DBBT
Digital Broadcasting & Broadband Technologies

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Procesiranje signala

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- Diskretni analitički signal
- Diskretni Hilbertov transformator
- Digitalne banke filtara
- QMF banke
- Višestepene filtarske banke

Diskretni analitički signal

- **Diskretni realni signal** ima simetričan amplitudski spektar u odnosu na učestanost 0
- **Diskretni analitički signal** nema simetričan spektar; sve spektralne komponente imaju vrednost nula na negativnim učestanostima
- U vremenskom domenu, **odbirci analitičkog signala imaju kompleksne vrednosti** (ne mogu da budu samo realni)
- Primena u digitalnim komunikacionim sistemima (SSB u FDM sistemima)

Spektar analitičkog signala (1)

$$y[n] = u[n] + j\,\hat{u}[n]$$

Signali u i $\hat{u}$ su realni

$$y[n] \rightarrow Y(e^{j\omega})$$

$$u[n] \rightarrow U(e^{j\omega})$$

$$\hat{u}[n] \rightarrow \hat{U}(e^{j\omega})$$

Furijeove transformacije

$$Y(e^{j\omega}) = U(e^{j\omega}) + j\,\hat{U}(e^{j\omega})$$

Spektar analitičkog signala (2)

$$U(e^{j\omega}) = U^*(e^{-j\omega})$$

$$\hat{U}(e^{j\omega}) = \hat{U}^*(e^{-j\omega})$$

Furijeove transformacije su konjugovano simetrične

$$U(e^{j\omega}) = \frac{1}{2}\left(Y(e^{j\omega}) + Y^*(e^{-j\omega})\right)$$

U ima kompleksne komponente

$$j\hat{U}(e^{j\omega}) = \frac{1}{2}\left(Y(e^{j\omega}) - Y^*(e^{-j\omega})\right)$$

$\hat{U}$ ima kompleksne komponente

Spektar analitičkog signala (3)

Predpostavka za analitički signal

$$Y(e^{j\omega}) = 0, \quad -\pi \le \omega < 0$$

$$Y(e^{j\omega}) = \begin{cases} 2U(e^{j\omega}) & 0 \le \omega < \pi \\ 0 & -\pi \le \omega < 0 \end{cases}$$

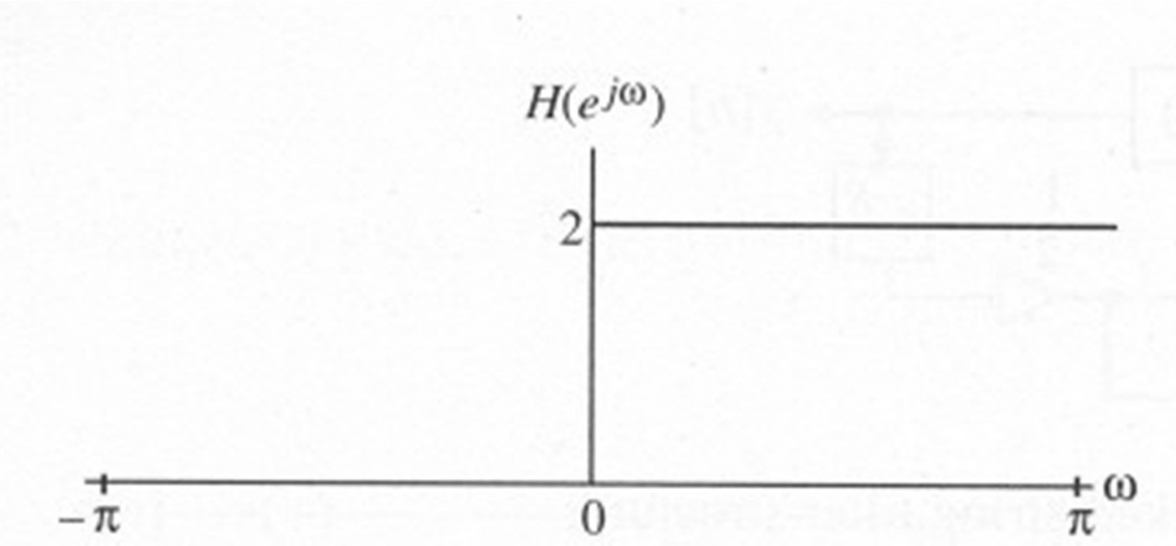
H je filtar kroz koji treba da se propusti signal u

$$H(e^{j\omega}) = \begin{cases} 2 & 0 \le \omega < \pi \\ 0 & -\pi \le \omega < 0 \end{cases}$$

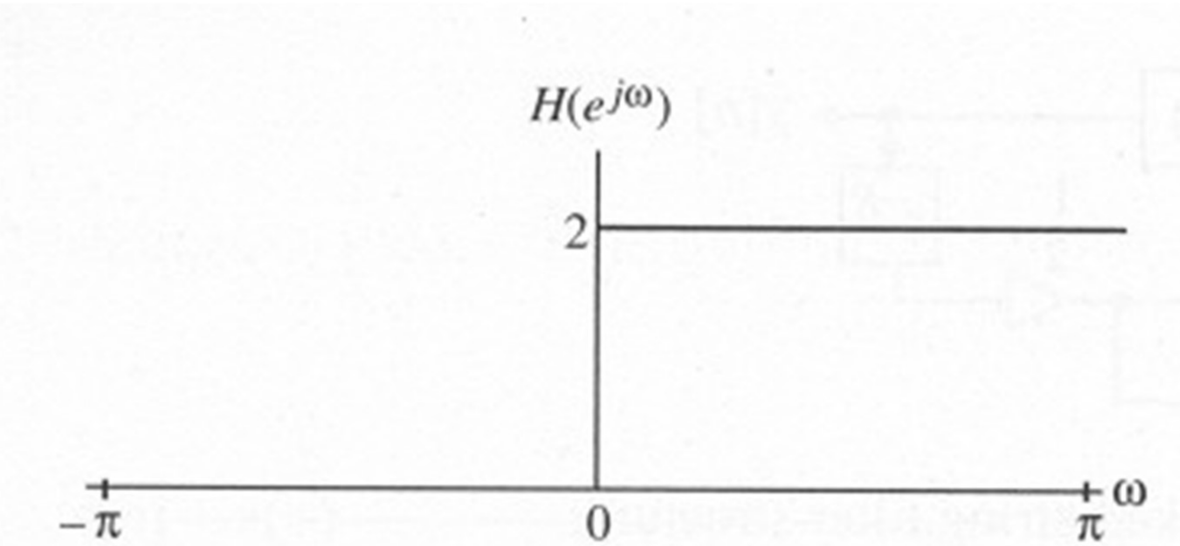
Spektar analitičkog signala (4)

$$H(e^{j\omega}) = \begin{cases} 2 & 0 \le \omega < \pi \\ 0 & -\pi \le \omega < 0 \end{cases}$$

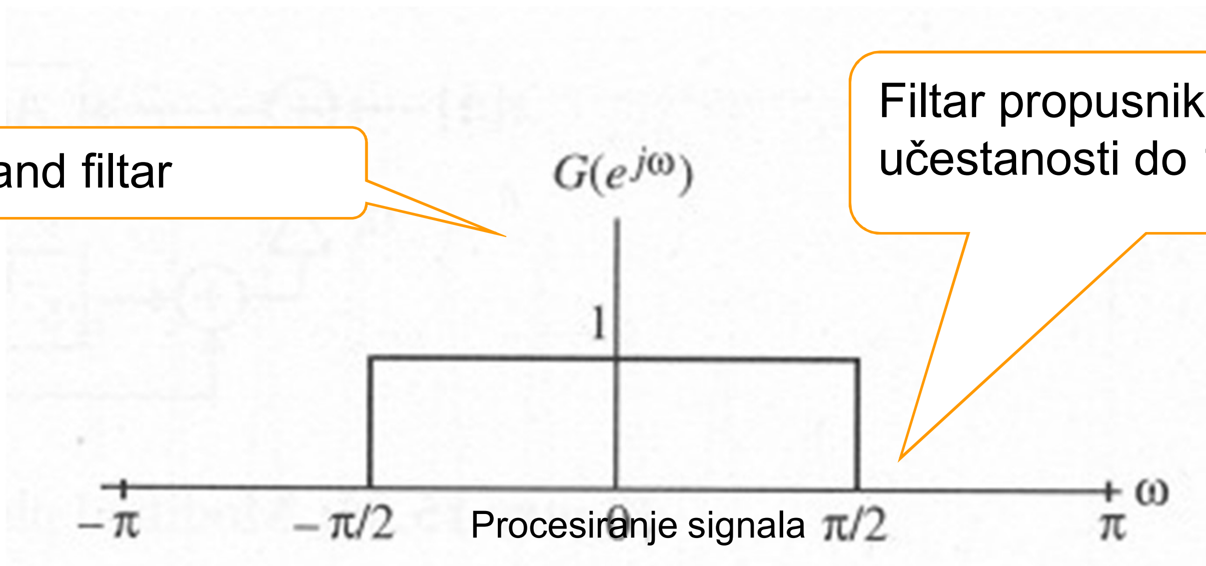
H je filtar kroz koji treba da se propusti signal u



Spektar analitičkog signala (5)

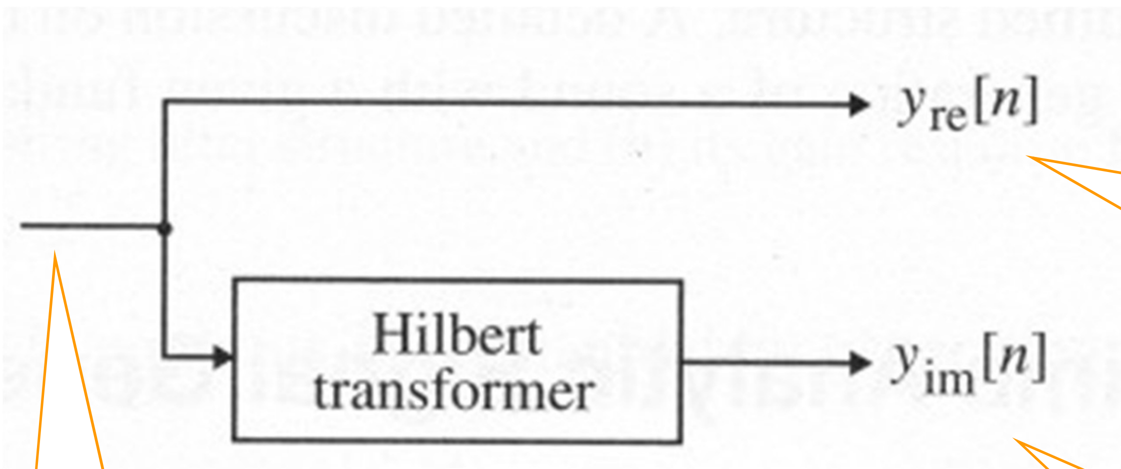


G je halfband filtar



Filtar propusnik niskih učestanosti do 1/2 opsega

Hilbertov transformator



Realni deo analitičkog signala čiji je realni deo u

Realni signali u

Sistem H_{HT} generiše imaginarni deo analitičkog signala za poznati realni deo analitičkog signala

Imaginardni deo $\hat{u}$ analitičkog signala $u+j\,\hat{u}$ čiji je realni deo u; signal $\hat{u}$ je realan

Diskretni Hilbertov transformator

$$j\hat{U}(e^{j\omega}) = \frac{1}{2}\left(Y(e^{j\omega}) - Y^{*}(e^{-j\omega})\right)$$

$$Y(e^{-j\omega}) = 0 \qquad 0 \le \omega < \pi$$
$$Y(e^{j\omega}) = 0 \qquad -\pi \le \omega < 0$$

$$\hat{U}(e^{j\omega}) = \begin{cases} -jU(e^{j\omega}) & 0 \le \omega < \pi \\ jU(e^{j\omega}) & -\pi \le \omega < 0 \end{cases}$$

Idealni Hilbertov transformator

$$\hat{U}(e^{j\omega}) = \begin{cases} -jU(e^{j\omega}) & 0 \le \omega < \pi \\ jU(e^{j\omega}) & -\pi \le \omega < 0 \end{cases}$$

$$H_{\mathrm{HT}}(e^{j\omega}) = \begin{cases} -j & 0 \le \omega < \pi \\ j & -\pi \le \omega < 0 \end{cases}$$

$$\left| H_{\mathrm{HT}}(e^{j\omega}) \right| = 1$$

Realni signali u kada se propusti kroz sistem H_{HT} dobija se imaginarni deo signala, $\hat{u}$

Sistem H_{HT} se naziva i *90-degree phase shifter*

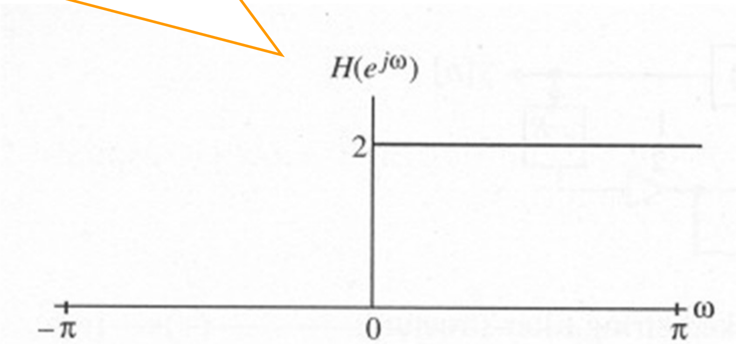
Hilbertov transformator (HT) i Half-band filter (HBF)

- Idealni Hilbertov transformator **ne može** da se realizuje
- Hilbertov transformator može da se realizuje kao half-band filter
- FIR half-band filter
- IIR half-band filter

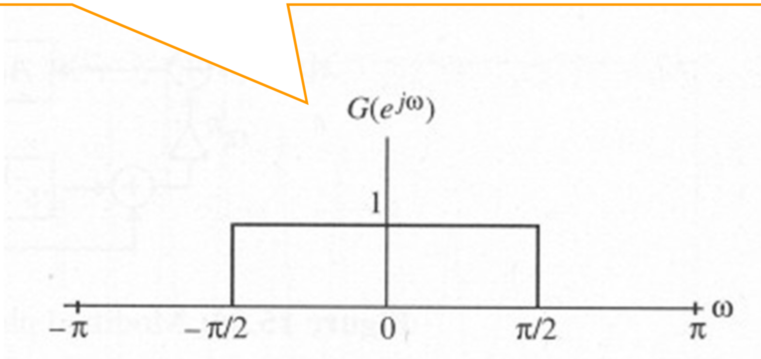
Relacije HT-HBF

- HT se dobija kada se frekvencijska karakteristika HBF pomeri za $\pi/2$ i skalira sa faktorom 1/2

H je kompleksni half-band filtar



G je filtar sa realnim koeficijentima



$$G(e^{j\omega}) = \frac{1}{2} H(e^{j\omega+\pi/2}) = \begin{cases} 1, & 0 \le |\omega| < \dfrac{\pi}{2} \\ 0, & \dfrac{\pi}{2} < |\omega| \le \pi \end{cases}$$

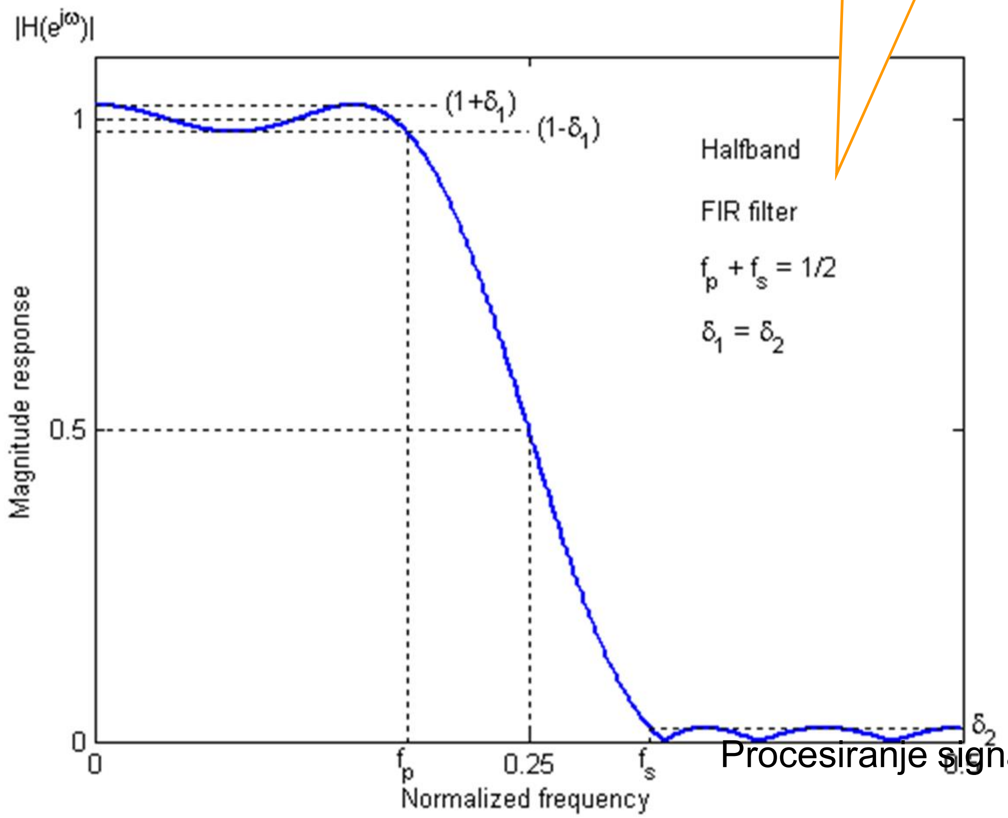
Funkcija prenosa HT

- Funkcija prenosa kompleksnog half-band filtra dobija se iz realnog half-band filtra propusnika niskih učestanosti

$$H(z) = j2G(-jz)$$

Half-band FIR filtar

Frekvencijska simetrija

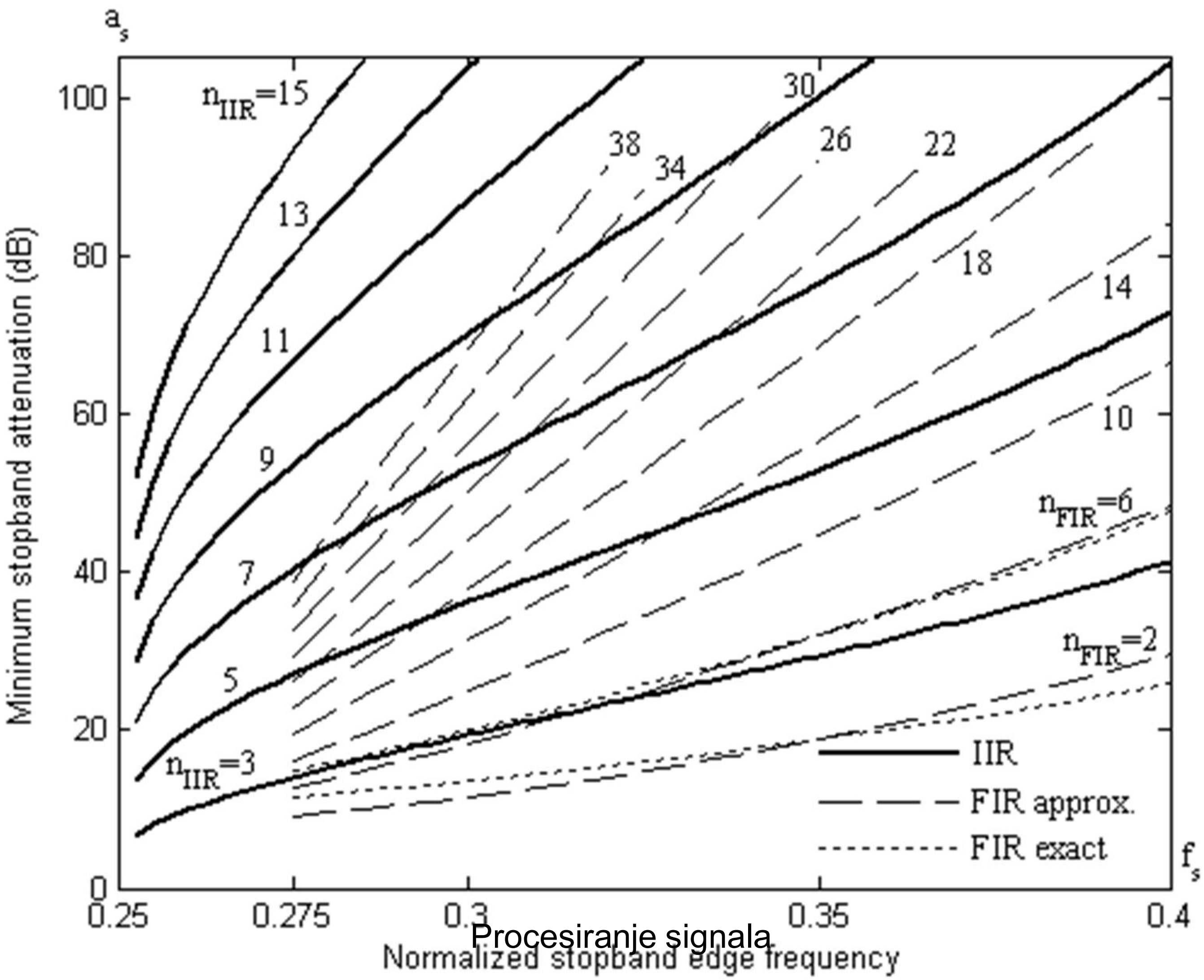


$$a_p = 20\log_{10}\frac{1+\delta_1}{1-\delta_1}$$

$$a_s = -20\log_{10}\frac{\delta_2}{1-\delta_1}$$

$$\left|H(e^{j\pi/2})\right| = \frac{1}{2}$$

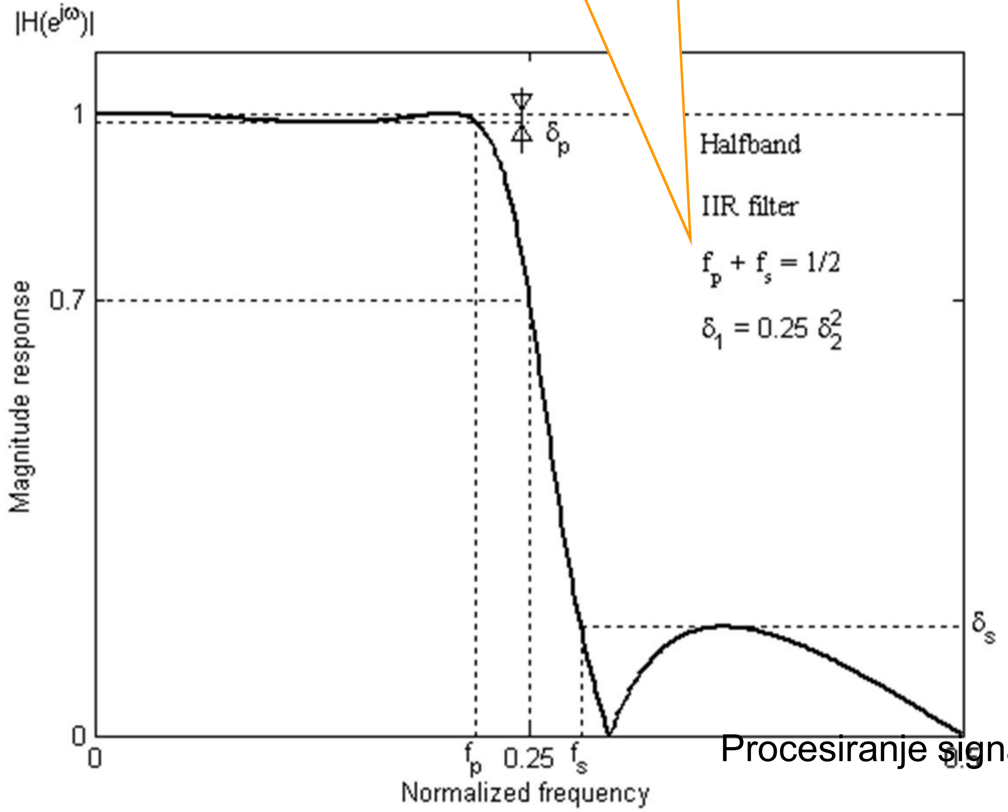
Half-band FIR ili IIR filtar?



Half-band IIR filtar

Frekvencijska simetrija

$$a_p = 10\log_{10}\left(1 + \frac{1}{10^{a_s/10} - 1}\right)$$



$$\delta_1 = \frac{1}{4}\delta_2^2$$

$$\delta_p = 1 - \sqrt{1 - \delta_s^2}$$

$$\left|H(e^{j\pi/2})\right|^2 = \frac{1}{2}$$

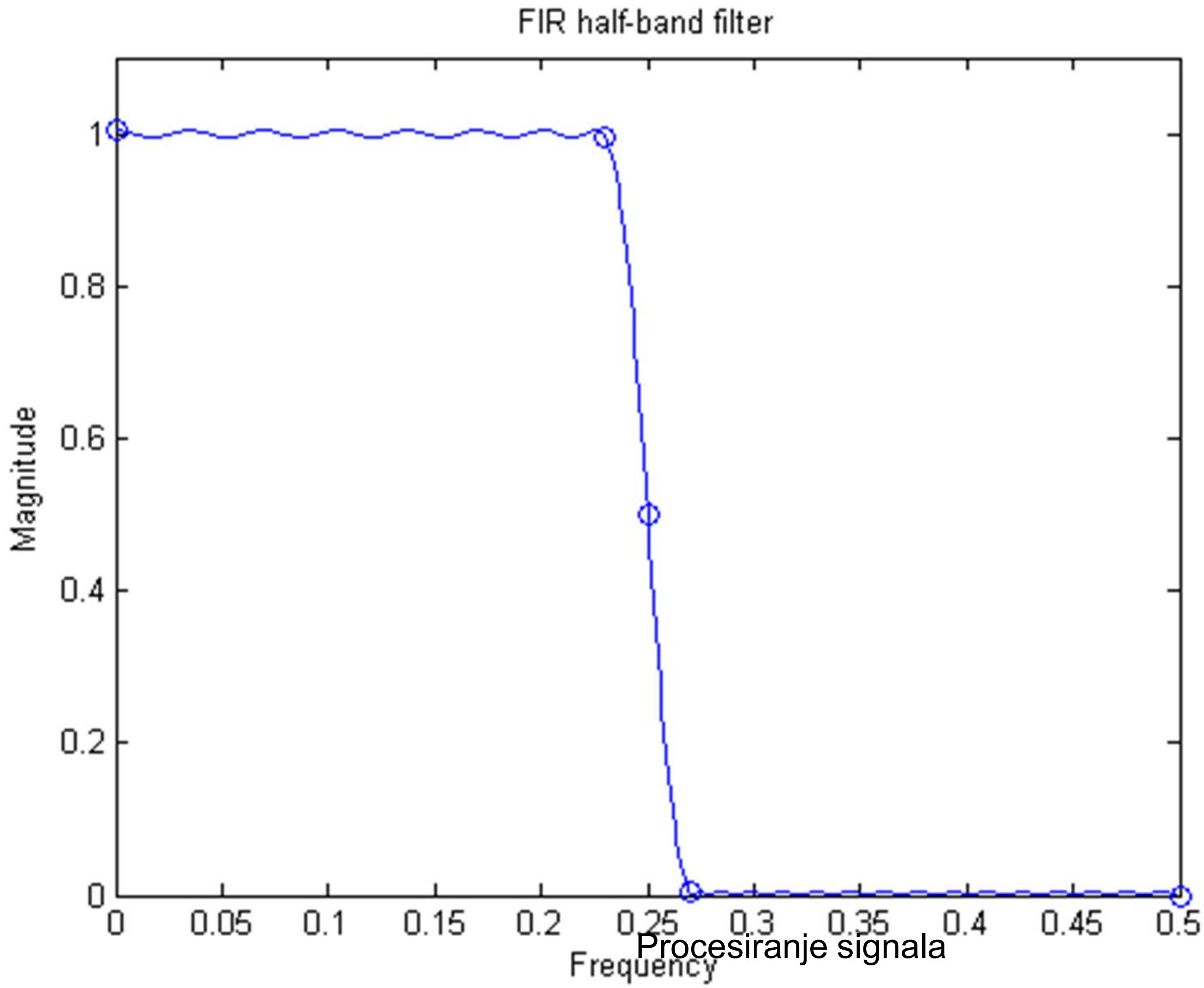
Half-band FIR filtar (1)

```
Fp = 0.23;   Fs = 0.5-Fp;
Ap = 0.07;   As = 48;
d1 = (1-10^(-Ap/20))/(1+10^(-Ap/20));
d2 = d1;

[n,fo,mo,w] = firpmord([2*Fp 2*Fs], [1 0], [d1 d2]);
b = firpm(n,fo,mo,w);
[h,w] = freqz(b,1,512);
plot(w/(2*pi),abs(h))
w01 = 2*[0 pi*Fp pi/4  pi*Fs pi/2];
h01 = freqz(b,1,w01);
hold on
  plot(w01/(2*pi),abs(h01),'o')
hold off
axis([0 0.5 0 1.1])
xlabel('Frequency'), ylabel('Magnitude'),
title('FIR half-band filter')
```

Hilbert_01.m

n = 61

Half-band FIR filtar (2)



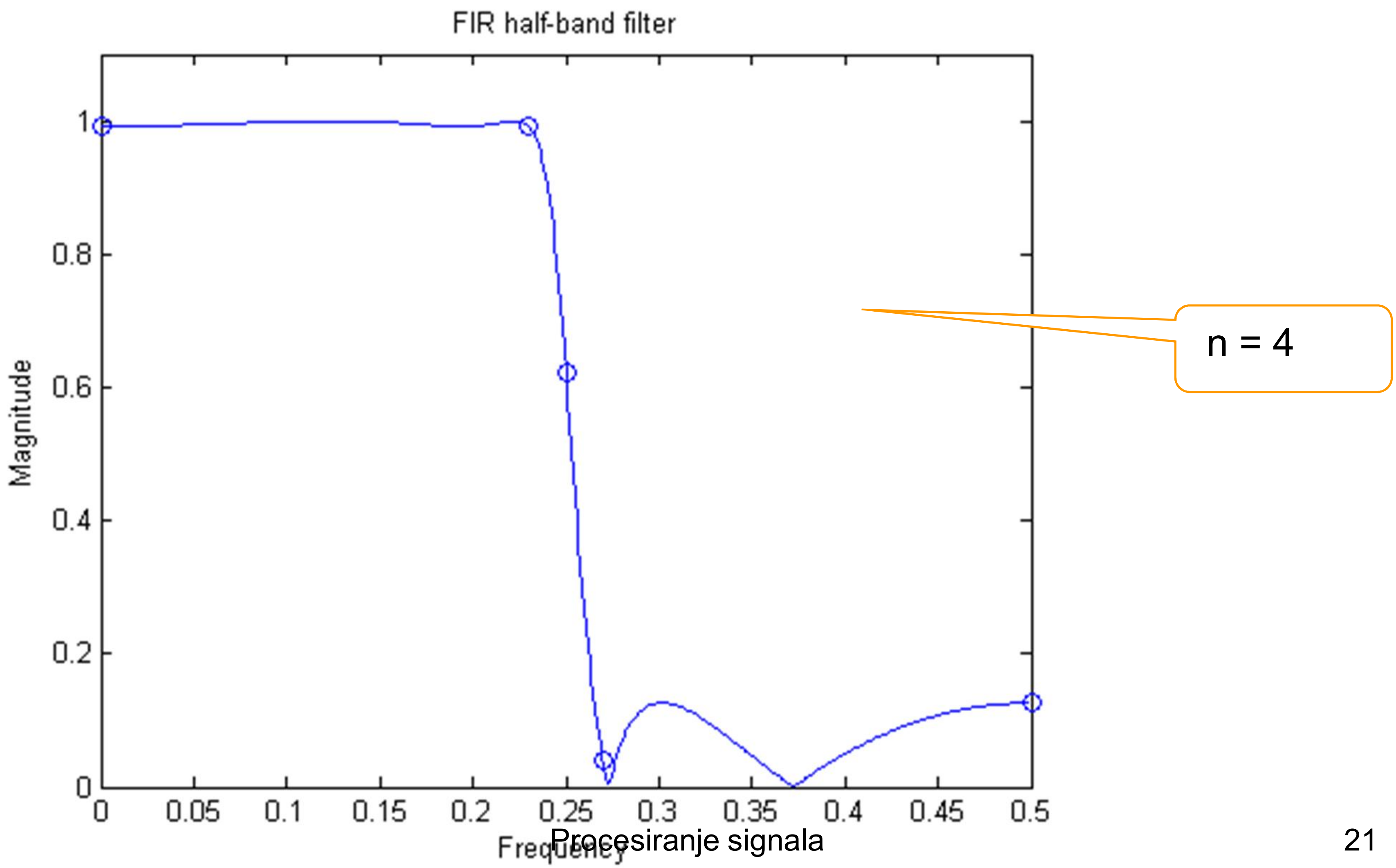
Half-band IIR filtar (1)

Hilbert_02.m

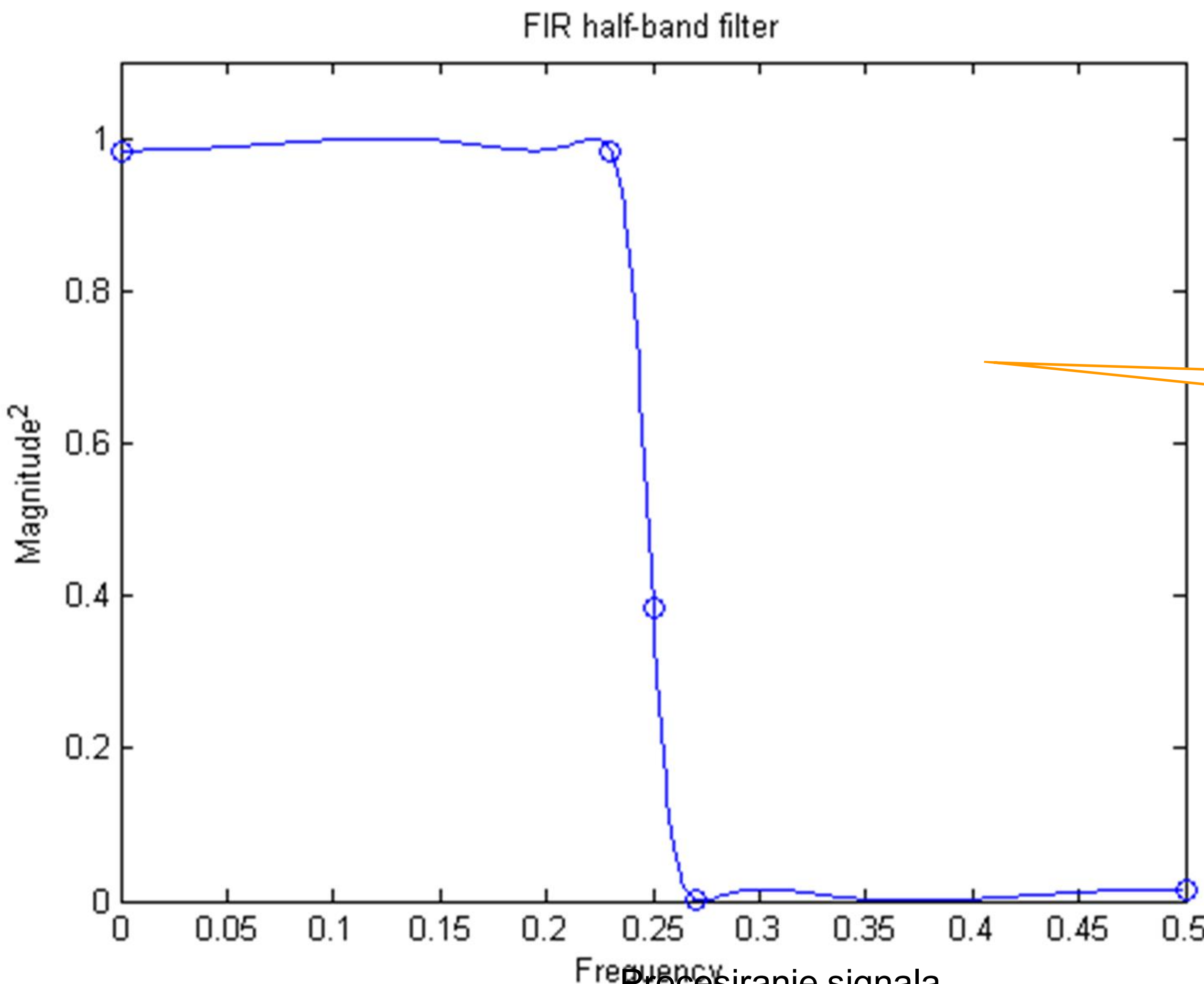
```
Fp = 0.23;   Fs = 0.5-Fp;
Ap = 0.07;   As = -10*log10(1-10^(-Ap/10))
d1 = (1-10^(-Ap/20))/(1+10^(-Ap/20));

n=ellipord(2*Fp, 2*Fs, Ap, As)
[a,b] = ellip(n,Ap,As,2*Fp);
[h,w] = freqz(a,b,512);
plot(w/(2*pi),abs(h))
w01 = 2*[0 pi*Fp pi/4  pi*Fs pi/2];
h01 = freqz(a,b,w01);
hold on
  plot(w01/(2*pi),abs(h01),'o')
hold off
```

n = 4

Half-band IIR filtar (2)



Half-band IIR filtar (3)

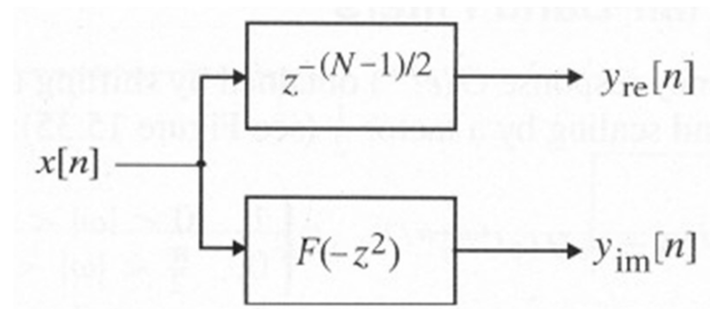


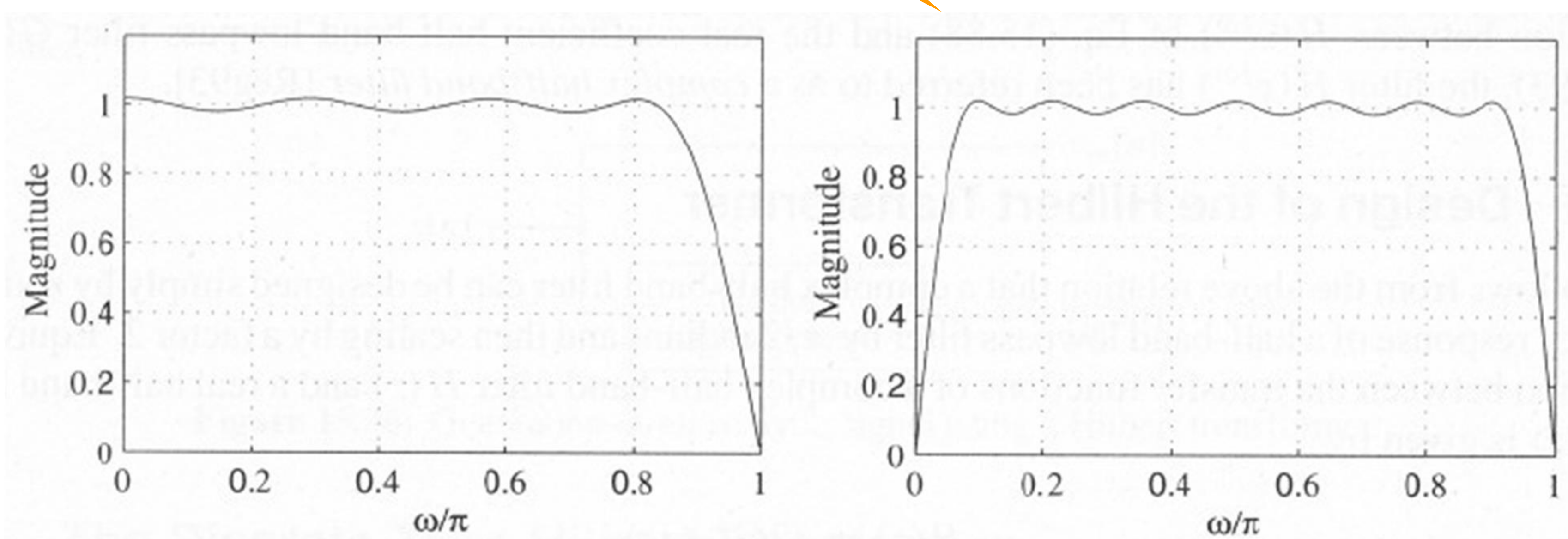
n = 2*4

Realizacija FIR HT

Hilbertov transformator kao kompleksni half-band filtar

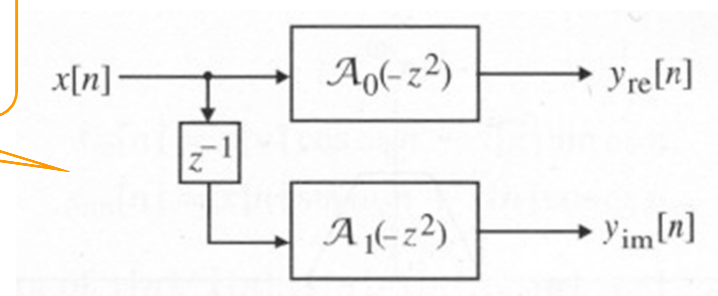
Half-band filtar



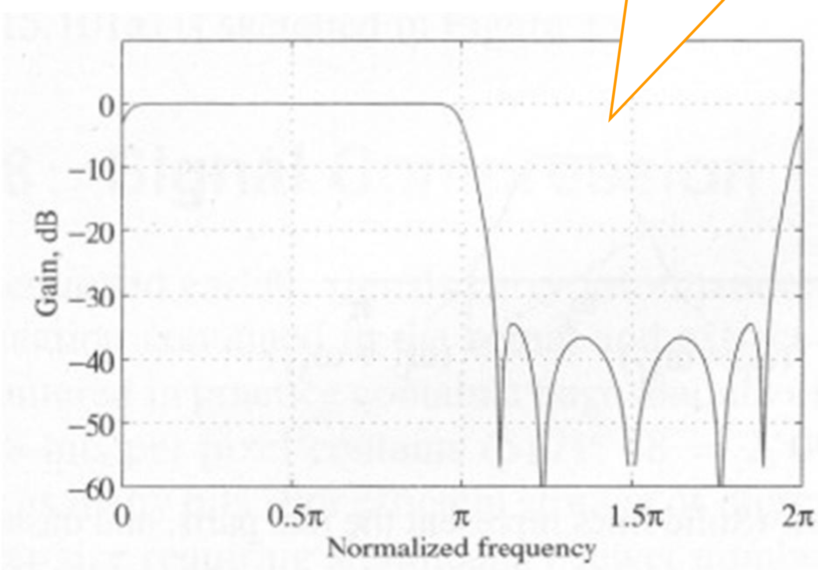


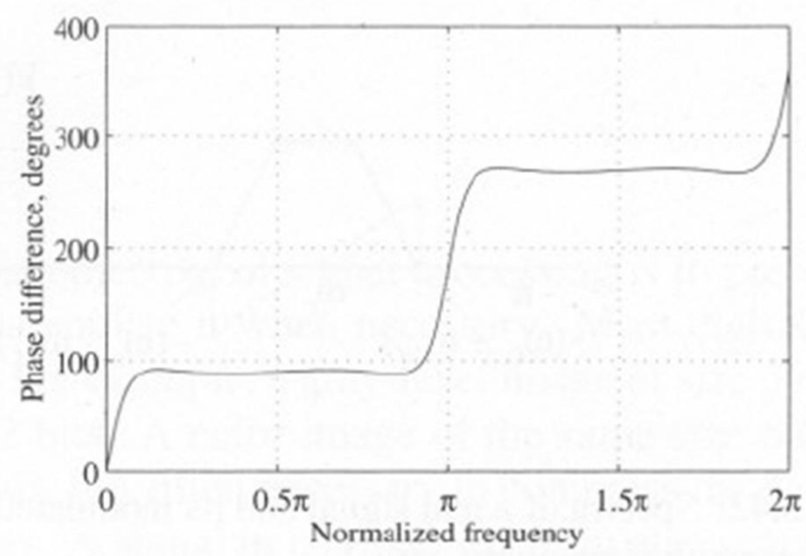
Realizacija IIR HT

Hilbertov transformator kao kompleksni half-band filtar



Half-band filtar

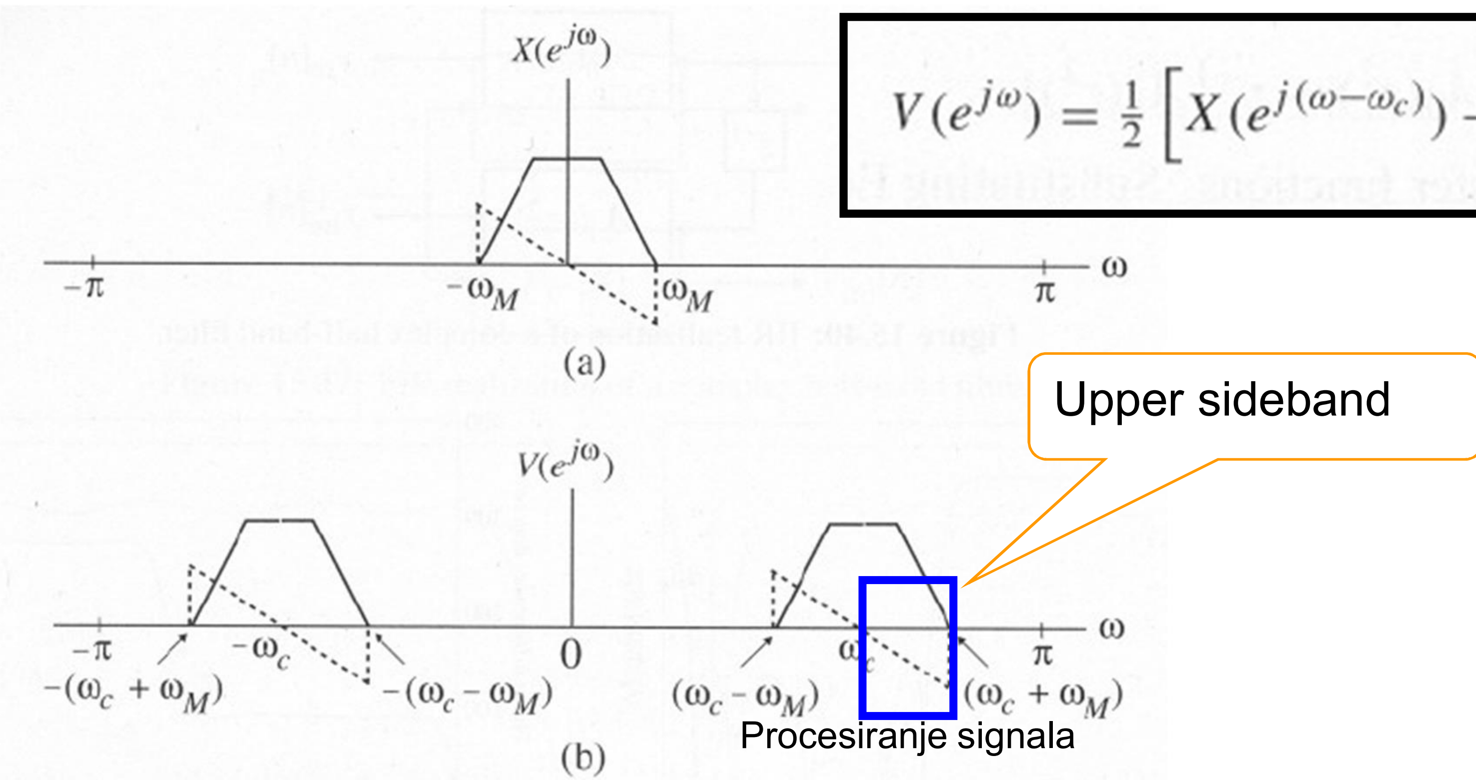




Modulacija

- Da bi se realan signal preneo na velike daljine, on se moduliše sa sinusnim signalom visoke učestanosti (nosioc - *carrier*)

$$V(e^{j\omega}) = \frac{1}{2}\left[X(e^{j(\omega-\omega_c)}) + X(e^{j(\omega+\omega_c)})\right]$$



SSB

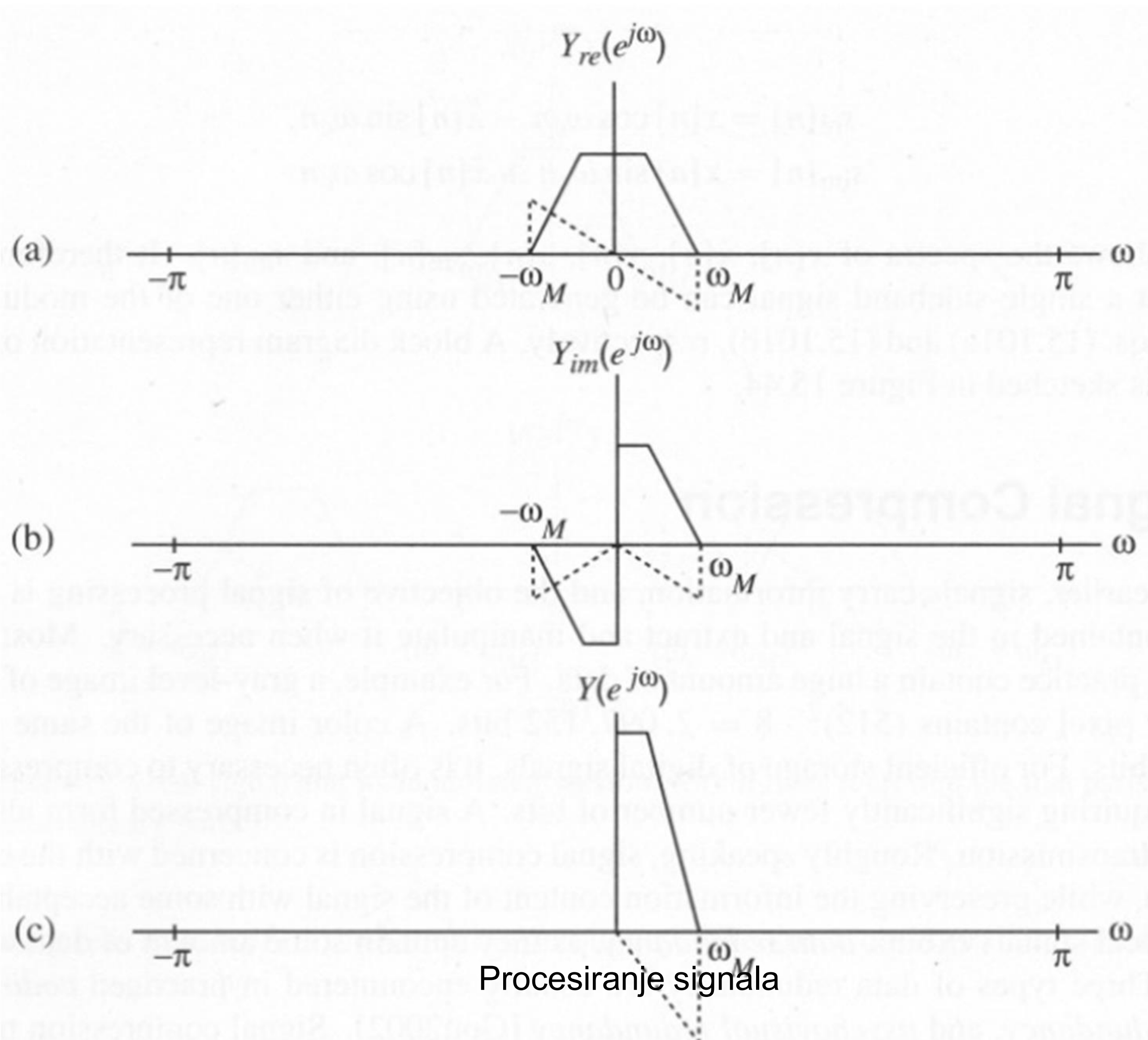
$$y[n] = u[n] + j\hat{u}[n]$$

Od realnog signala u, generiše se analitički signal koji ima isti takav realni deo, i odgovarajući imaginarni deo $\hat{u}$ koji se generiše Hilbertovim transformatorom H_{HT}

Modulacija analitičkog signala sinusnim signalom

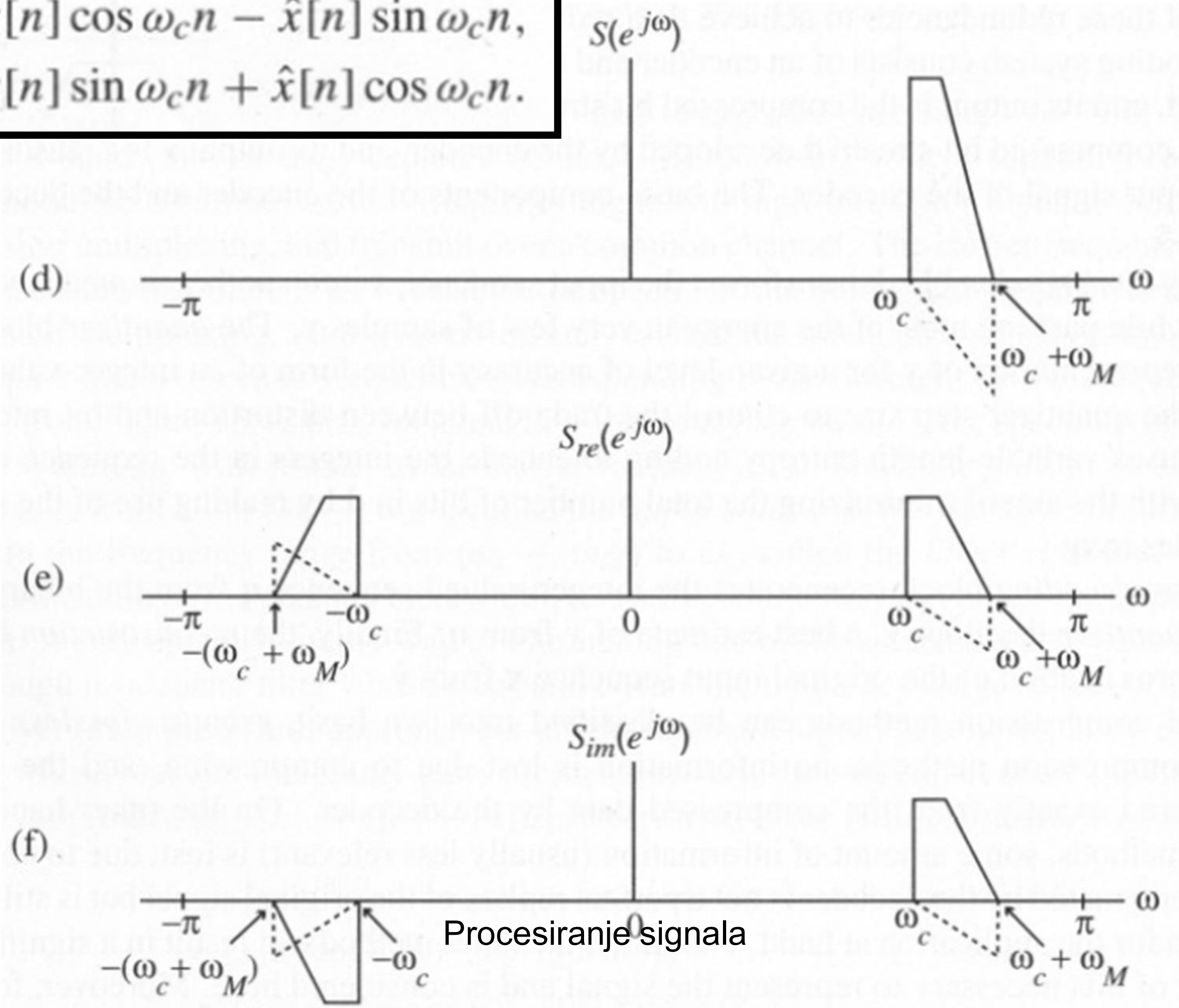
$$\begin{aligned} s[n] = y[n]e^{j\omega_c n} &= (y_{\mathrm{re}}[n] + j y_{\mathrm{im}}[n])(\cos \omega_c n + j \sin \omega_c n) \\ &= (x[n]\cos \omega_c n - \hat{x}[n]\sin \omega_c n) \\ &\quad + j\,(x[n]\sin \omega_c n + \hat{x}[n]\cos \omega_c n)\,. \end{aligned}$$

Generisanje SSB (1)



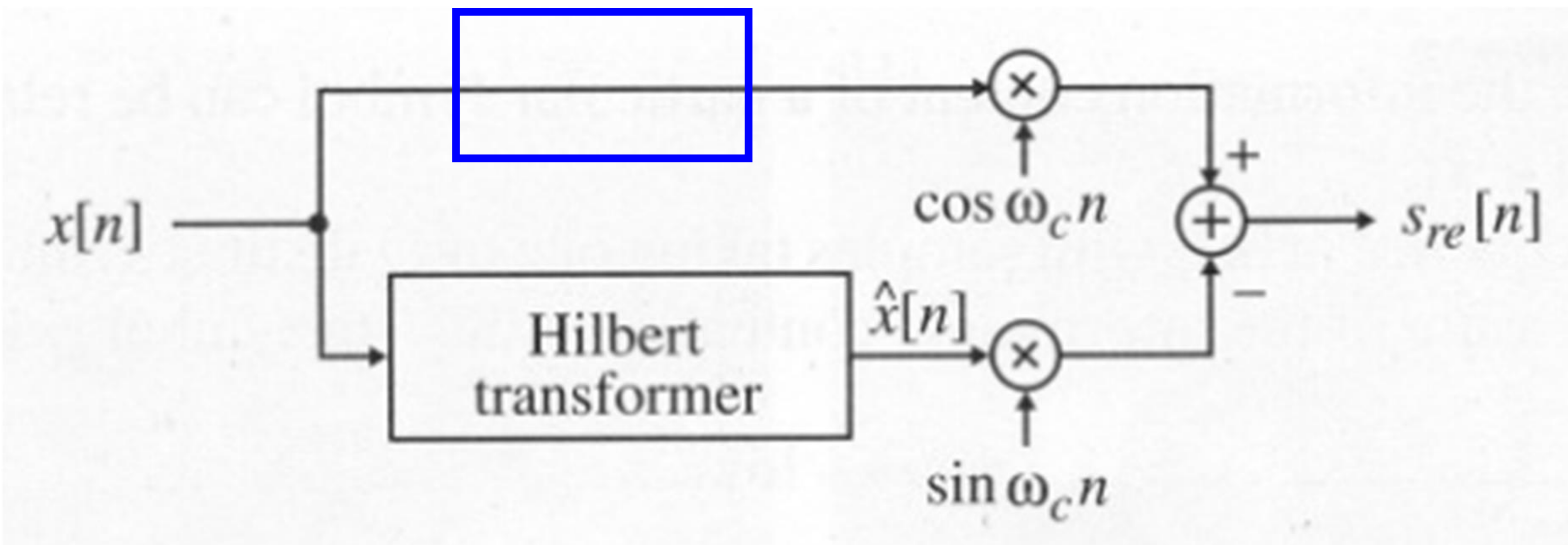
Generisanje SSB (2)

$$s_{\mathrm{re}}[n] = x[n]\cos\omega_c n - \hat{x}[n]\sin\omega_c n,$$
$$s_{\mathrm{im}}[n] = x[n]\sin\omega_c n + \hat{x}[n]\cos\omega_c n.$$



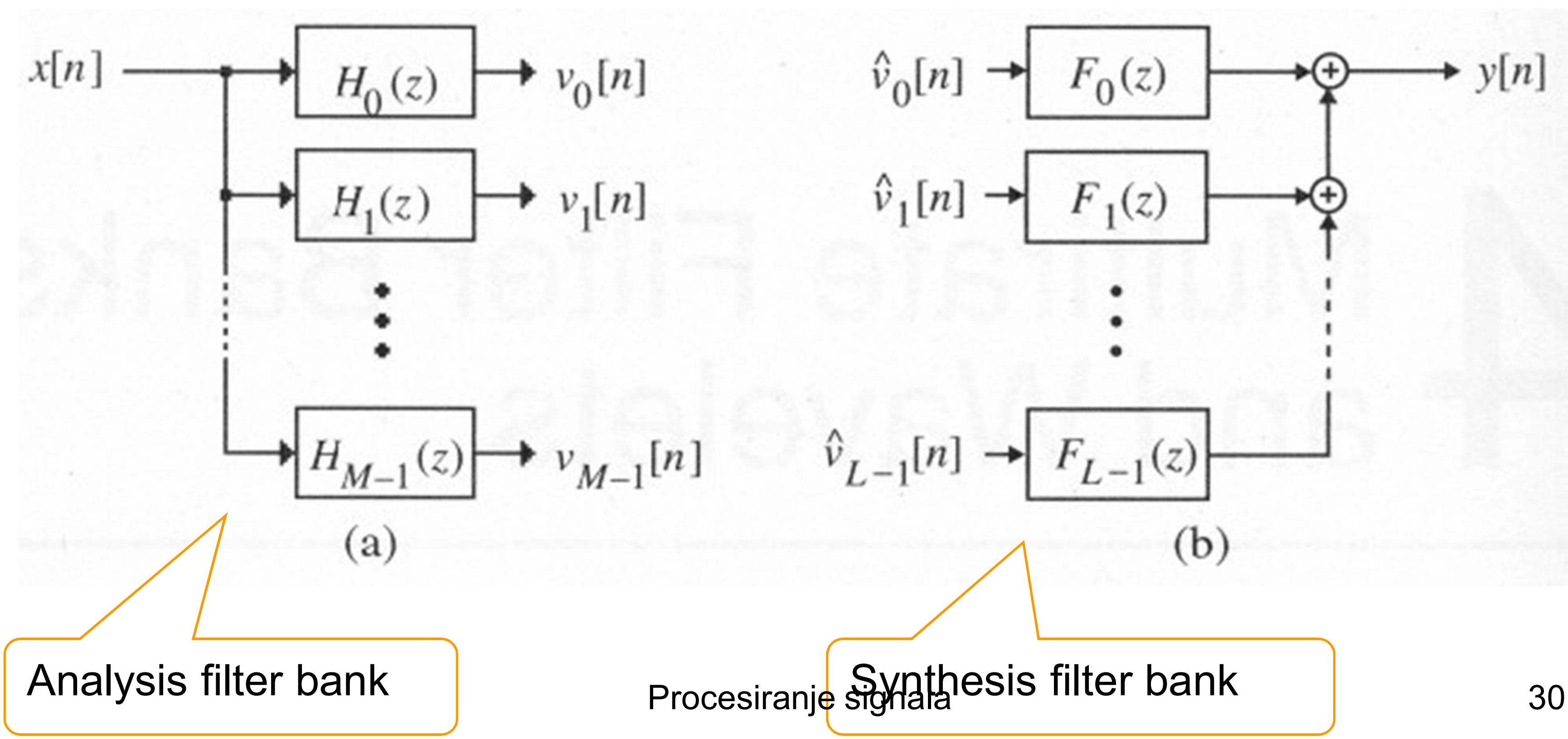
Generisanje SSB signala

Kašnjenje koje se pravi u HT



Filtarske banke

- Digitalna filtarska banka je set digitalnih filtara propusnika opsega učestanosti koji imaju (a) zajednički ulaz ili (b) zbirni izlaz



Analysis filter bank

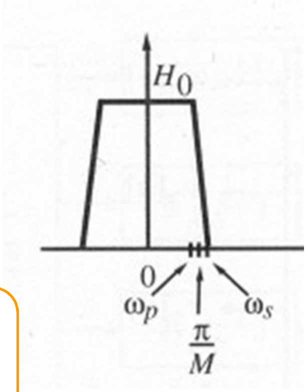
- ***Analysis filter bank*** se koriste za dekompoziciju ulaznog signala u set podopsežnih signala, gde svaki od njih zauzima deo originalnog frekvencijskog opsega
- Ulazni signal se **analizira** i razdvaja u set uskopojasnih spektralnih opsega

Synthesis filter bank

- ***Synthesis filter bank*** se koriste da se set podopsežnih signala kombinuje da se napravi jedan signal koji zauzima širi frekvencijski opseg
- Najčešće se koristi da se od signala dobijenih bankom filtara analize pobnovo napravi originalni signal koji zauzima kontinualni frekvencijski opseg
- Od ulaznih signala, **sintezom** se generiše originalni signal

Uniformna DFT filtarska banka

Uniformna filtarska banka ima istu širinu propusnog opsega svih filtara



$$H_0(z) = \sum_{n=0}^{\infty} h_0[n] z^{-n}$$

H_0 je lowpass filtar od 0 do ω_p, h_0 je realni impulsni odziv

$$h_k[n] = h_0[n] W_M^{-kn}, \quad W_M = e^{-j2\pi/M}$$

Modulacijom h_0 sa eksponencijalnom sekvencom, $0<k\leq M-1$

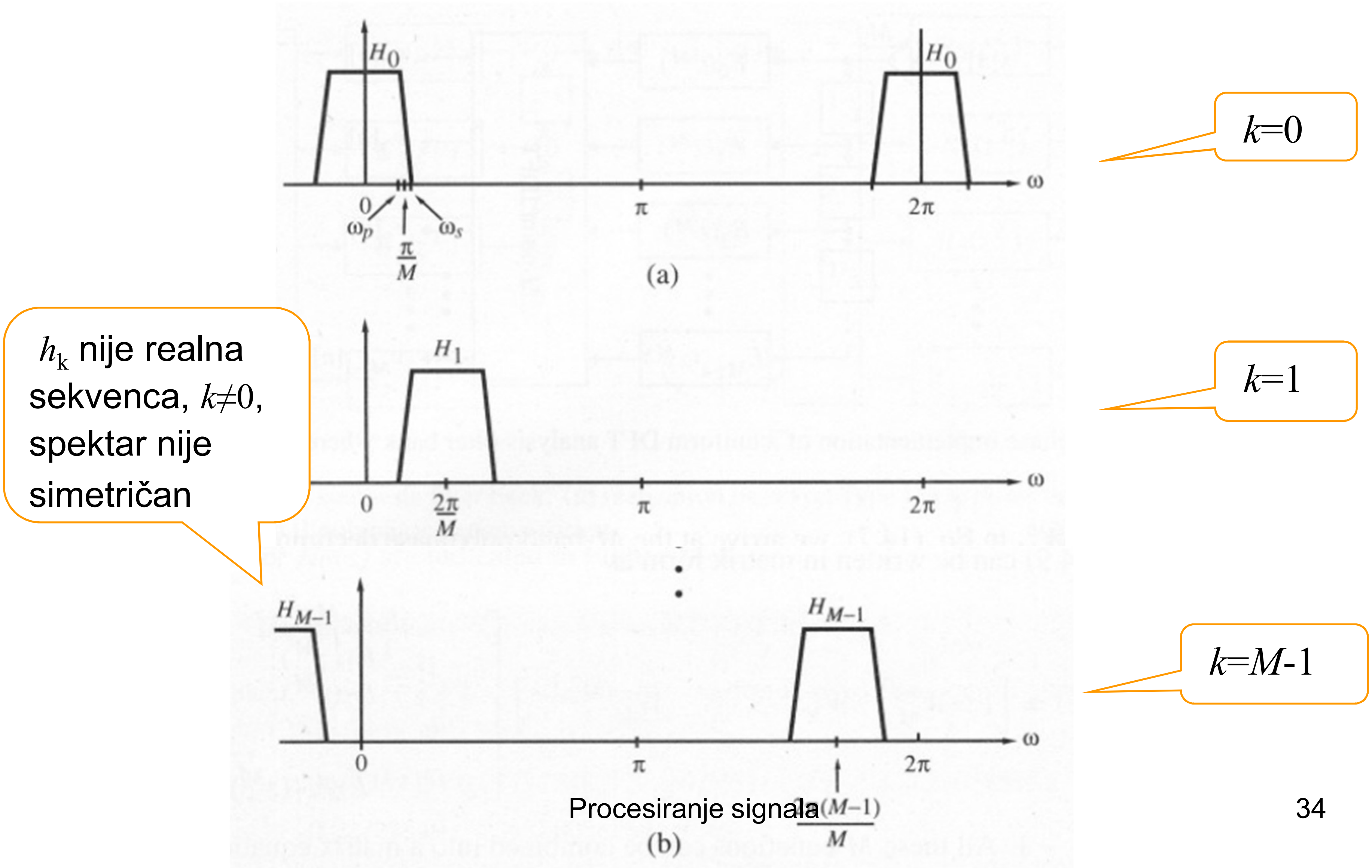
$$H_k(z) = H_0\left(zW_M^k\right), \quad 0 \leq k \leq M-1$$

Funkcija prenosa

$$H_k(e^{j\omega}) = H_0\left(e^{j(\omega-2\pi k/M)}\right)$$

Frekvencijski odziv

Uniformna filtarska banka



Polifazna implementacija

$$H_0(z) = \sum_{l=0}^{M-1} z^{-l} E_l(z)$$

H_0 je lowpass prototip filtar

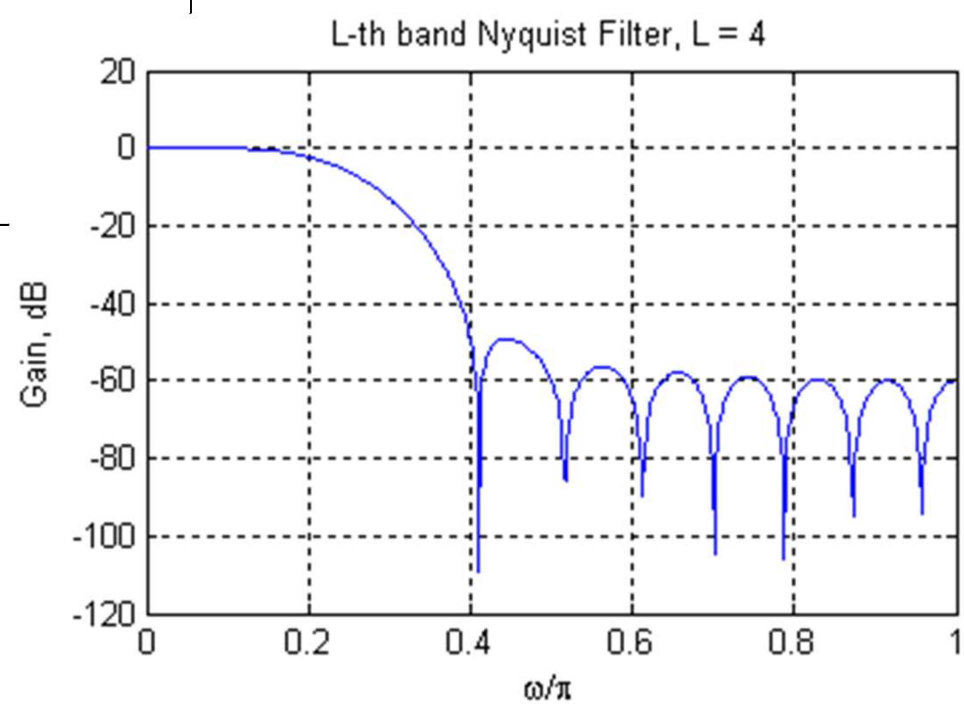
E_l je polifazna komponenta

$$E_l(z) = \sum_{n=0}^{\infty} e_l[n] z^{-n} = \sum_{n=0}^{\infty} h_0[l + nM] z^{-n}, \; 0 \le l \le M-1$$

Implementacija: Program_14 _3

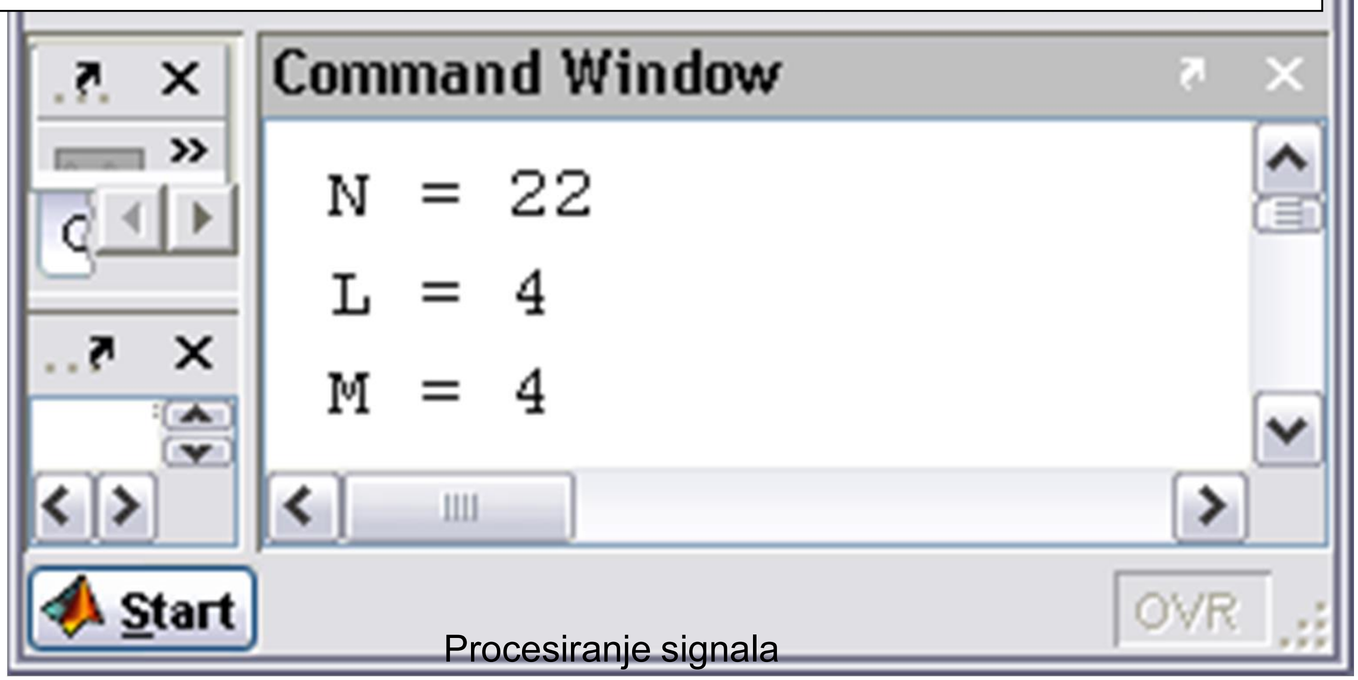
```
N = 22;
L = 4;
n = -N/2:N/2;
b = sinc(n/L)/L;
win = hamming(N+1);
c = b.*win';
[h,w] = freqz(c,1,512);
plot(w/pi,20*log10(abs(h)));
```

c je prototip filtar



Program_14 _3 (2)

```
M = 4;
disp([' N = ' num2str(N)])
disp([' L = ' num2str(L)])
disp([' M = ' num2str(M)])
```



Program_14 _3 (3)

```
E(1,:) = c(1:M:end);
for ind =2:M-1
    E(ind,:) = c(ind:M:end);
end
EM1 = c(4:M:end);
if length(EM1)<length(E(1,:))
    EM1(length(E(1,:)))=0;
end
E(M,:) = EM1;
disp('E = ')
disp(E)
```

E su polifazne komponente

Command Window

```
E =
    0.0016   -0.0112    0.0631    0.2209   -0.0273    0.0038
    0.0031   -0.0252    0.1475    0.1475   -0.0252    0.0031
    0.0038   -0.0273    0.2209    0.0631   -0.0112    0.0016
   -0.0000    0.0000    0.2500    0.0000   -0.0000         0
```

Program_14 _3 (4)

$$E_0(z) = 0.00163694 - 0.01121888\,z^{-1} + 0.06311487\,z^{-2} + 0.22088513\,z^{-3} - 0.02725549\,z^{-4} + 0.00382693\,z^{-5},$$

$$E_1(z) = 0.00313959 - 0.025174873\,z^{-1} + 0.147532912\,z^{-2} + 0.147532912\,z^{-3} - 0.025174873\,z^{-4} + 0.00313959\,z^{-5},$$

$$E_2(z) = 0.00382693 - 0.02725549\,z^{-1} + 0.22088513\,z^{-2} + 0.06311487\,z^{-3} - 0.01121888\,z^{-4} + 0.00163694\,z^{-5},$$

$$E_3(z) = 0.25\,z^{-2}.$$

Command Window

```
E =
    0.0016   -0.0112    0.0631    0.2209   -0.0273    0.0038
    0.0031   -0.0252    0.1475    0.1475   -0.0252    0.0031
    0.0038   -0.0273    0.2209    0.0631   -0.0112    0.0016
   -0.0000    0.0000    0.2500    0.0000   -0.0000         0
```

Program_14 _3 (5)

$$H_0(z) = \sum_{n=0}^{\infty} h_0[n] z^{-n}$$

H_0 je lowpass filtar od 0 do ω_p,
h_0 je realni impulsni odziv

$$H_k(z) = H_0\left(z W_M^k\right), \quad 0 \le k \le M-1$$

Funkcija prenosa

$$H_k(z) = \sum_{l=0}^{M-1} z^{-l} W_M^{-kl} E_l(z^M), \quad 0 \le k \le M-1$$

Program_14 _3 (6)

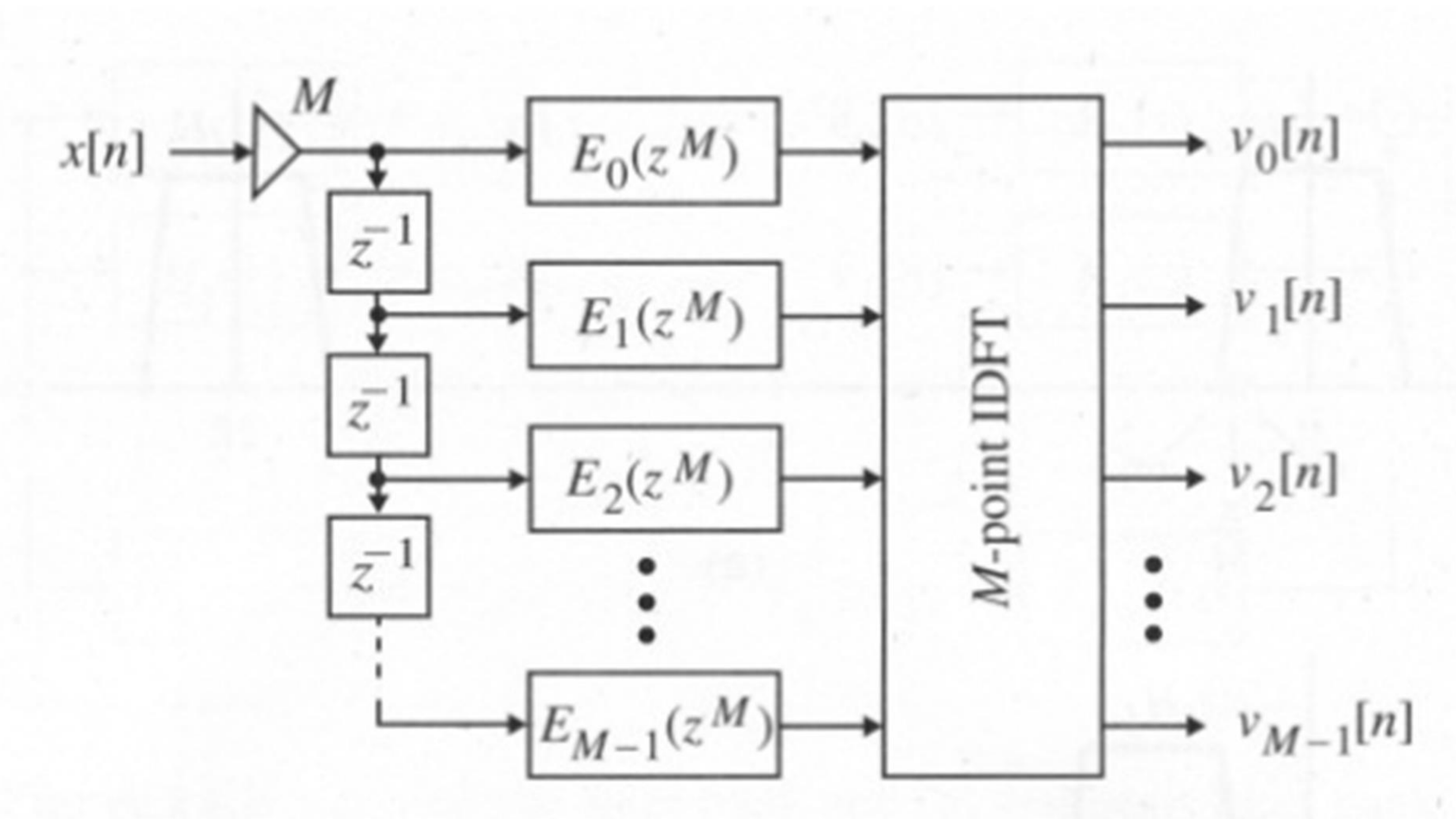
$$H_k(z) = \begin{bmatrix} 1 & W_M^{-k} & W_M^{-2k} & \cdots & W_M^{-(M-1)k} \end{bmatrix} \begin{bmatrix} E_0(z^M) \\ z^{-1}E_1(z^M) \\ z^{-2}E_2(z^M) \\ \vdots \\ z^{-(M-1)}E_{M-1}(z^M) \end{bmatrix}$$

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ H_2(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} = M\mathbf{D}^{-1} \begin{bmatrix} E_0(z^M) \\ z^{-1}E_1(z^M) \\ z^{-2}E_2(z^M) \\ \vdots \\ z^{-(M-1)}E_{M-1}(z^M) \end{bmatrix}$$

Program_14 _3 (7)

$$\mathbf{D} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_M^1 & W_M^2 & \cdots & W_M^{(M-1)} \\ 1 & W_M^2 & W_M^4 & \cdots & W_M^{2(M-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W_M^{(M-1)} & W_M^{2(M-1)} & \cdots & W_M^{(M-1)^2} \end{bmatrix}$$

Program_14 _3 (8)



Program_14 _3 (9)

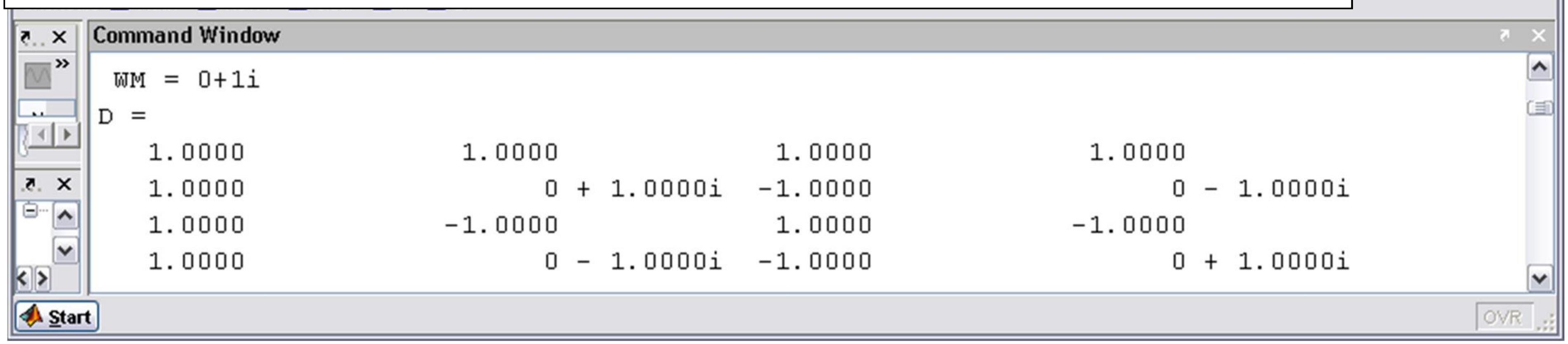
$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ H_2(z) \\ H_3(z) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} E_0(z^4) \\ z^{-1}E_1(z^4) \\ z^{-2}E_2(z^4) \\ z^{-3}E_3(z^4) \end{bmatrix}$$

$$H_0(z) = E_0(z^4) + z^{-1}E_1(z^4) + z^{-2}E_2(z^4) + z^{-3}E_3(z^4),$$
$$H_1(z) = E_0(z^4) + j\,z^{-1}E_1(z^4) - z^{-2}E_2(z^4) - j\,z^{-3}E_3(z^4),$$
$$H_2(z) = E_0(z^4) - z^{-1}E_1(z^4) + z^{-2}E_2(z^4) - z^{-3}E_3(z^4),$$
$$H_3(z) = E_0(z^4) - j\,z^{-1}E_1(z^4) - z^{-2}E_2(z^4) + j\,z^{-3}E_3(z^4).$$

Program_14 _3 (10)

```
Wm = exp(i*2*pi/M);
Wm = round(10^12*Wm)/10^12;
disp([' WM = ' num2str(Wm)])
wm1 = 0:M-1;
D = Wm.^(0*wm1);
for ind =1:M-1
    D = [D;Wm.^(ind*wm1)];
end
disp('D = ')
disp(D)
```

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$



Program_14 _3 (11)

```
E =
  Columns 1 through 9

   0.0016         0         0         0   -0.0112         0         0         0    0.0631
        0    0.0031         0         0         0   -0.0252         0         0         0
        0         0    0.0038         0         0         0   -0.0273         0         0
        0         0         0   -0.0000         0         0         0    0.0000         0

  Columns 10 through 18

        0         0         0    0.2209         0         0         0   -0.0273         0
   0.1475         0         0         0    0.1475         0         0         0   -0.0252
        0    0.2209         0         0         0    0.0631         0         0         0
        0         0    0.2500         0         0         0    0.0000         0         0

  Columns 19 through 24

        0         0    0.0038         0         0
        0         0         0    0.0031         0
  -0.0112         0         0         0    0.0016
        0   -0.0000         0         0         0
```

```
e0=E(1,:); e0(4,1)=0;
Em(1,:) = e0(:)';
for ind =2:M
  e0=E(ind,:); e0(4,1)=0;
  e0(ind,:)=e0(1,:);
  e0(1,:)=0*e0(1,:);
  Em(ind,:) = e0(:)';
end
disp(Em)
```

Program_14 _3 (12)

```
Hk =
  Columns 1 through 4

   0.0016              0.0031              0.0038
   0.0016                   0 + 0.0031i   -0.0038
   0.0016             -0.0031              0.0038
   0.0016                   0 - 0.0031i   -0.0038

  Columns 5 through 8

  -0.0112             -0.0252             -0.0
  -0.0112                   0 - 0.0252i    0.0
  -0.0112              0.0252             -0.0
  -0.0112                   0 + 0.0252i    0.0

  Columns 9 through 12

   0.0631              0.1
   0.0631
   0.0631             -0.1
   0.0631

  Columns 13 through 16
```

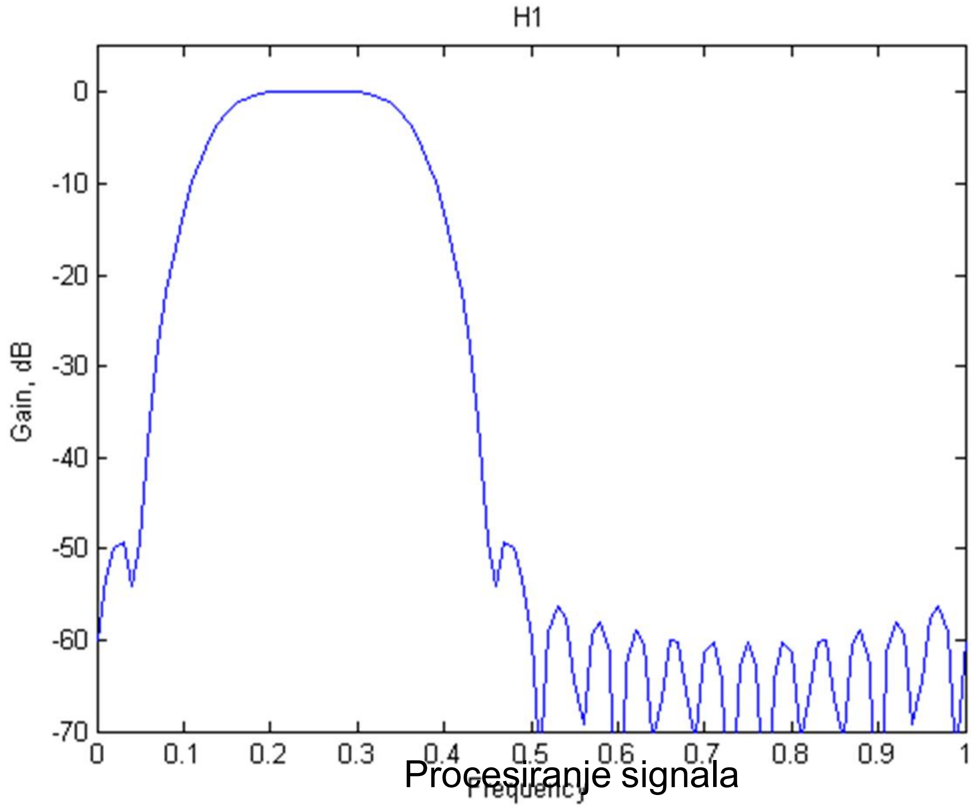
```
H=D*Em;
disp('Hk = ')
disp(H)
H0=H(1,:);
H1=H(2,:);
H2=H(3,:);
H3=H(4,:);
```

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ H_2(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} = M\mathbf{D}^{-1} \begin{bmatrix} E_0(z^M) \\ z^{-1}E_1(z^M) \\ z^{-2}E_2(z^M) \\ \vdots \\ z^{-(M-1)}E_{M-1}(z^M) \end{bmatrix}$$

$$H_0(z) = E_0(z^4) + z^{-1}E_1(z^4) + z^{-2}E_2(z^4) + z^{-3}E_3(z^4),$$
$$H_1(z) = E_0(z^4) + j\,z^{-1}E_1(z^4) - z^{-2}E_2(z^4) - j\,z^{-3}E_3(z^4),$$
$$H_2(z) = E_0(z^4) - z^{-1}E_1(z^4) + z^{-2}E_2(z^4) - z^{-3}E_3(z^4),$$
$$H_3(z) = E_0(z^4) - j\,z^{-1}E_1(z^4) - z^{-2}E_2(z^4) + j\,z^{-3}E_3(z^4).$$

Program_14 _3 (13)

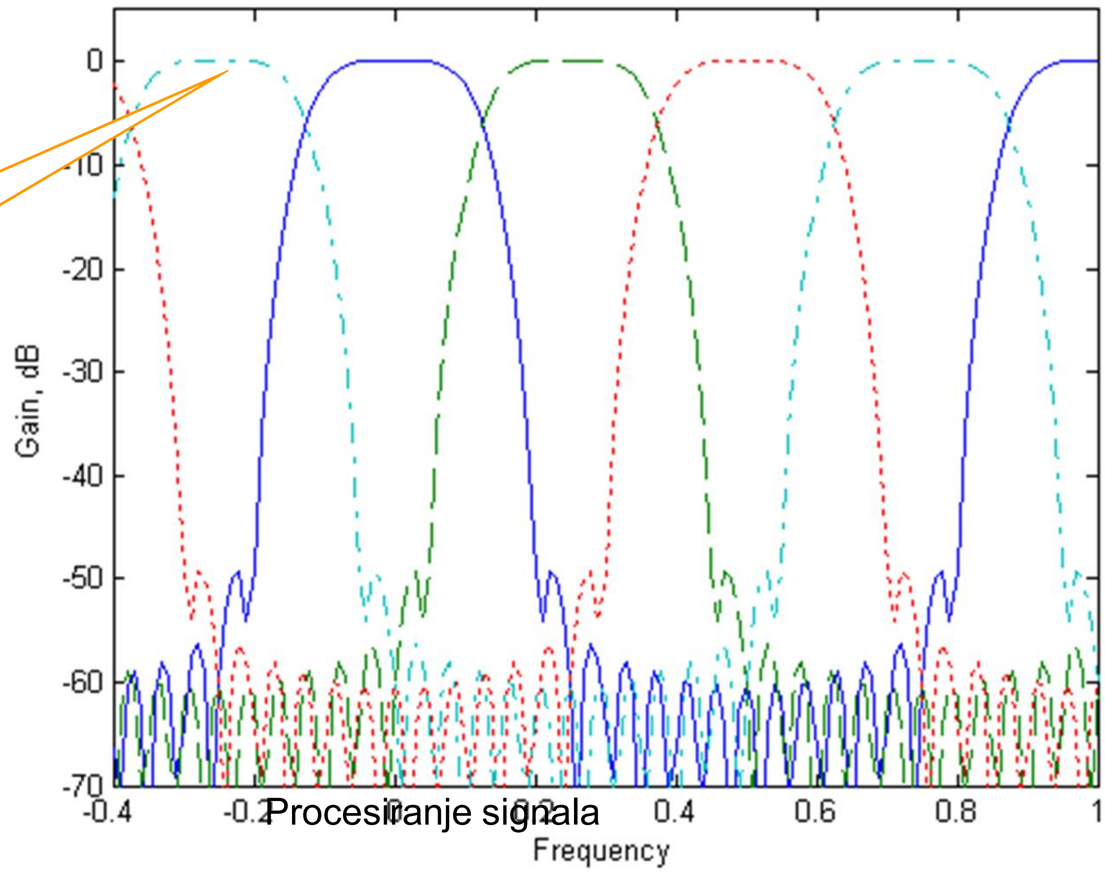
```
f= -1:0.01:1;
w = 2*pi*f;
ha1 = 20*log10(abs(freqz(H1,1,w)));
figure; plot(f,ha1); axis([0 1 -70 5]); title('H1');
```



Program_14 _3 (14)

```
figure; plot(f,ha0,'-',f,ha1,'--',f,ha2,':',f,ha3,'-.');
axis([-0.4 1 -70 5])
xlabel('Frequency');ylabel('Gain, dB');
```

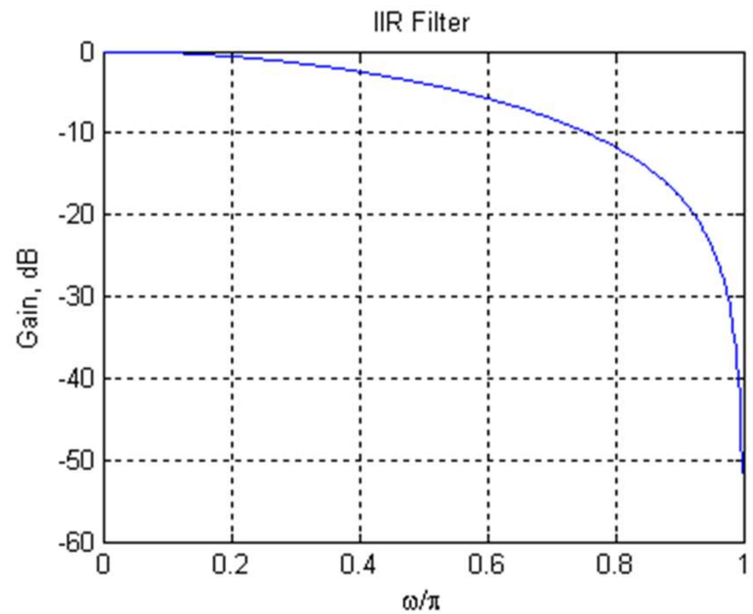
h_3 nije realna sekvenca, spektar nije simetričan



IIR prototip: Program_14 _4

c je prototip filtar

```
[num,den]=cheby1(1,0.1,0.08);
a = num(1)
b = num(2)
c = den(2)
num0 = [a b];
den0 = [1 c];
[h,w] = freqz(num0,den0,512);
plot(w/pi,20*log10(abs(h)));
xlabel('\omega/\pi');
ylabel('Gain, dB');
title(['IIR Filter']);
```



Program_14 _4 (2)

E su polifazne komponente

```
numE0 = [a 0 0 b*c^2];
numE1 = [0 b-a*c 0 0];
numE2 = [0 0 -b*c+a*c^2 0];
denE0 = [1 0 0 c^3];
```

```
a =    0.4529
b =    0.4529
c =   -0.0943
numE0 =    0.4529         0         0    0.0040
numE1 =         0    0.4956         0         0
numE2 =         0         0    0.0467         0
denE0 =    1.0000         0         0   -0.0008
```

Program_14 _4 (Mitra)

$$H(z) = \frac{a + b z^{-1}}{1 + c z^{-1}}, \qquad |c| < 1.$$

$$\begin{bmatrix} E_0(z^3) \\ z^{-1}E_1(z^3) \\ z^{-2}E_2(z^3) \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^1 & W_3^2 \\ 1 & W_3^2 & W_3^1 \end{bmatrix}\begin{bmatrix} H(z) \\ H(zW_3^1) \\ H(zW_3^2) \end{bmatrix}$$

$$\begin{aligned} z^{-2}E_2(z^3) &= \tfrac{1}{3}\left[H(z) + W_3^2 H(z\,W_3^1) + W_3^1 H(z\,W_3^2)\right] \\ &= \tfrac{1}{3}\left[\tfrac{a+b\,z^{-1}}{1+c\,z^{-1}} + e^{j4\pi/3}\left(\tfrac{a+b\,e^{j2\pi/3}z^{-1}}{1+c\,e^{j2\pi/3}z^{-1}}\right) + e^{j2\pi/3}\left(\tfrac{a+b\,e^{j4\pi/3}z^{-1}}{1+c\,e^{j4\pi/3}z^{-1}}\right)\right] \\ &= z^{-2}\left(\frac{-bc + ac^2}{1 + c^3 z^{-3}}\right). \end{aligned}$$

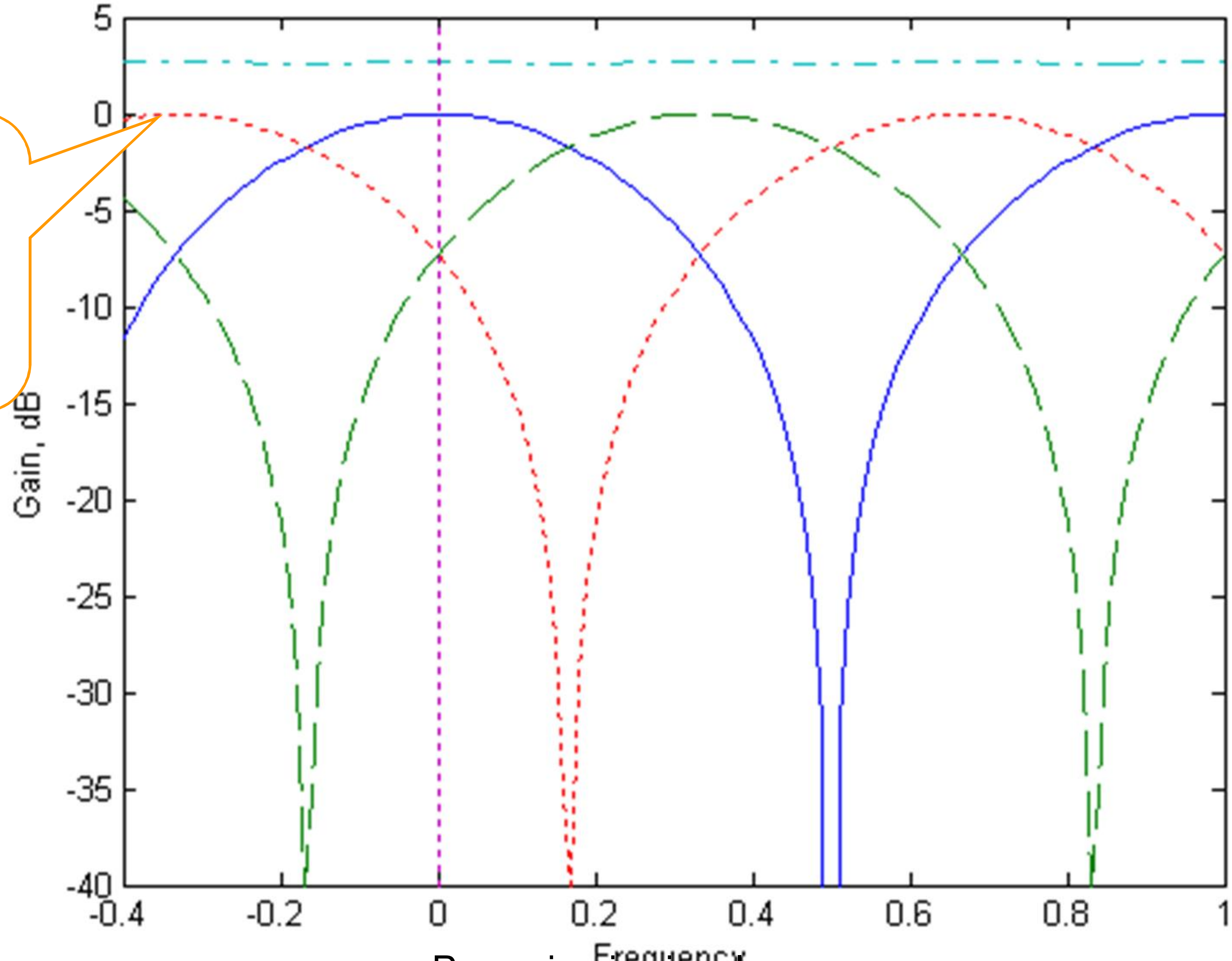
$$E_0(z) = \frac{a + bc^2 z^{-1}}{1 + c^3 z^{-1}}, \qquad E_1(z) = \frac{b - ac}{1 + c^3 z^{-1}}, \qquad E_2(z) = \frac{-bc + ac^2}{1 + c^3 z^{-1}}.$$

Program_14 _4 (3)

```
Wm = exp(i*2*pi/3);

numH1 = numE0+Wm*numE1+Wm^2*numE2;
ha1m = freqz(numH1,denE0,w);
ha1 = 20*log10(abs(freqz(numH1,denE0,w)));
figure; plot(f,ha1); axis([0 1 -40 5])
xlabel('Frequency');ylabel('Gain, dB');title('H1');

numH2 = numE0+Wm^2*numE1+Wm^4*numE2;
ha2m = freqz(numH2,denE0,w);
ha2 = 20*log10(abs(freqz(numH2,denE0,w)));
figure; plot(f,ha2); axis([0 1 -40 5])
xlabel('Frequency');ylabel('Gain, dB');title('H2');
```

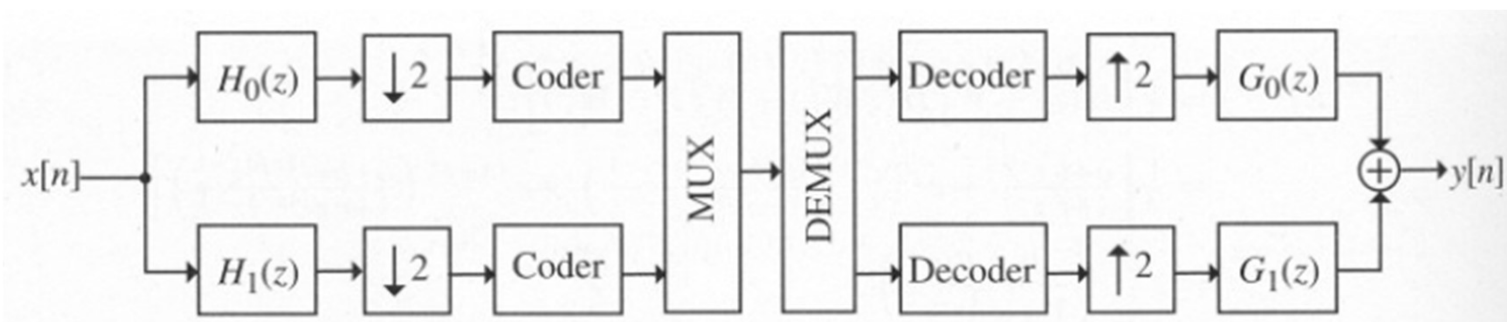
Program_14 _4 (4)

H_1 nije realan filtar, spektar nije simetričan

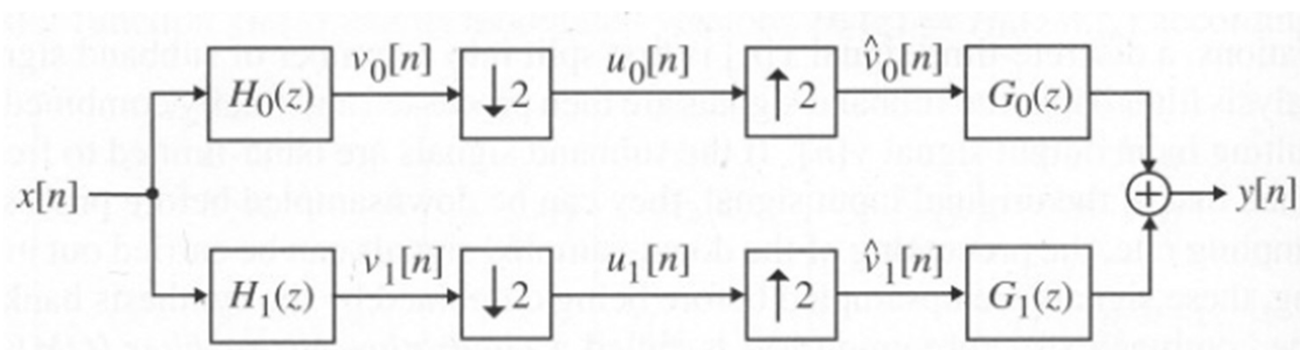


Quadrature-Mirror sistem

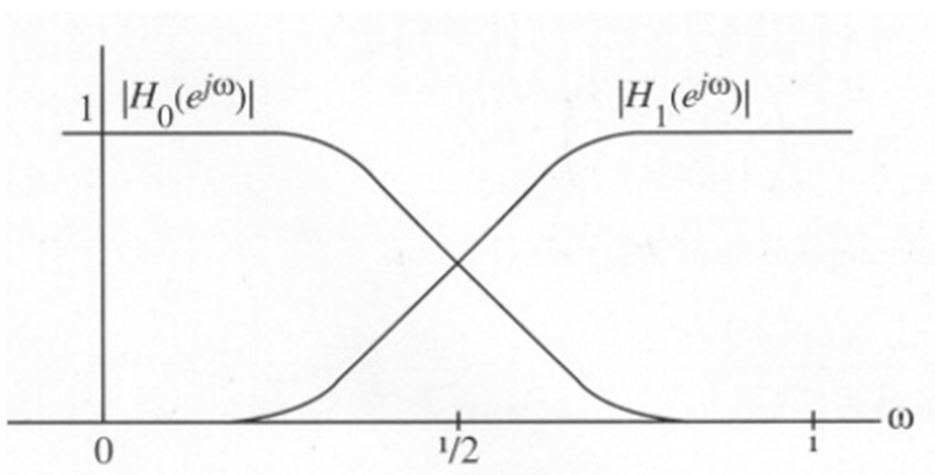
- Ulazni signal se propušta kroz filtre propusnike niskih i visokih učestanosti, sa graničnim frekvencijama na sredini celog opsega, zatim se izvrši downsampling pa kodovanje, i u tom obliku prenosi, arhivira ili obrađuje
- Da bi se regenerisao originalni signal, posle dekodovanja i upsamplinga, propušta se kroz dva filtra, čiji zbir sadrži informaciju iz originalnog signala



QMF banka

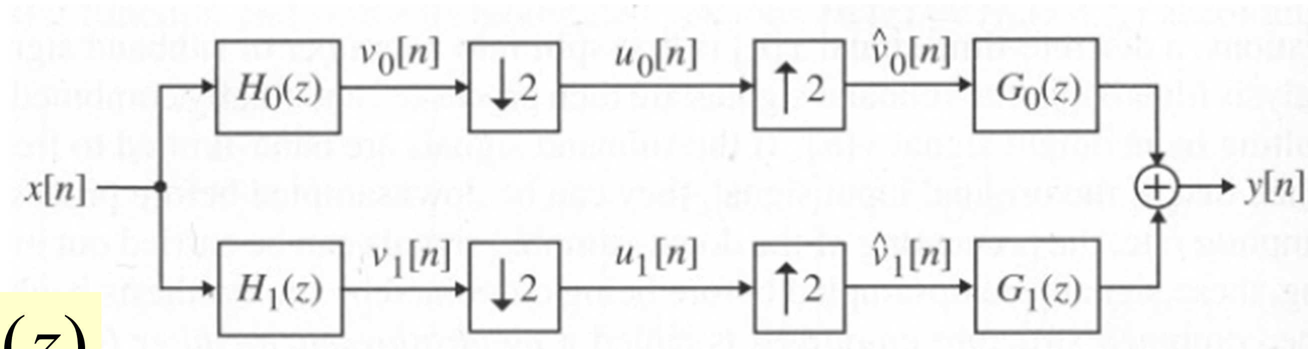


x[n]
$H_0(z)$
$v_0[n]$
↓2
$u_0[n]$
↑2
$\hat{v}_0[n]$
$G_0(z)$
y[n]
$H_1(z)$
$v_1[n]$
↓2
$u_1[n]$
↑2
$\hat{v}_1[n]$
$G_1(z)$



1
$|H_0(e^{j\omega})|$
$|H_1(e^{j\omega})|$
0
1/2
1
ω

QMF banka



$$V_k(z) = H_k(z)X(z)$$

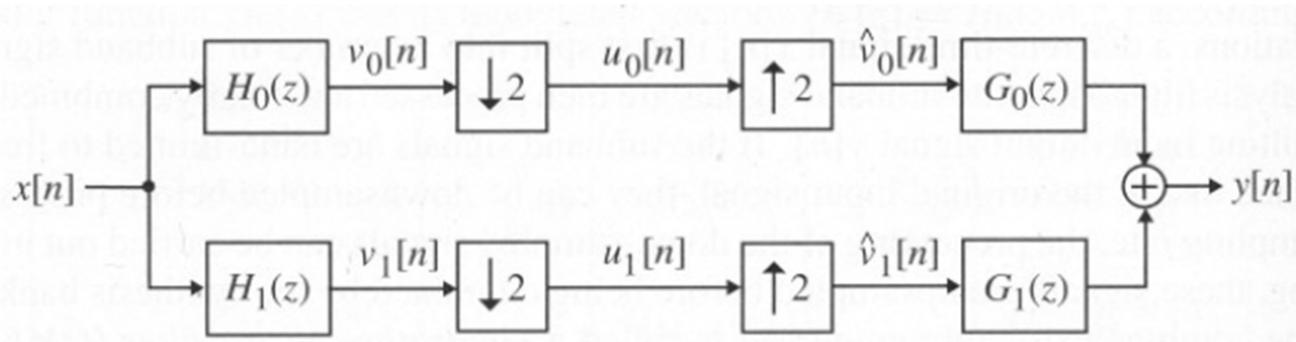
$$U_k(z) = \frac{1}{2}\left(V_k(z^{1/2}) + V_k(-z^{1/2})\right)$$

$$V_k(z) = U_k(z^2)$$

$$\hat{V}_k(z) = \frac{1}{2}\left(H_k(z)X(z) + H_k(-z)X(-z)\right)$$

$$Y(z) = T(z)X(z) + A(z)X(-z)$$

QMF banka



$$Y(z) = T(z)X(z) + A(z)X(-z)$$

$$T(z) = \frac{1}{2}\left(H_0(z)G_0(z) + H_1(z)G_1(z)\right)$$

Distortion Transfer Function

$$A(z) = \frac{1}{2}\left(H_0(-z)G_0(z) + H_1(-z)G_1(z)\right)$$

Aliasing Term

Preklapanje spektra

- Uslov da ne postoji preklapanje spektra (aliasing)

$$A(z) = 0$$

$$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0$$

$$Y(z) = T(z)X(z)$$

Funkcije prenosa

$$Y(z) = T(z)X(z)$$

$$\left|T(e^{j\omega})\right| = d \neq 0$$

Magnitude preserving

$$T(e^{j\omega}) = d\ e^{j(\alpha\omega+\beta)}$$

Phase preserving

$$T(z) = z^{-l}$$

Perfect reconstruction

QMF banka sa perfektnom rekonstrukcijom

$$H_0(z) = \frac{1}{\sqrt{2}}\left(1 + z^{-1}\right)$$

$$H_1(z) = \frac{1}{\sqrt{2}}\left(1 - z^{-1}\right)$$

$$G_0(z) = \frac{1}{\sqrt{2}}\left(1 + z^{-1}\right)$$

$$G_1(z) = \frac{1}{\sqrt{2}}\left(-1 + z^{-1}\right)$$

$$T(z) = \frac{1}{2} z^{-1}$$

$$A(z) = 0$$

QMF banka bez preklapanja spektra

$$G_0(z) = H_1(-z)$$

$$G_1(z) = -H_0(-z)$$

$$T(z) = \frac{1}{2}\left(H_0(z)H_1(-z) - H_0(-z)H_1(z)\right)$$

$$A(z) = 0$$

Parni stepeni po z se poništavaju

QMF banka
bez preklapanja spektra

$$G_0(z) = H_0(z)$$

H_0 je filtar propusnik niskih učestanosti

$$H_1(z) = H_0(-z)$$

H_1 je filtar propusnik visokih učestanosti

$$G_1(z) = -H_0(-z)$$

$$\left|H_1(e^{j\omega})\right| = \left|H_0(e^{j(\pi-\omega)})\right|$$

Slika u ogledalu u odnosu na $\pi/2$
Ova učestanost se naziva kvadraturna učestanost

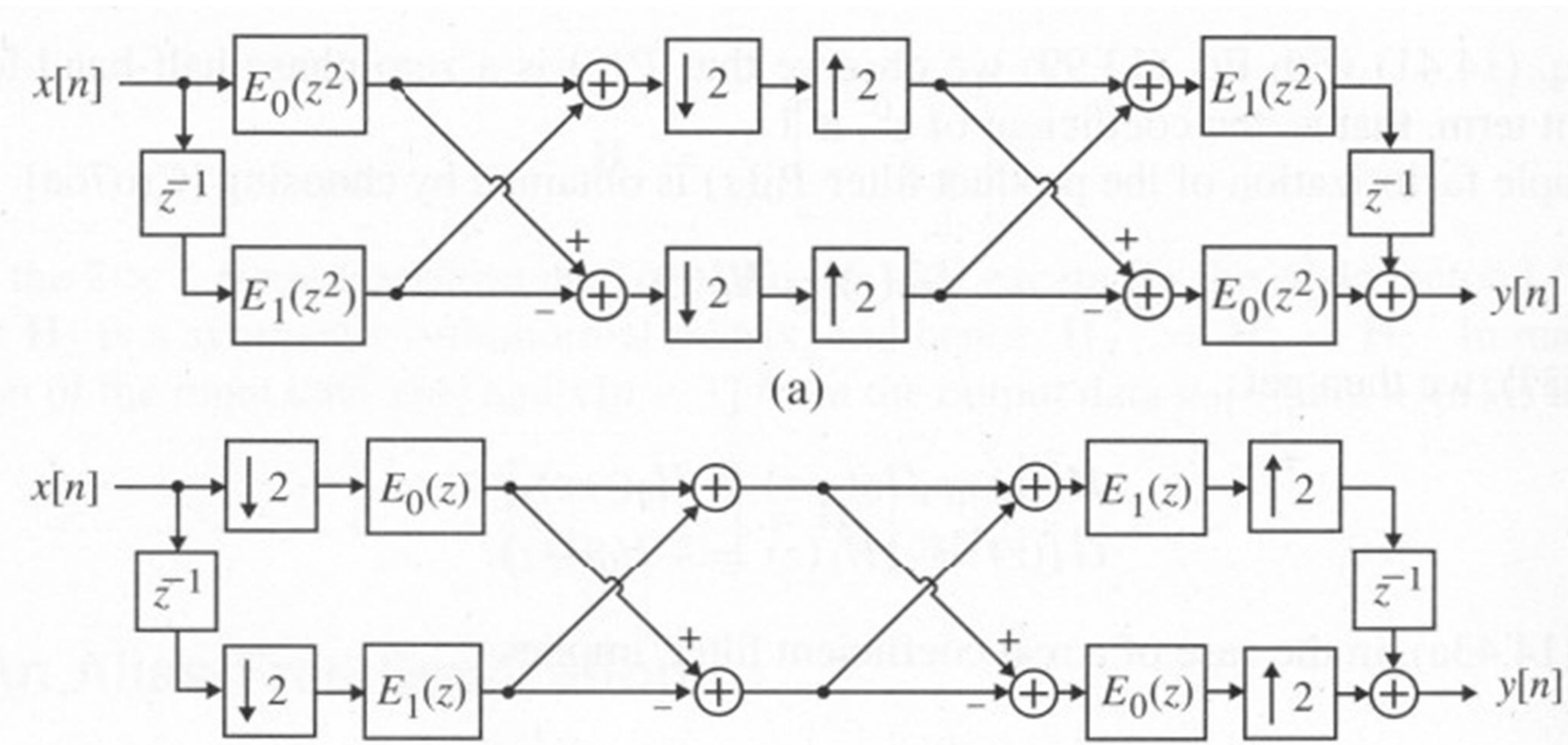
Polifazna realizacija

$$H_0(z) = E_0(z^2) + z^{-1}E_1(z^2)$$

$$H_1(z) = E_0(z^2) - z^{-1}E_1(z^2)$$

$$T(z) = 2z^{-1}E_0(z^2)E_1(z^2)$$

Efikasna realizacija



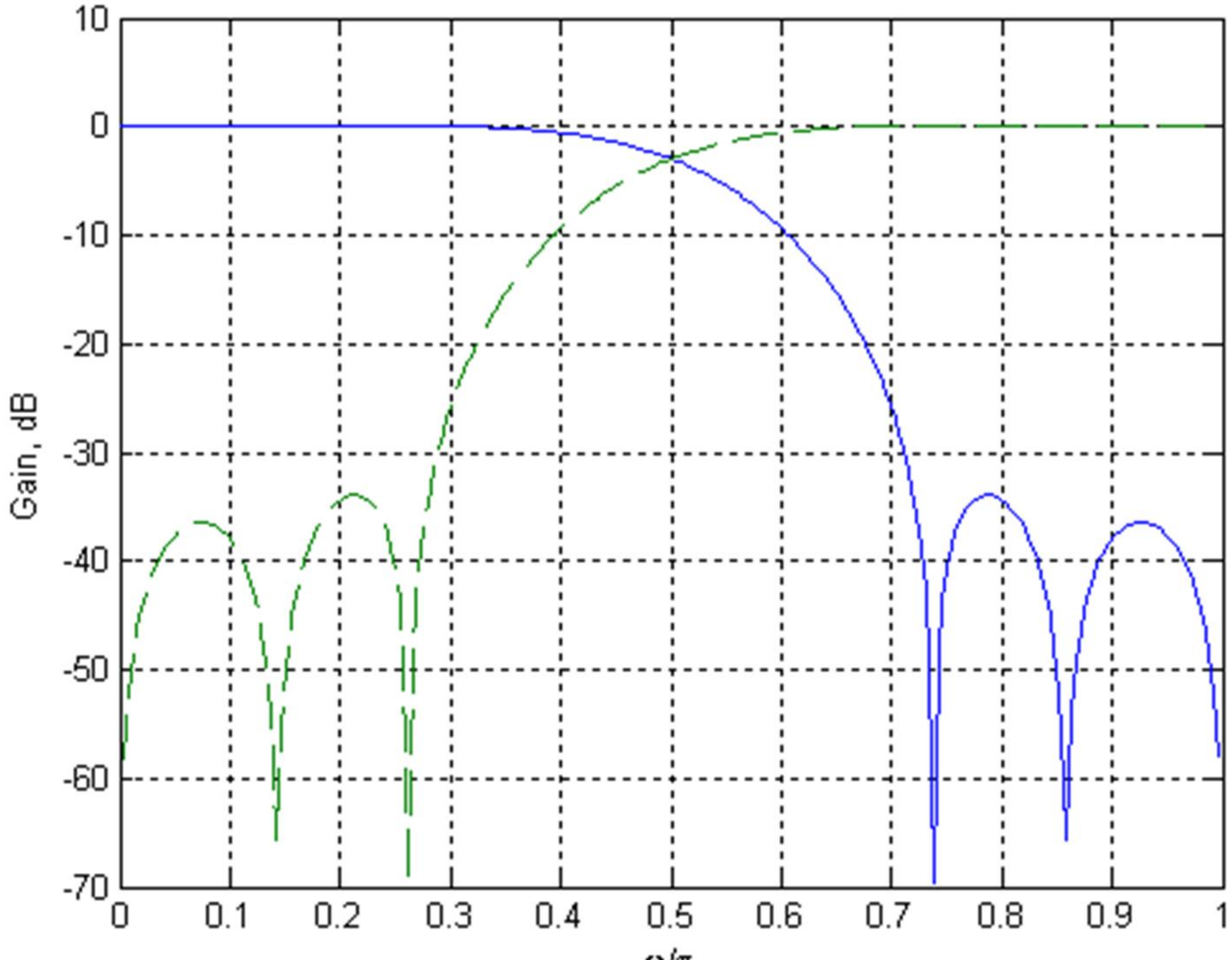
FIR QMF: Program_14 _1

```
B1 = [-0.006443977 0.02745539 -0.00758164 ...
     -0.0913825 0.09808522 0.4807962];
B1 = [B1 fliplr(B1)];
L = length(B1);
for k = 1:L
   B2(k) = ((-1)^k)*B1(k);
end
[H1z, w] = freqz(B1, 1, 256);
h1 = abs(H1z); g1 = 20*log10(h1);
[H2z, w] = freqz(B2, 1, 256);
h2 = abs(H2z); g2 = 20*log10(h2);
```

$$H_1(z) = H_0(-z)$$

H_0 je filtar propusnik niskih učestanosti

Program_14 _1 (2)



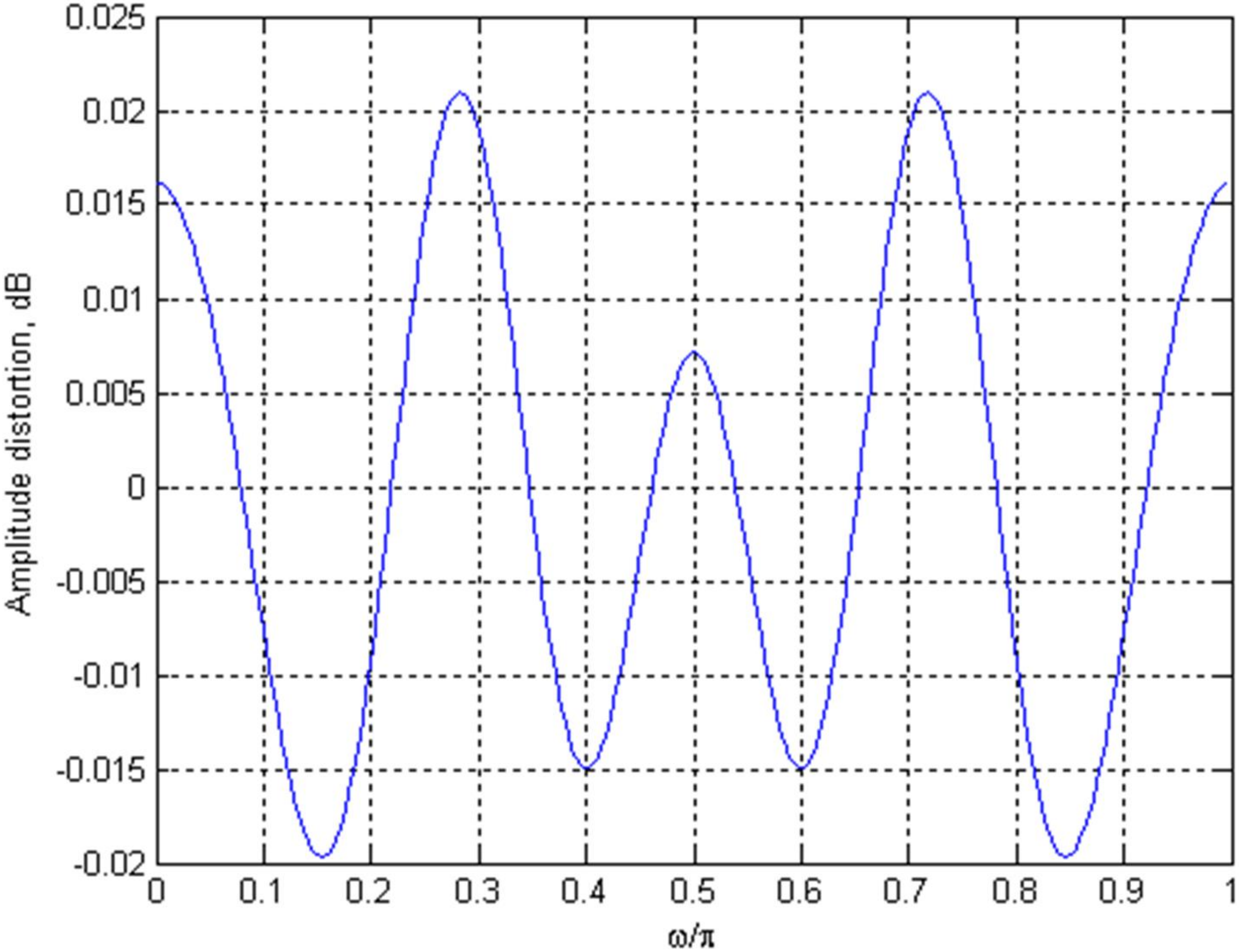
Program_14 _1 (3)

```
for i = 1:256,
    sum(i) = (h1(i)*h1(i)) + (h2(i)*h2(i));
end
d =10*log10(sum);
plot(w/pi,d);grid;
xlabel('\omega/\pi');
ylabel('Amplitude distortion, dB')
```

$$\left|H_0(e^{j\omega})\right|^2 + \left|H_1(e^{j\omega})\right|^2$$

Distorzija

Program_14 _1 (3)



Samo ovaj FIR ima perfektnu rekonstrukciju

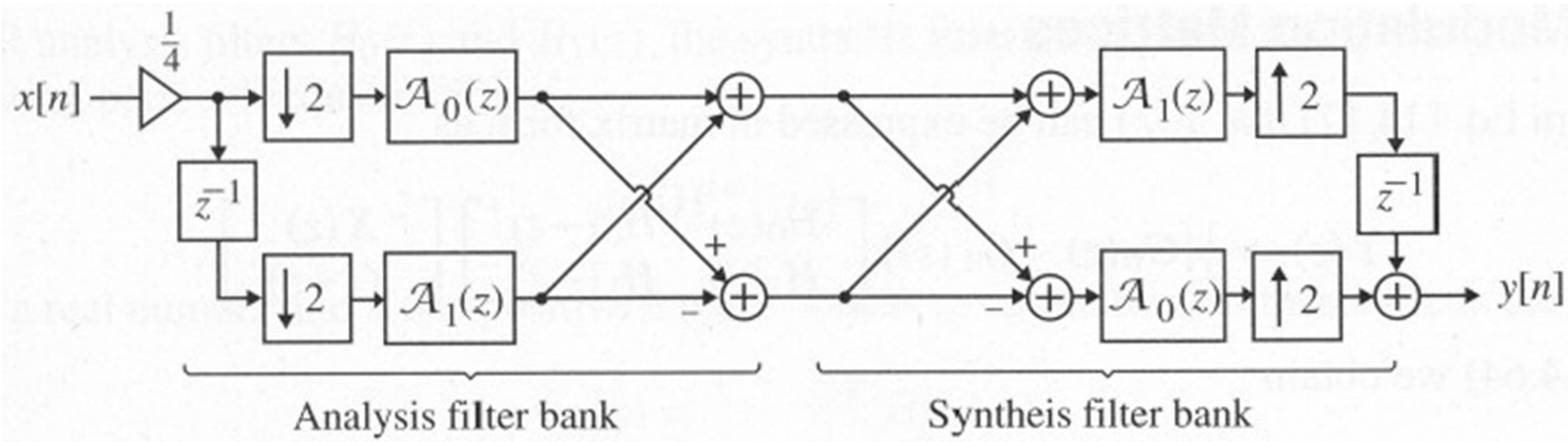
$$H_0(z) = \frac{1}{\sqrt{2}}\left(1 + z^{-1}\right)$$

$$G_0(z) = \frac{1}{\sqrt{2}}\left(1 + z^{-1}\right)$$

$$H_1(z) = \frac{1}{\sqrt{2}}\left(1 - z^{-1}\right)$$

$$G_1(z) = \frac{1}{\sqrt{2}}\left(-1 + z^{-1}\right)$$

IIR realizacija



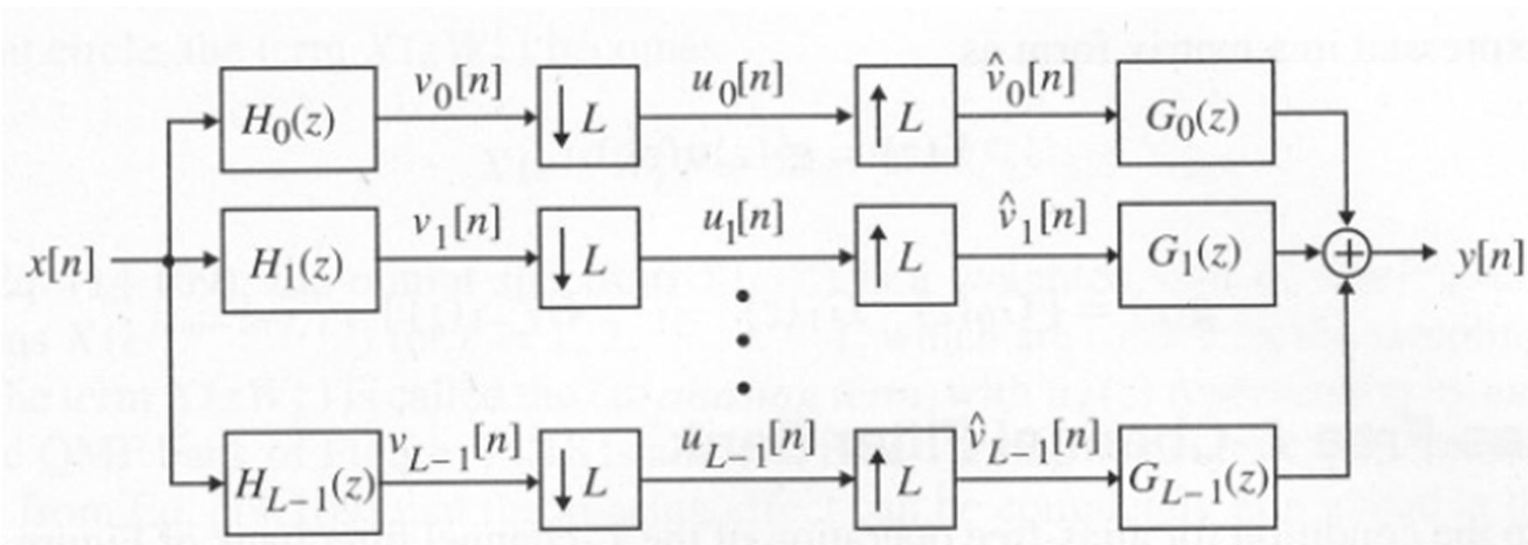
$$H_0(z) = A_0(z^2) + z^{-1}A_1(z^2)$$

$$H_1(z) = A_0(z^2) - z^{-1}A_1(z^2)$$

$$T(z) = 2z^{-1}A_0(z^2)A_1(z^2)$$

Butterworth i eliptički filtri imaju allpass filtre u granama.
Nema preklapanja spektra

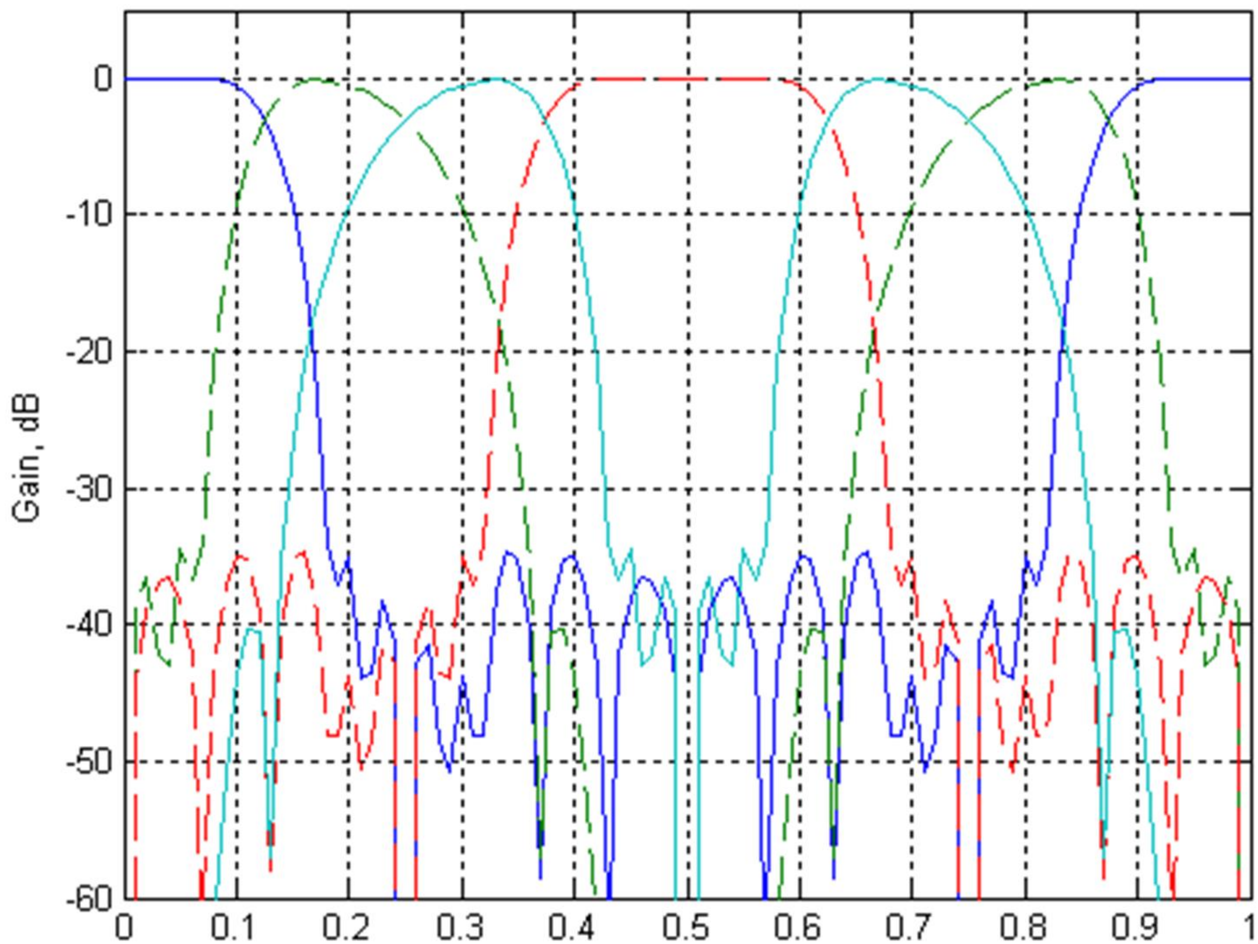
L-kanalna filtarska banka



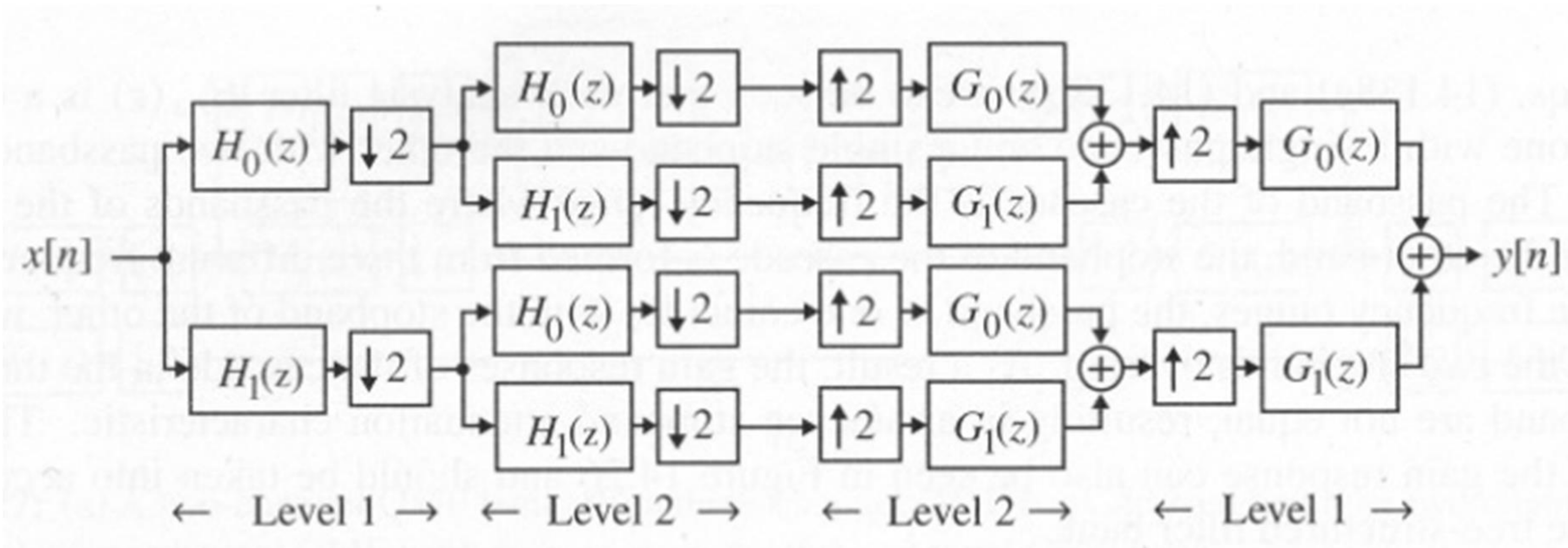
Program_14 _2

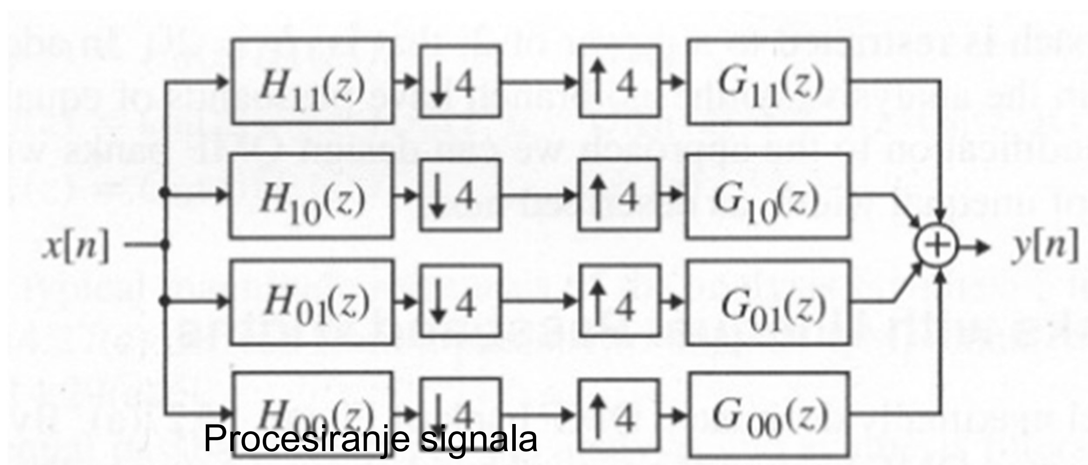
```
B1=[-0.00644 0.0274554 -0.00758 -0.09138 0.098085 0.4808];
B1 = [B1 fliplr(B1)];  L = length(B1);
for k = 1:L
   B2(k) = ((-1)^k)*B1(k);
end
B10 = zeros(1, 2*length(B1));
B10([1: 2: length(B10)]) = B1;
B11 = zeros(1, 2*length(B2));
B11([1: 2: length(B11)]) = B2;
C0 = conv(B1, B10);C1 = conv(B1, B11);
C2 = conv(B2, B10);C3 = conv(B2, B11);
f = 0:0.01:1; w = 2*pi*f;
H00z=freqz(C0,1,w);  h00=abs(H00z); M00=20*log10(h00);
H01z=freqz(C1,1,w);  h01=abs(H01z); M01=20*log10(h01);
H10z=freqz(C2,1,w);  h10=abs(H10z); M10=20*log10(h10);
H11z=freqz(C3,1,w);  h11=abs(H11z); M11=20*log10(h11);
plot(f, M00,'-',f, M01,'--',f, M10,'--',f,M11,'-');
```

Program_14 _2

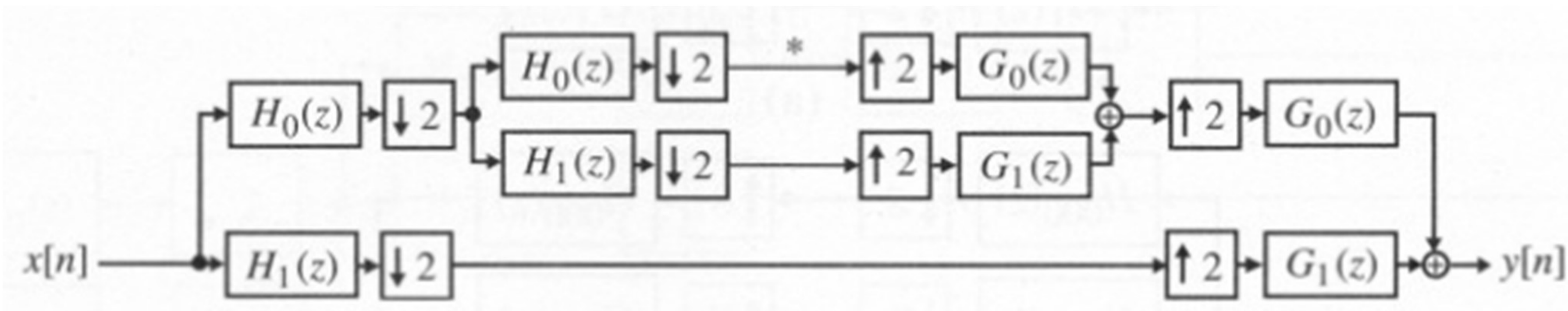


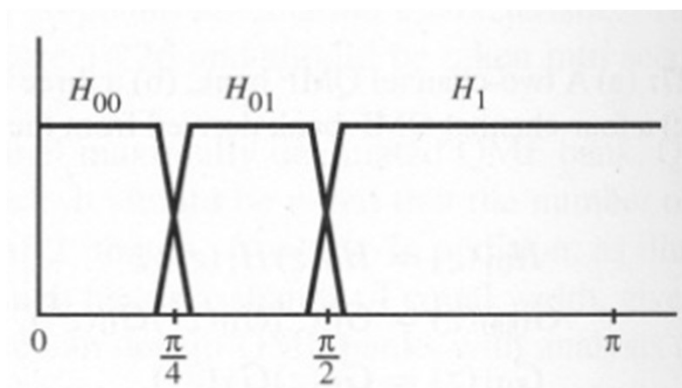
4-kanalna filtarska banka



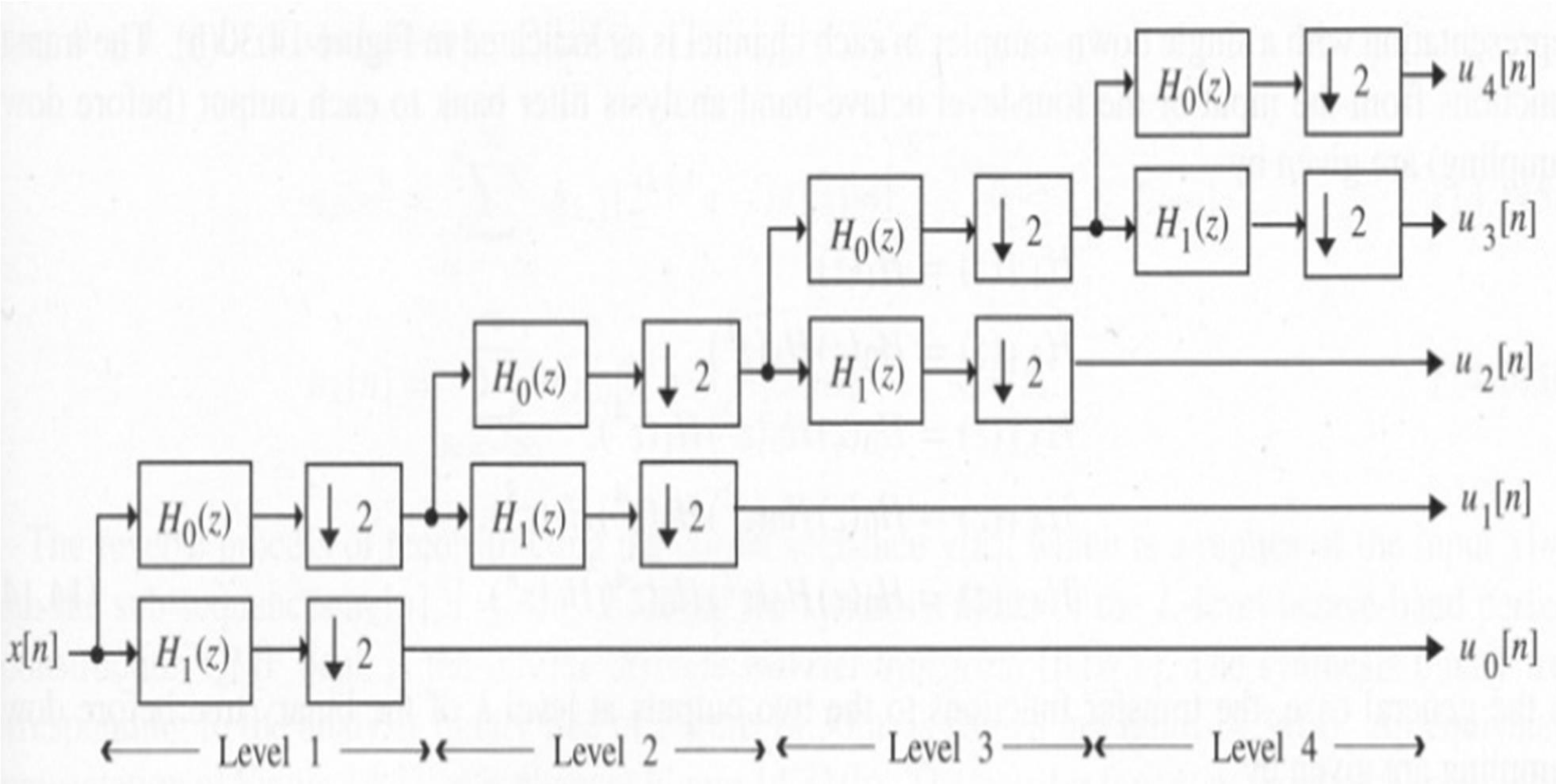


Banke nejednake širine opsega



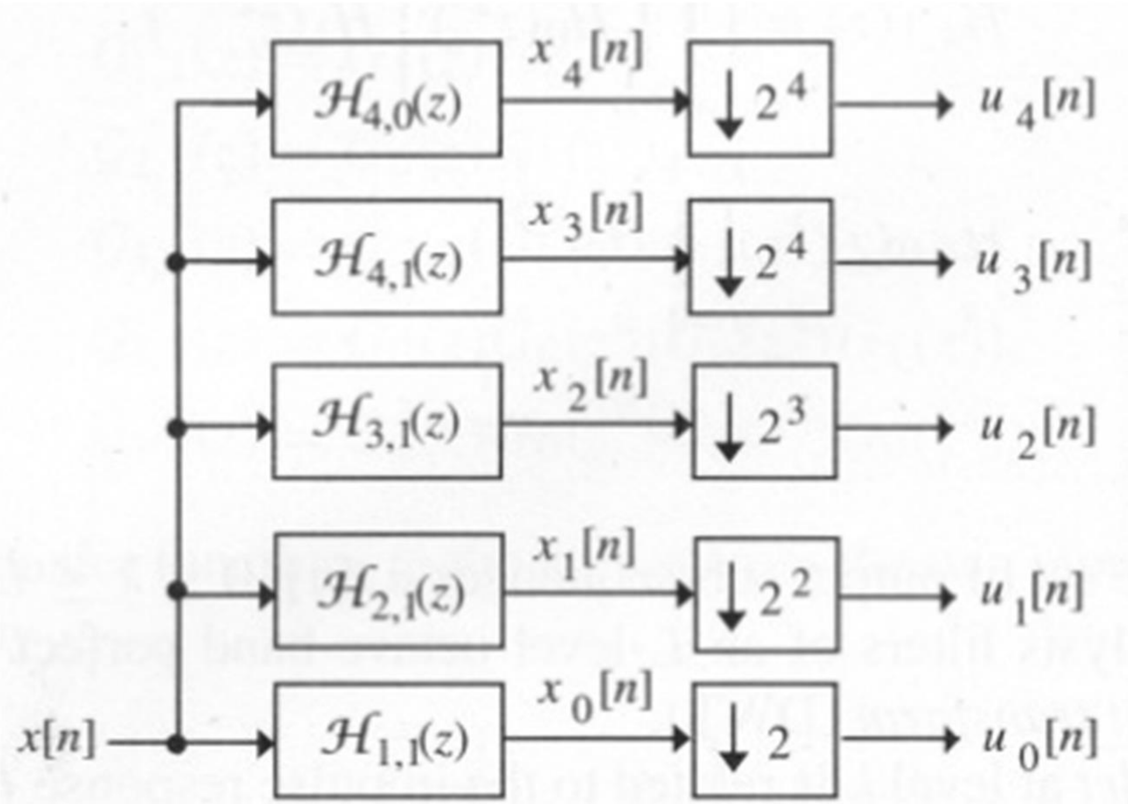


Višestepene filtarske banke



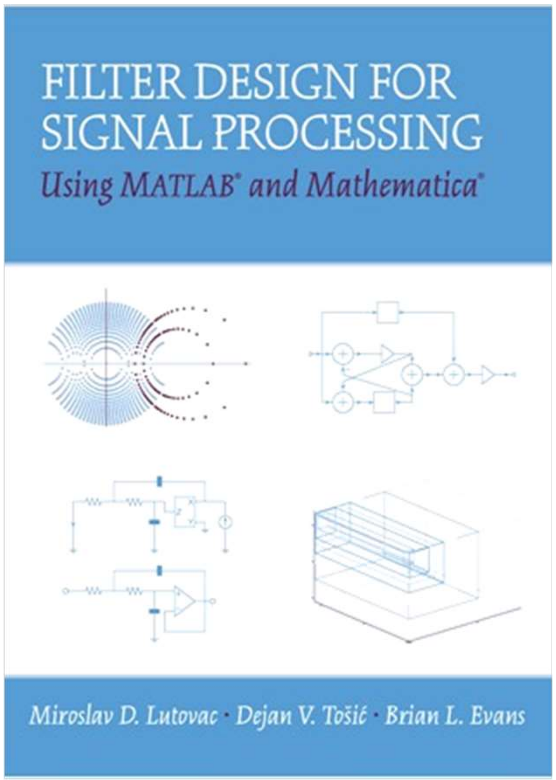
(a)

Filtarska banka:
4 nivoa oktavnih opsega

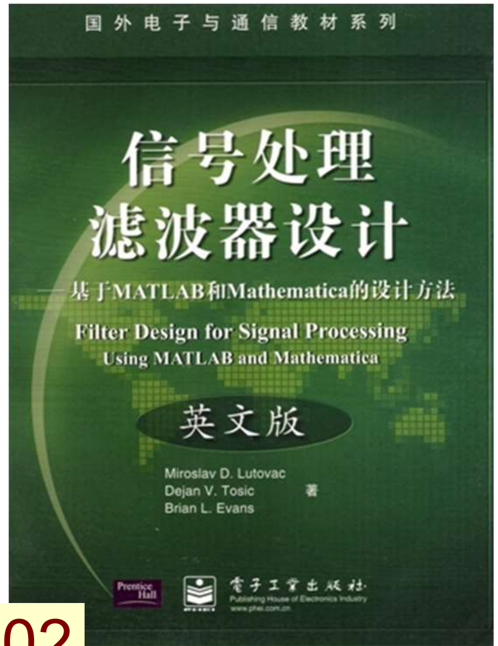


$$\mathcal{H}_{1,1}(z) = H_1(z),$$
$$\mathcal{H}_{2,1}(z) = H_0(z)H_1(z^2),$$
$$\mathcal{H}_{3,1}(z) = H_0(z)H_0(z^2)H_1(z^4),$$
$$\mathcal{H}_{4,1}(z) = H_0(z)H_0(z^2)H_0(z^4)H_1(z^8),$$
$$\mathcal{H}_{4,0}(z) = H_0(z)H_0(z^2)H_0(z^4)H_0(z^8).$$

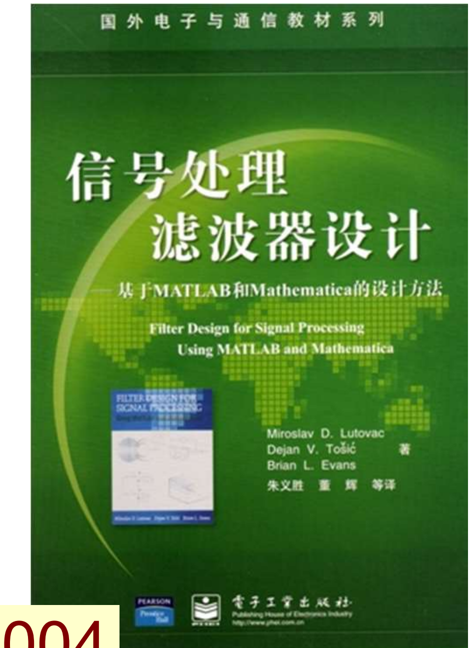
Further reading



2001



2002



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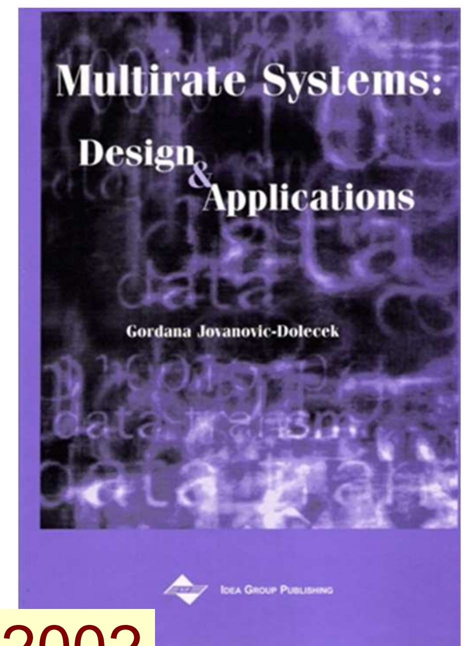

2002

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