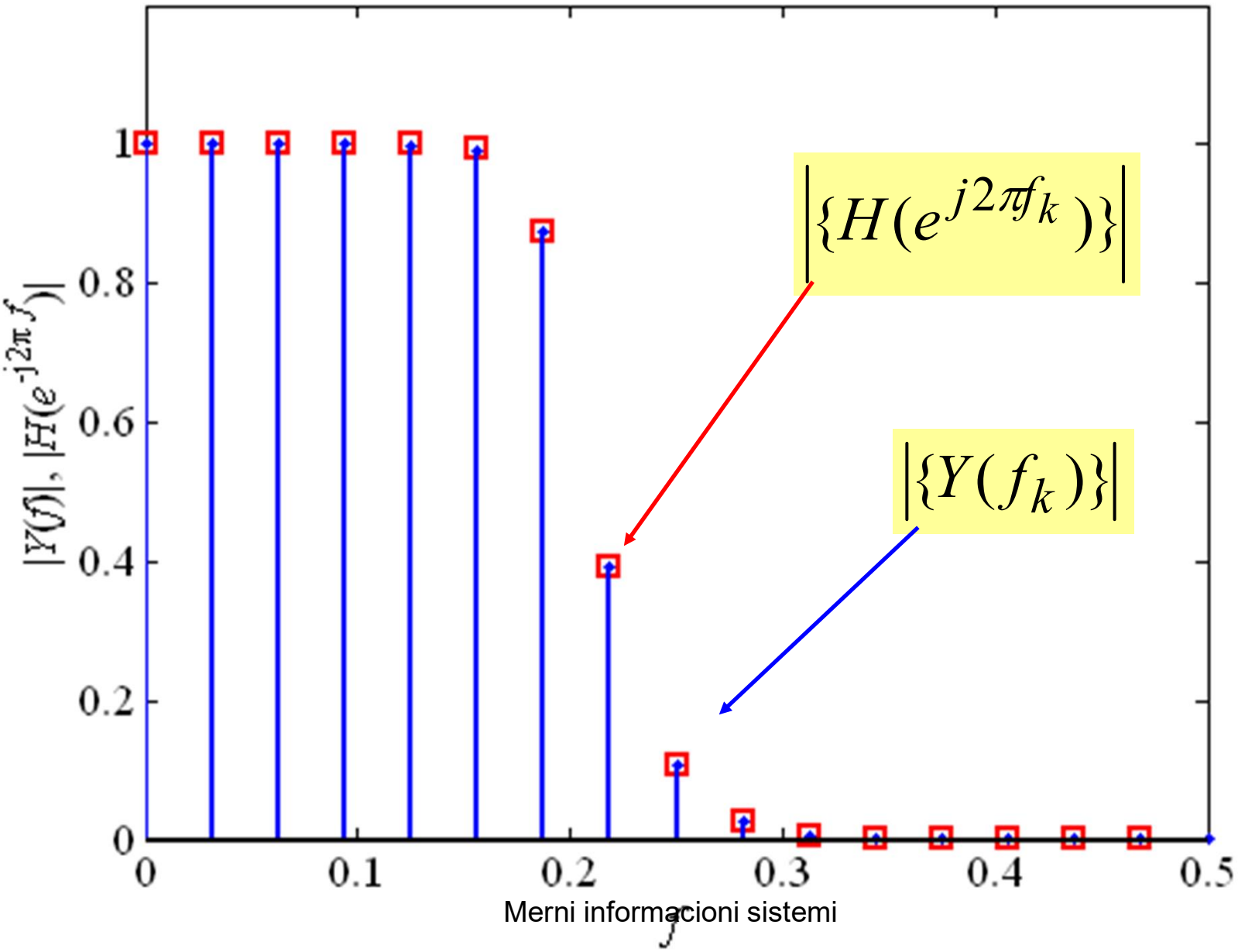
$$H(e^{j\omega}) = \frac{\sum_{l=0}^{L} b_l e^{-j\omega l}}{1 + \sum_{m=1}^{M} a_m e^{-j\omega m}}$$

DTFT - freqz

MATLAB: **doc freqz**

- [h,w] = freqz(b,a,l) returns the frequency response vector h and the corresponding angular frequency vector w for the digital filter whose transfer function is determined by the (real or complex) numerator and denominator polynomials represented in the vectors b and a, respectively
- The vectors h and w are both of length l
- The angular frequency vector w has values ranging from 0 to radians per sample
- When you don't specify the integer l, or you specify it as the empty vector [], the frequency response is calculated using the default value of 512 samples

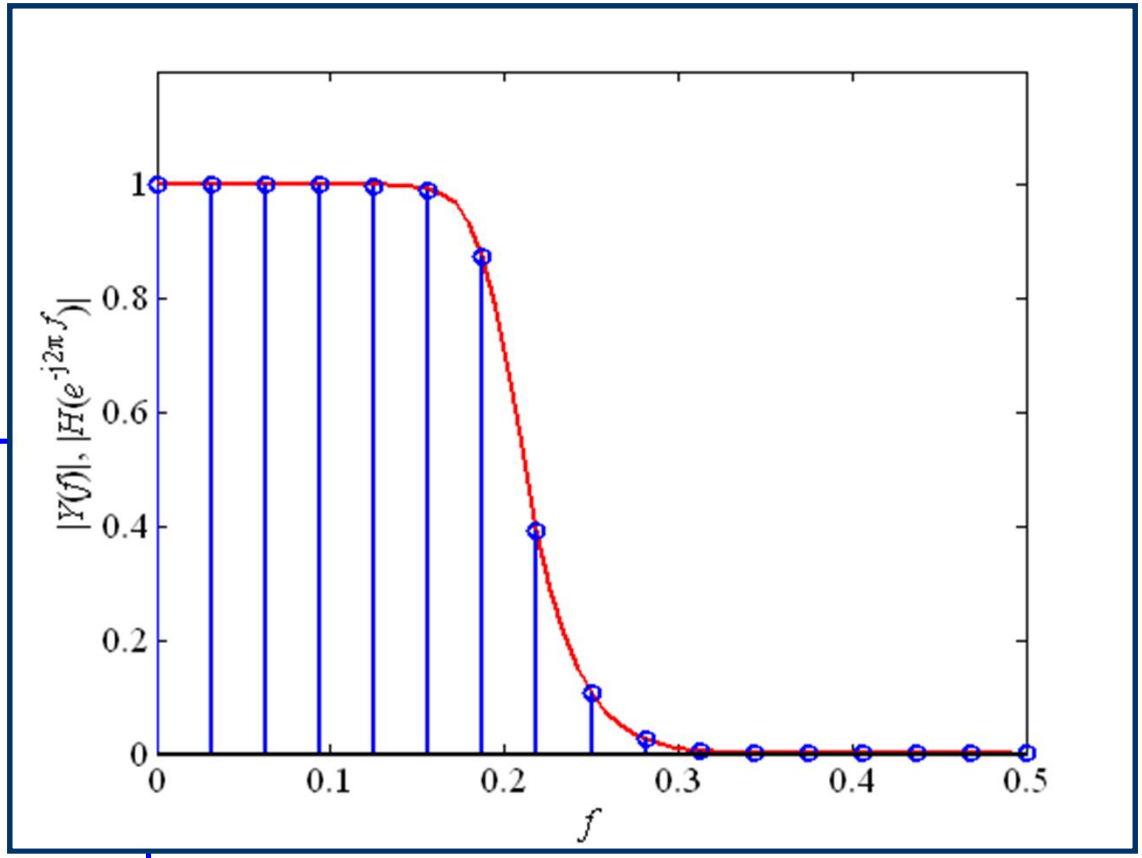

$|Y(f)|, |H(e^{-j2\pi f})|$
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
f
$\left|H(e^{j2\pi f})\right|$
$\left|\{Y(f_k)\}\right|$


$\left|\{H(e^{j2\pi f_k})\}\right|$
$\left|\{Y(f_k)\}\right|$
$|Y(f)|, |H(e^{-j2\pi f})|$
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
f

Frekvencijska karakteristika sistema

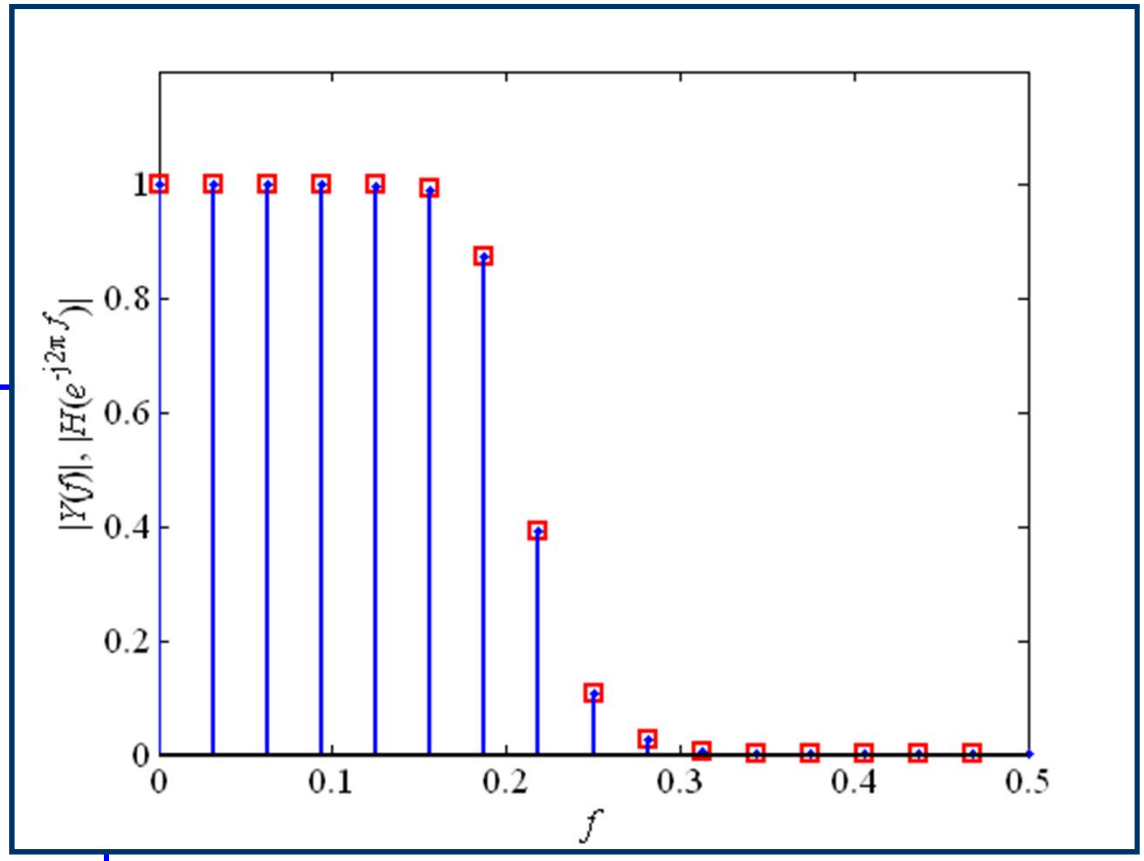
Frekvencijska karakteristika računa se funkcijom **`freqz`**

```
fn = 0.2;
[b,a] = butter(7,2*fn)
N = 32;
n = 0:N-1;
x = (n==0);
y = filter(b,a,x);
Y = fft(y);
[h,w] = freqz(b,a,2*N);
plot(w/(2*pi),abs(h),'r-')
hold on
 stem(n/N,abs(Y))
hold off
```



freqz i fft

```
fn = 0.2;
[b,a] = butter(7,2*fn)
N = 32;
n = 0:N-1;
x = (n==0);
y = filter(b,a,x);
Y = fft(y);
[h,w] = freqz(b,a,N/2);
plot(w/(2*pi),abs(h),'rs')
hold on
 stem(n/N,abs(Y))
hold off
```

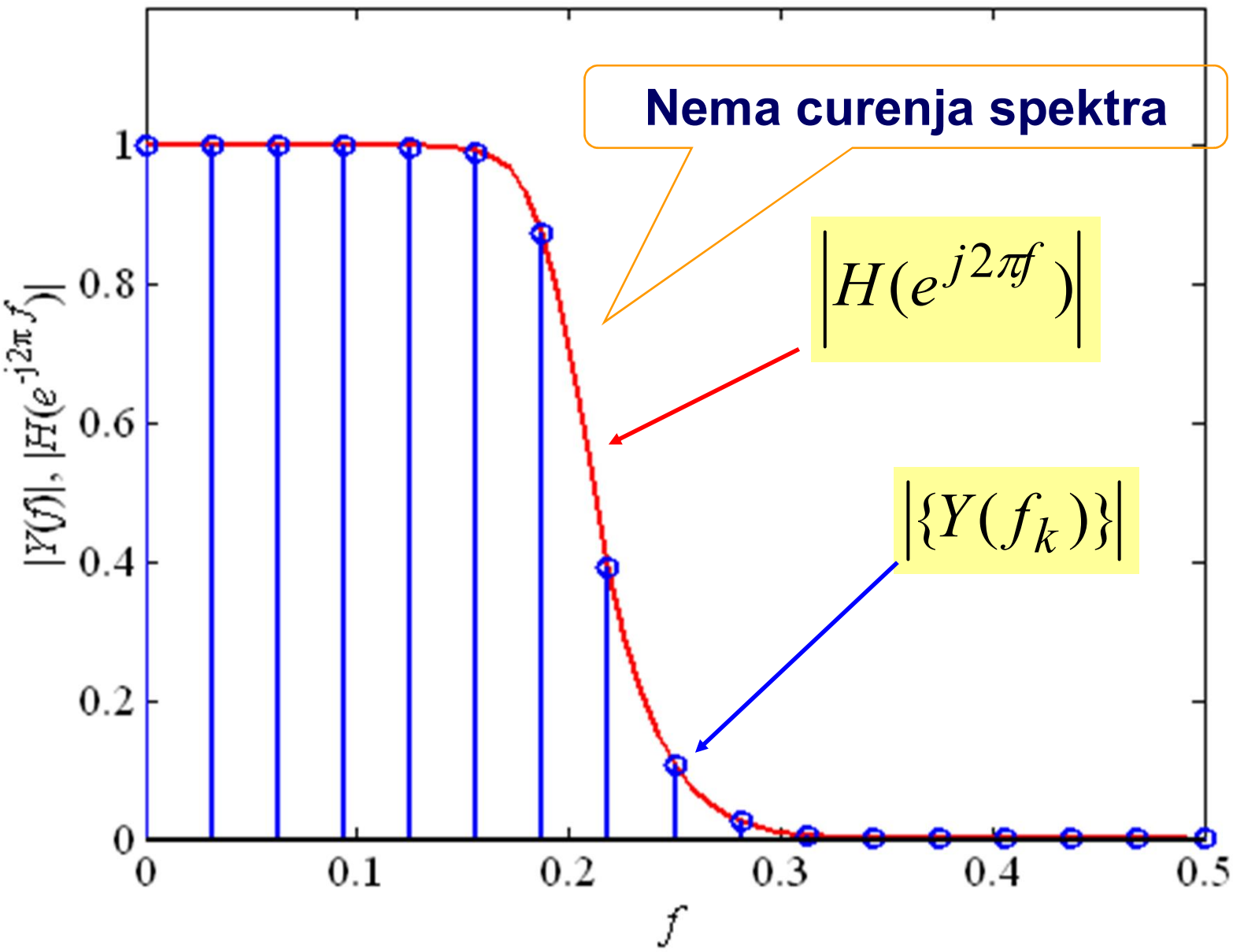


`freqz`

Da bi odredili
šta će da se dogodi sa
sinusoidalnim signalom na ulazu u filtar,
nije potrebno da procesiramo taj
signal sa filtrom, već
da nacrtamo **kontinualnu** karakteristiku
sa **`freqz`** koja
zavisi samo od koeficijenata filtra

`fft`

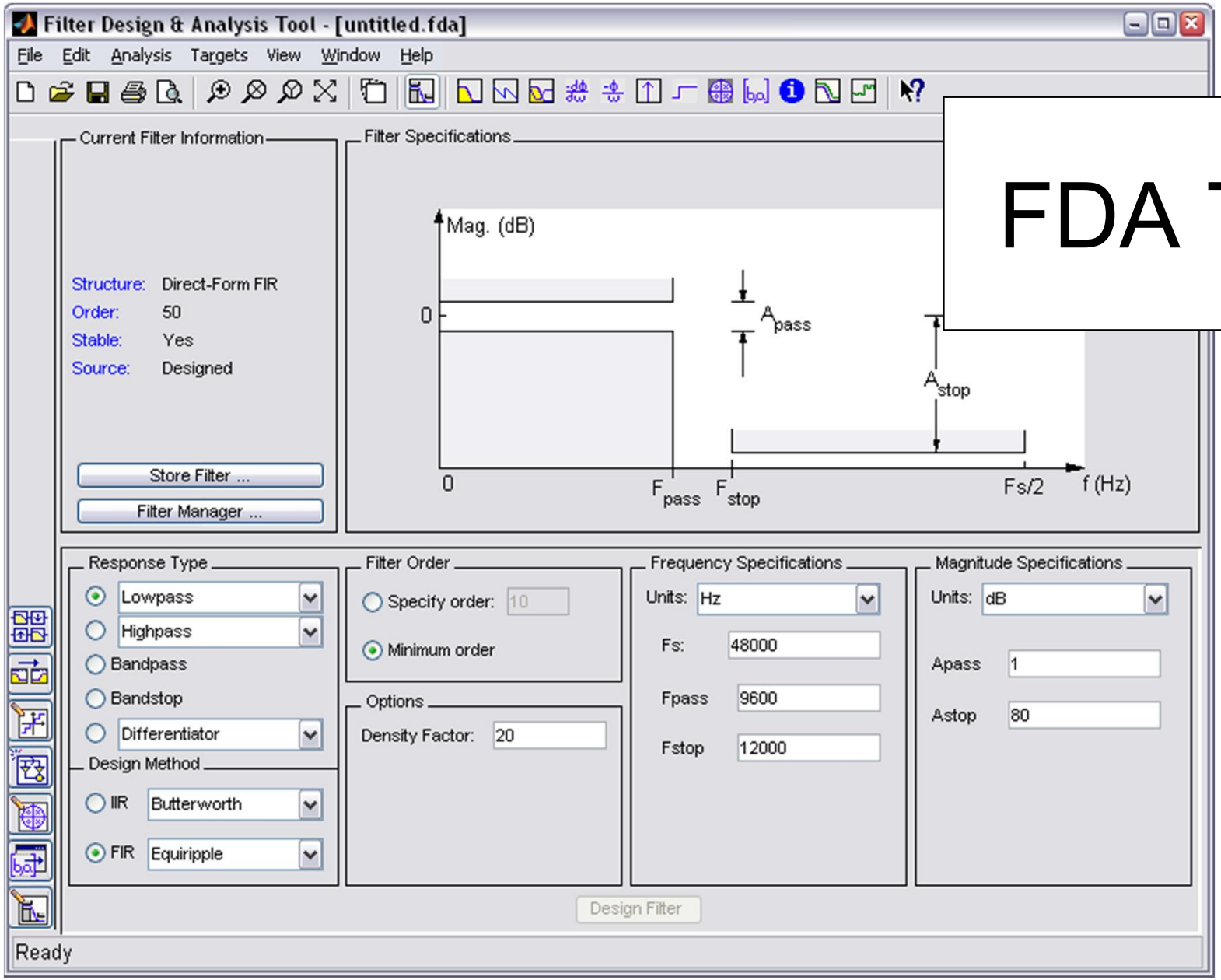
Da bi odredili
šta će se dogoditi sa
sinusoidalnim signalom na ulazu u filtar,
možemo da procesiramo taj
signal sa filtrom (funkcija **`filter`**),
i da nacrtamo **diskretni** spektar
sa **`fft`** koji tada
zavisi od koeficijenata filtra i dužine niza
(i od učestanosti i amplitude signala)


Nema curenja spektra
$\left|H(e^{j2\pi f})\right|$
$\left|\{Y(f_k)\}\right|$
$|Y(f)|, |H(e^{-j2\pi f})|$
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
f

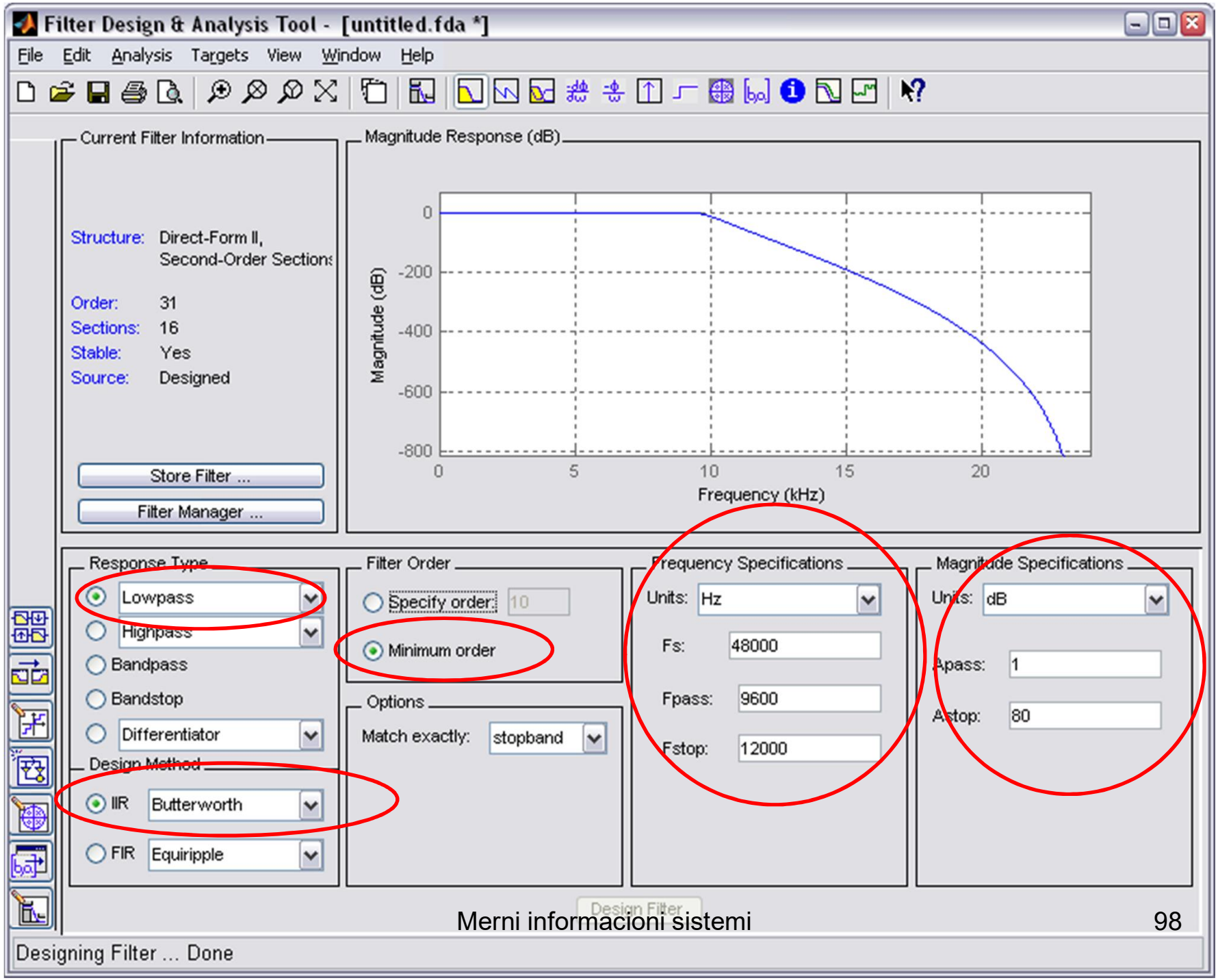
Analiza filtara

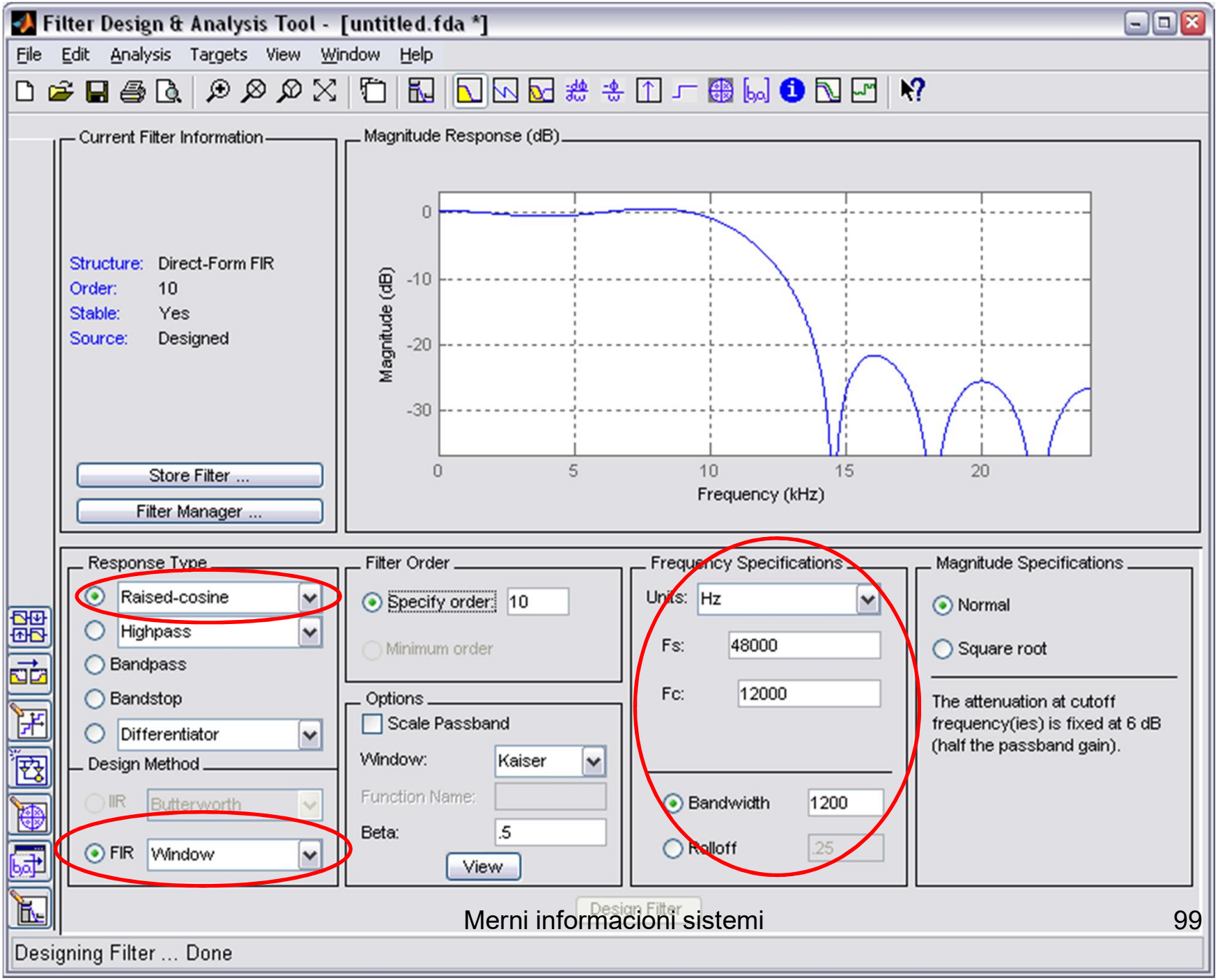
Filter analysis

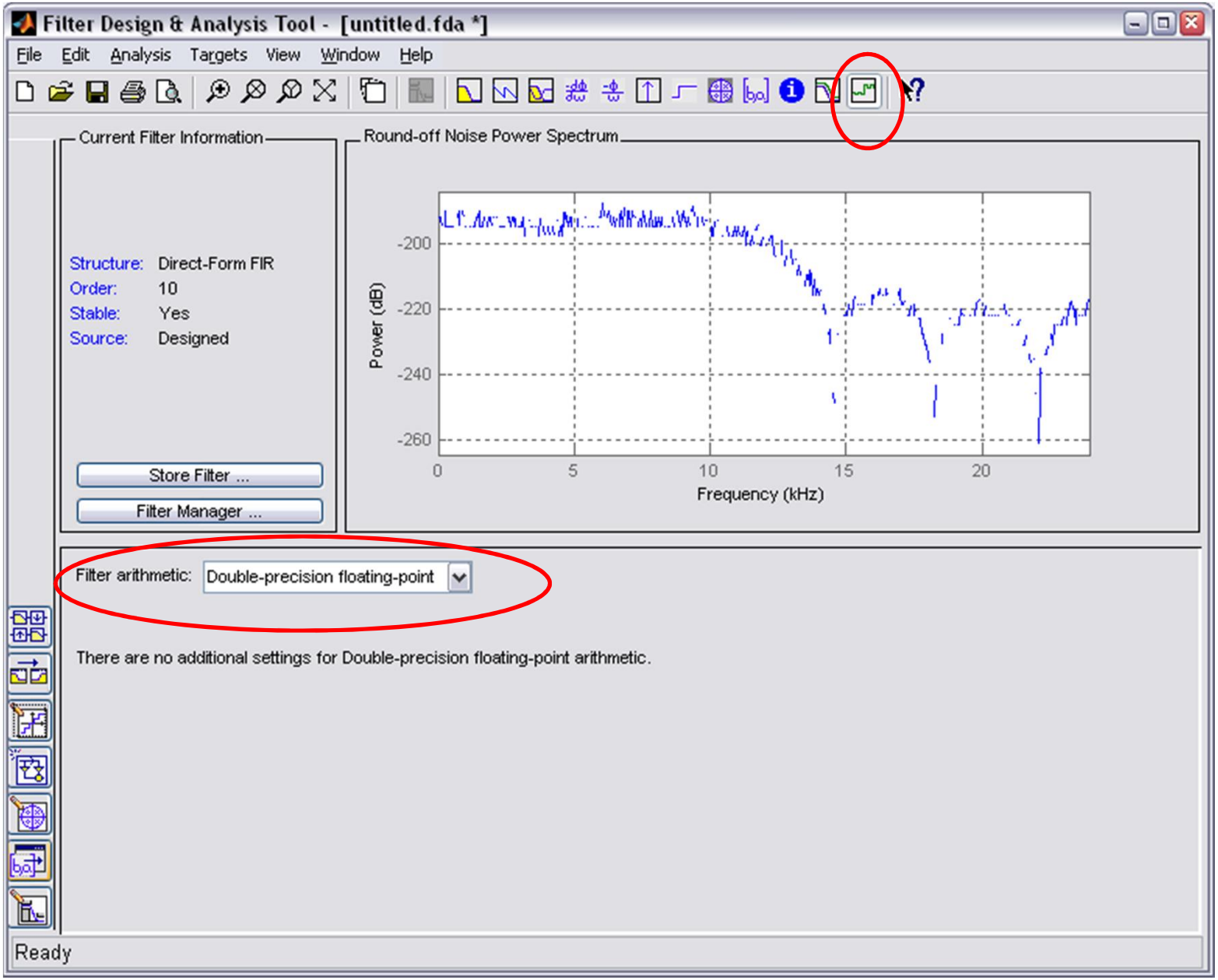
abs - Magnitude
angle - Phase angle
filternorm - Compute the 2-norm or inf-norm of digital filter
freqz - Z-transform frequency response
fvtool - Filter Visualization Tool
grpdelay - Group delay
impz - Discrete impulse response
phasedelay - Phase delay of a digital filter
phasez - Digital filter phase response (unwrapped)
zerophase - Zero-phase response of a real filter
zplane - Discrete pole-zero plot

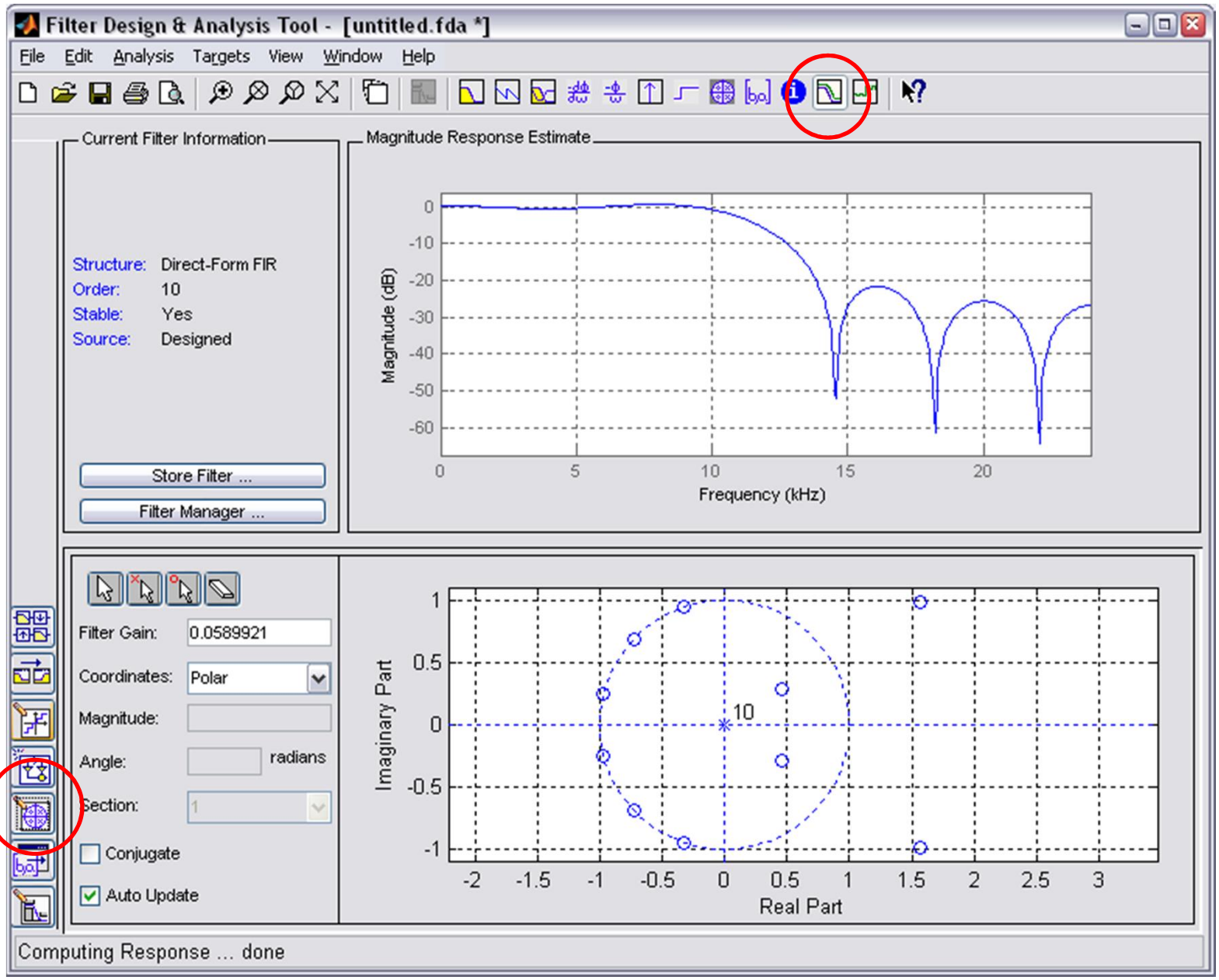

Filter Design & Analysis Tool - [untitled.fda]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 50
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Filter Specifications
Mag. (dB)
0
Apass
Astop
0
Fpass
Fstop
Fs/2
f (Hz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR
Butterworth
FIR
Equiripple
Filter Order
Specify order: 10
Minimum order
Options
Density Factor: 20
Frequency Specifications
Units: Hz
Fs: 48000
Fpass 9600
Fstop 12000
Magnitude Specifications
Units: dB
Apass 1
Astop 80
Design Filter
Ready

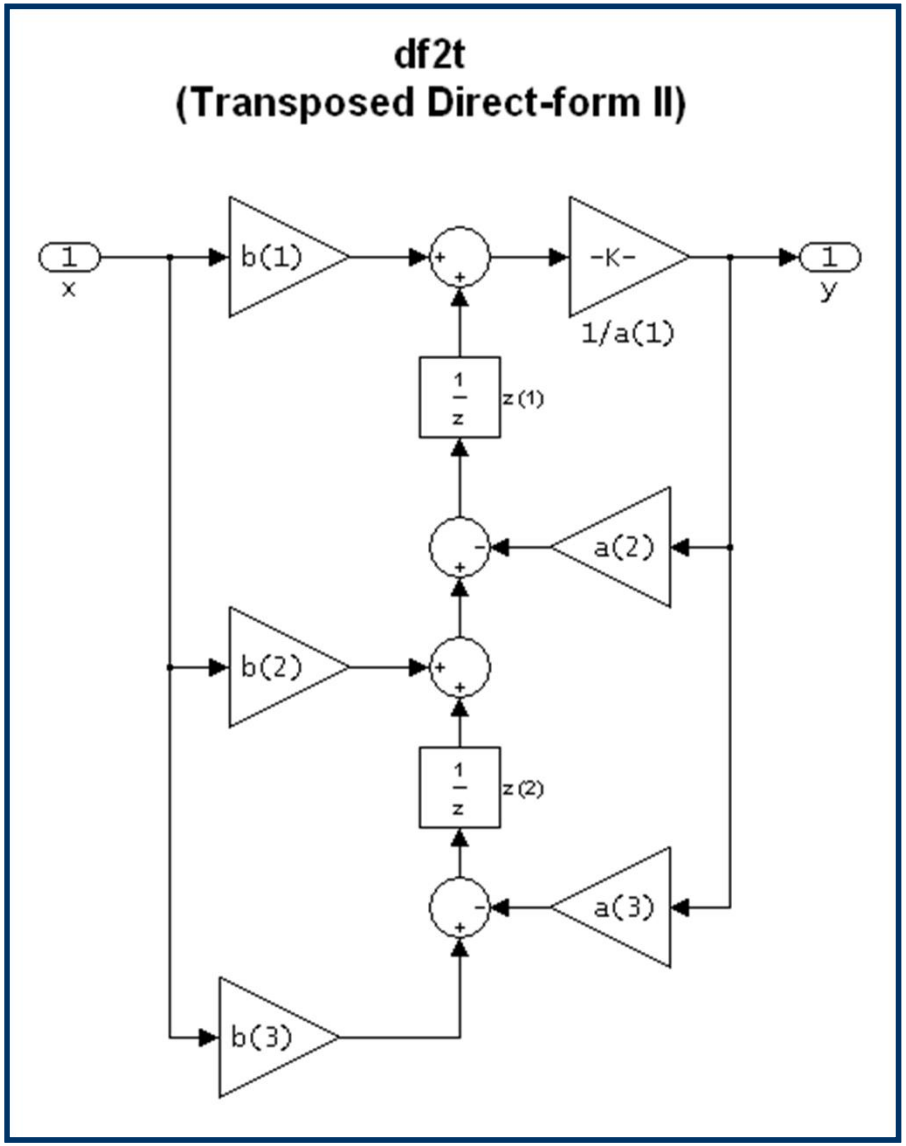
FDA Tool


Filter Design & Analysis Tool - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form II, Second-Order Sections
Order: 31
Sections: 16
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
Magnitude (dB)
0
-200
-400
-600
-800
0
5
10
15
20
Frequency (kHz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR
Butterworth
FIR
Equiripple
Filter Order
Specify order:
10
Minimum order
Options
Match exactly:
stopband
Frequency Specifications
Units:
Hz
Fs:
48000
Fpass:
9600
Fstop:
12000
Magnitude Specifications
Units:
dB
Apass:
1
Astop:
80
Design Filter
Designing Filter ... Done


Filter Design & Analysis Tool - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 10
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
Magnitude (dB)
0
-10
-20
-30
0
5
10
15
20
Frequency (kHz)
Response Type
Raised-cosine
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR
Butterworth
FIR
Window
Filter Order
Specify order:
10
Minimum order
Options
Scale Passband
Window:
Kaiser
Function Name:
Beta:
.5
View
Frequency Specifications
Units:
Hz
Fs:
48000
Fc:
12000
Bandwidth
1200
Rolloff
.25
Magnitude Specifications
Normal
Square root
The attenuation at cutoff frequency(ies) is fixed at 6 dB (half the passband gain).
Design Filter
Designing Filter ... Done


Filter Design & Analysis Tool - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 10
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Round-off Noise Power Spectrum
Power (dB)
-200
-220
-240
-260
0
5
10
15
20
Frequency (kHz)
Filter arithmetic: Double-precision floating-point
There are no additional settings for Double-precision floating-point arithmetic.
Ready

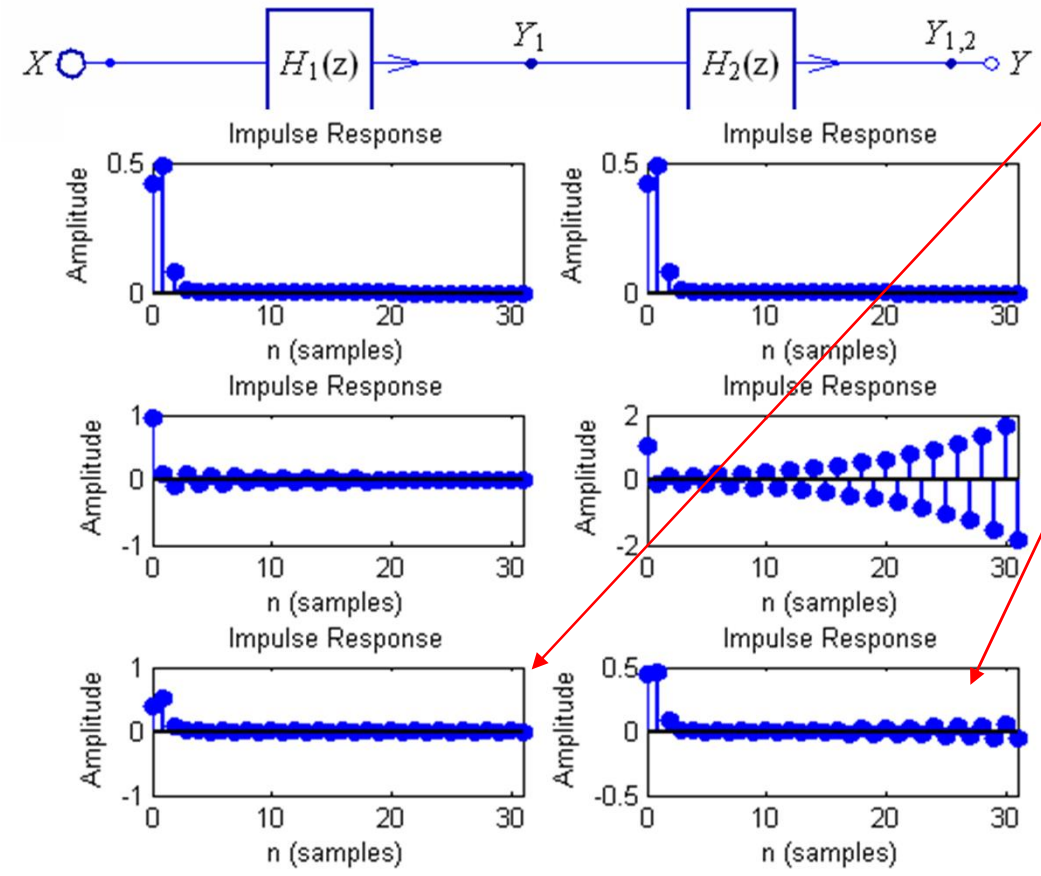

Filter Design & Analysis Tool - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 10
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response Estimate
Magnitude (dB)
Frequency (kHz)
Filter Gain: 0.0589921
Coordinates: Polar
Magnitude:
Angle: radians
Section: 1
Conjugate
Auto Update
Imaginary Part
Real Part
Computing Response ... done



Implementacija

Hd = dfilt.df2t(b,a)

Hd = dfilt.df2t




```
a1= 1. -0.1584
a2= 1.  0.9000
a3= 1.  0.7416 -0.1426
a4= 1.  1.1000
a5= 1.  0.9416 -0.1742
```

```
subplot(3,2,1)
impz(b1,a1,N)
subplot(3,2,3)
impz(b2,a2,N)
subplot(3,2,5)
impz(b3,a3,N)
subplot(3,2,2)
impz(b1,a1,N)
subplot(3,2,4)
impz(b4,a4,N)
subplot(3,2,6)
impz(b5,a5,N)
```

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