DBBT
Digital Broadcasting &
Broadband Technologies

Co-funded by the
Erasmus+ Programme
of the European Union

Procesiranje signala

Profesor dr Miroslav Lutovac

"This project has been funded with support from the European Commission. This publication [communication] reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein"

Parseval-ova teorema

Transformacija je simetrična
pri čemu * označava
konjugovano kompleksnu vrednost

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

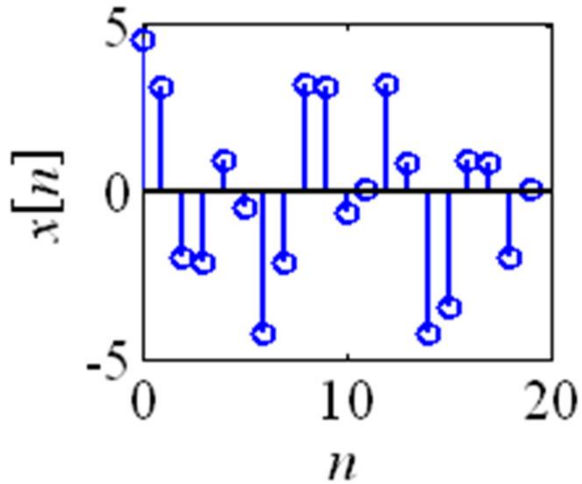
Pojam snage

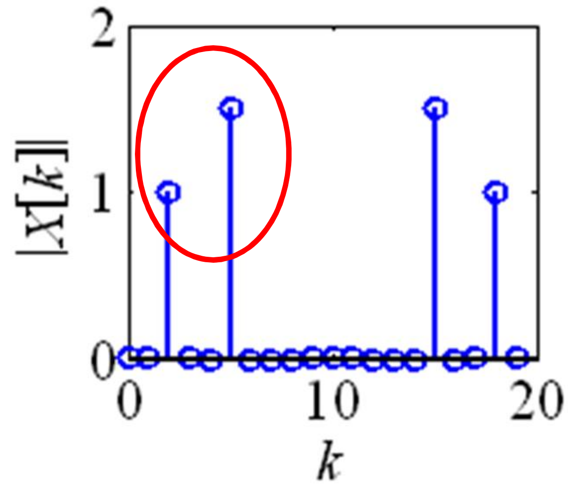
Normalizovana transformacija

Normalizovana transformacija da bi moduo u transformaciji odgovarao amplitudi sinusoide

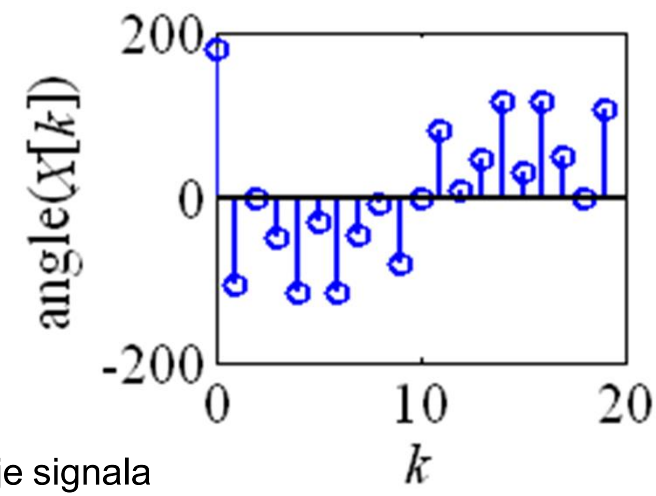
$$\{X_{\mathrm{norm}}[k]\} = \{\frac{1}{N} X[k]\}$$

FFT – sa normalizacijom

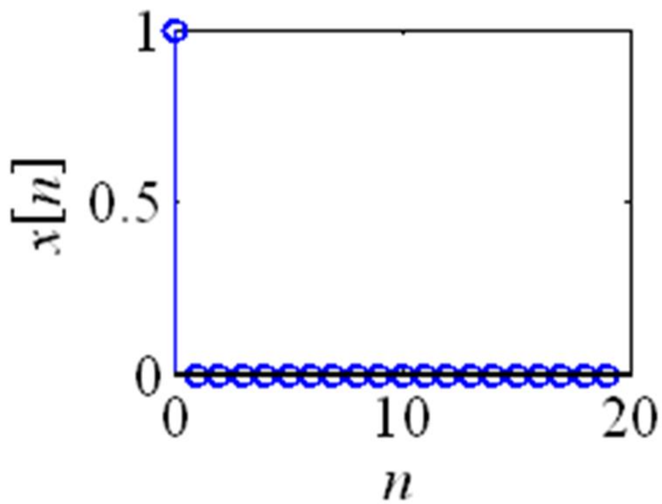


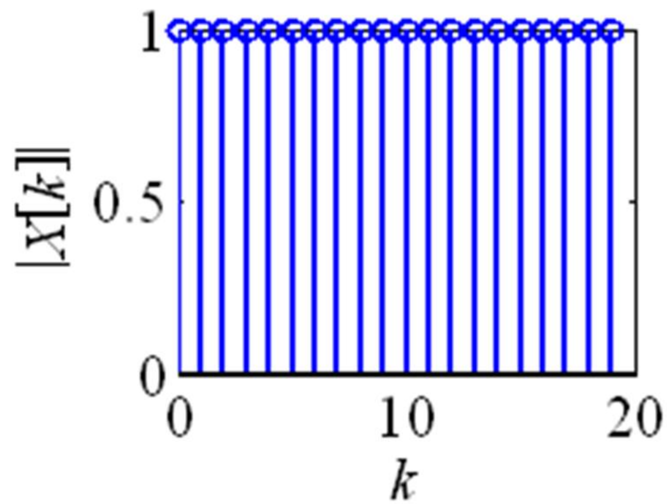



```
N=20; n=0:N-1;
k1=2;    k2=5;
f1=k1/N; f2=k2/N;
fi2=-pi/6;
x = 2*cos(2*pi*f1*n+0)+...
    3*cos(2*pi*f2*n+fi2);
X = fft(x)
. . .
stem(n,abs(X)/N)
```

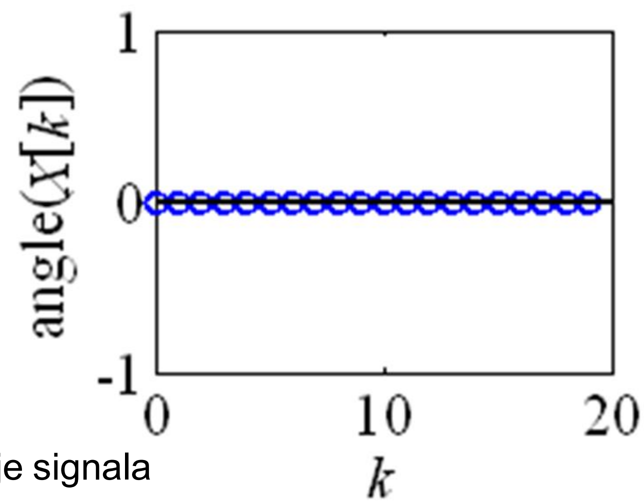


DFT jedinične funkcije

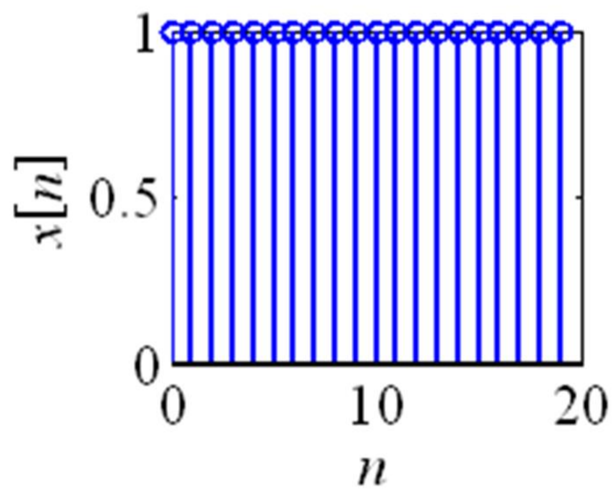


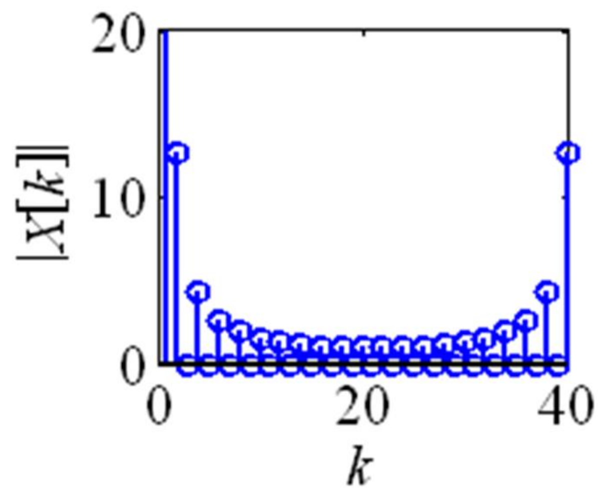



```
N=20; n=0:N-1;
x = (n==0);
X = fft(x)
. . .
stem(n,abs(X)/N)
```

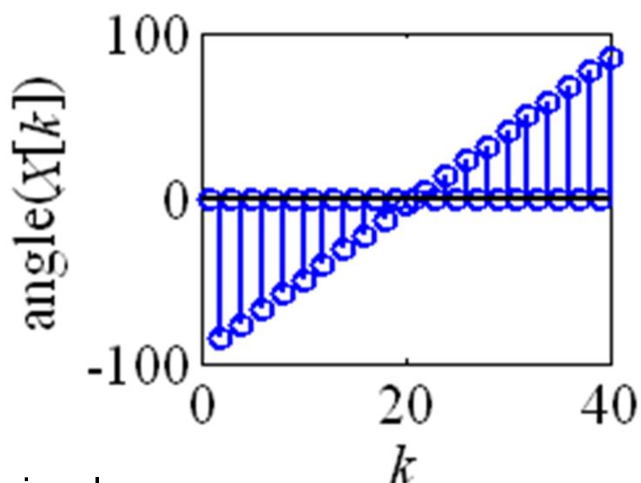


DFT odskočne funkcije

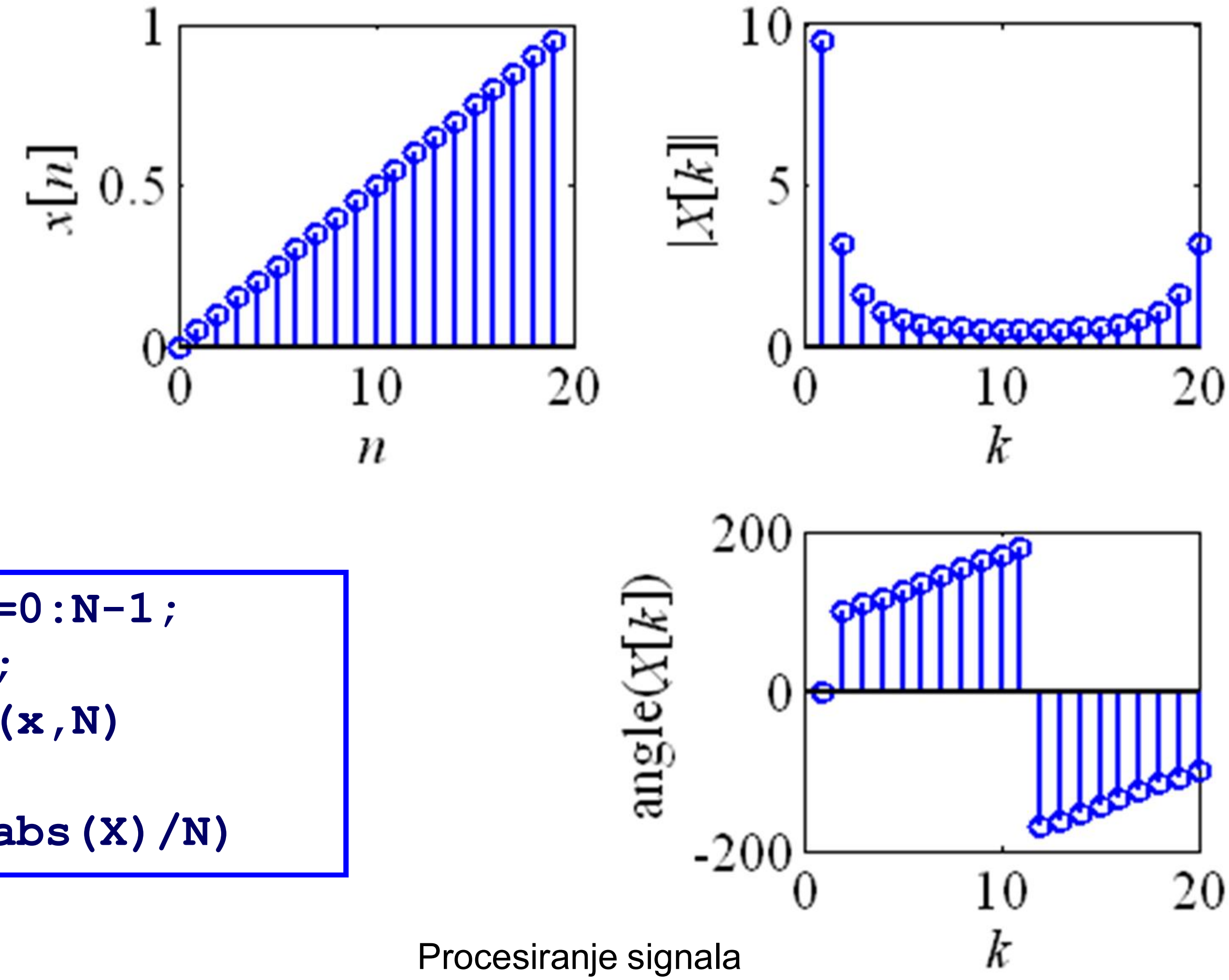





```
N=20; n=0:N-1;
x = (n>=0);
X = fft(x,2*N)
. . .
stem(n,abs(X)/N)
```

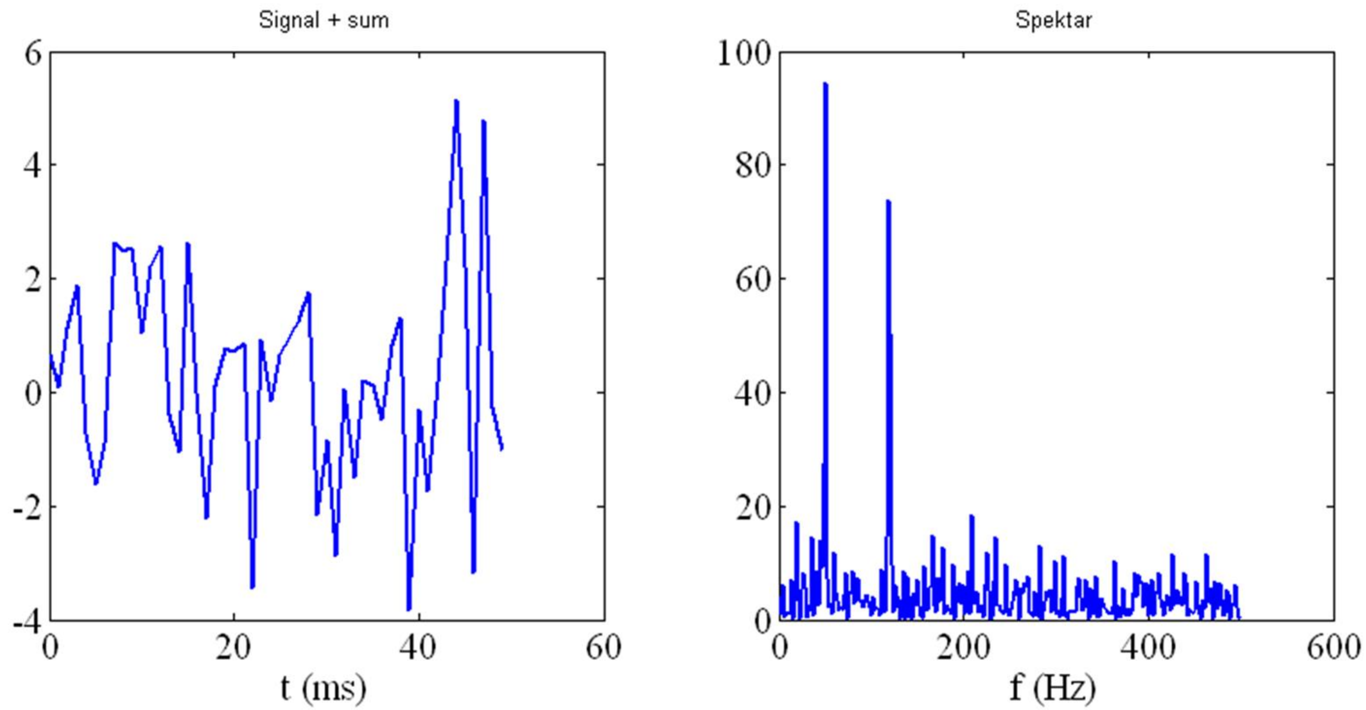


DFT rastuće funkcije




```
N=20; n=0:N-1;
x = n/N;
X = fft(x,N)
. . .
stem(n,abs(X)/N)
```

DFT kontinualne funkcije




```
t = 0:0.001:0.6;
x = sin(2*pi*50*t)+sin(2*pi*120*t);
y = x + 2*randn(size(t));
Y = fft(y,512);
Pyy = Y.* conj(Y) / 512;
f = 1000*(0:256)/512;
```

Inverzna DFT

Sekvenca u vremenskom domenu $x[n]$ dužine N preslikava se u sekvencu $X[k]$, iste dužine gde se članovi računaju po formuli

$$\boxed{x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn}}$$

Matlab **`ifft`** realizuje IDFT

x=ifft[X]

```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128]
X = fft(x)
y = ifft(X)
```

```
x =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

```
X =
  Columns 1 through 4
   1.9922  1.1861-0.6487i 0.7969-0.3984i 0.6889-0.1799i
  Columns 5 through 8
   0.6641  0.6889+0.1799i 0.7969+0.3984i 1.1861+0.6487i
```

```
y =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

DFT u više tačaka

- Y=fft(X,n) returns the n-point DFT
- If the length of X is less than n,
  X is padded with trailing zeros to length n
- If the length of X is greater than n,
  the sequence X is truncated
- When X is a matrix,
  the length of the columns
  are adjusted in the same manner

Matlab **ifft** realizuje IDFT

```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128]
X = fft(x,10)
y = ifft(X)
```

```
x =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

```
X =
  Columns 1 through 4
   1.9922  1.1861-0.6487i 0.7969-0.3984i 0.6889-0.1799i
  Columns 5 through 8
   0.6641  0.6889+0.1799i 0.7969+0.3984i 1.1861+0.6487i
```

```
y =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078 0 0
```

Matlab **`fft`** i **`ifft`**

```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128]
X = fft(x,10)
y = ifft(X)
```

```
x =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

```
X =
  Columns 1 through 4
   1.9922  1.1861-0.6487i 0.7969-0.3984i 0.6889-0.1799i
  Columns 5 through 8
   0.6641  0.6889+0.1799i 0.7969+0.3984i 1.1861+0.6487i
```

```
y =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078 0 0
```

Matlab `fft` i `ifft`

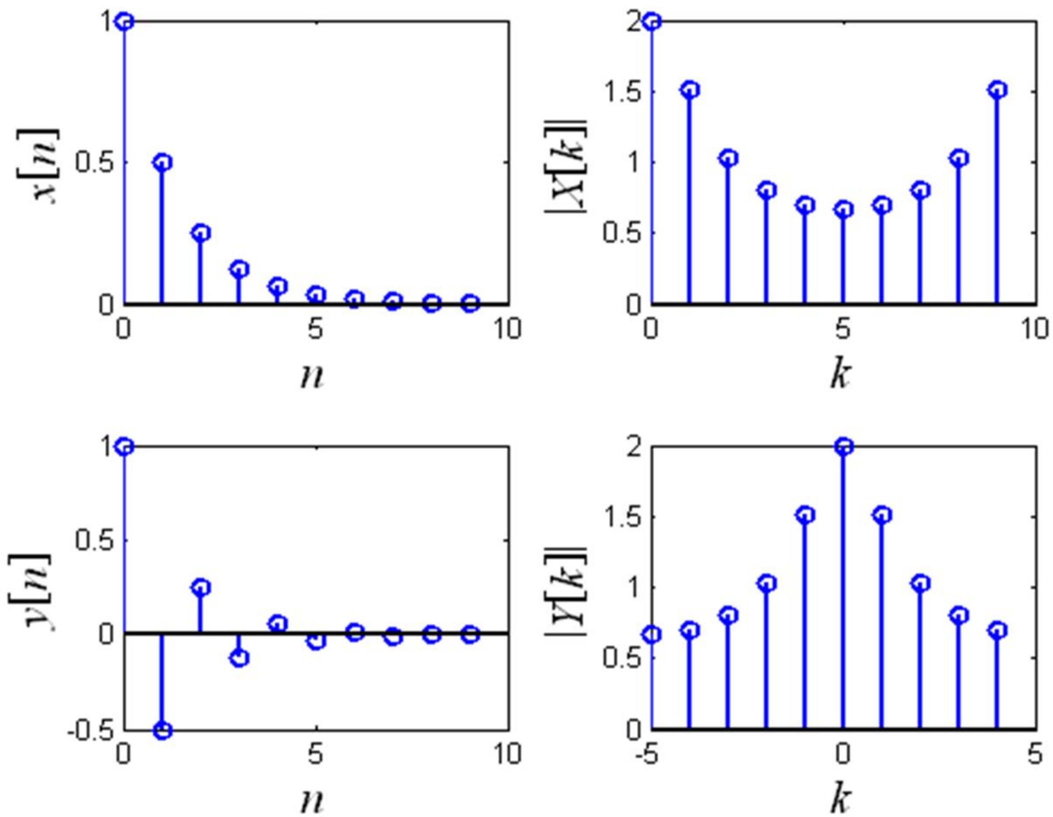
```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128]
X = fft(x,6)
y = ifft(X)
```

```
x =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

```
X =
  Columns 1 through 4
   1.9922  1.1861-0.6487i 0.7969-0.3984i 0.6889-0.1799i
  Columns 5 through 8
   0.6641  0.6889+0.1799i 0.7969+0.3984i 1.1861+0.6487i
```

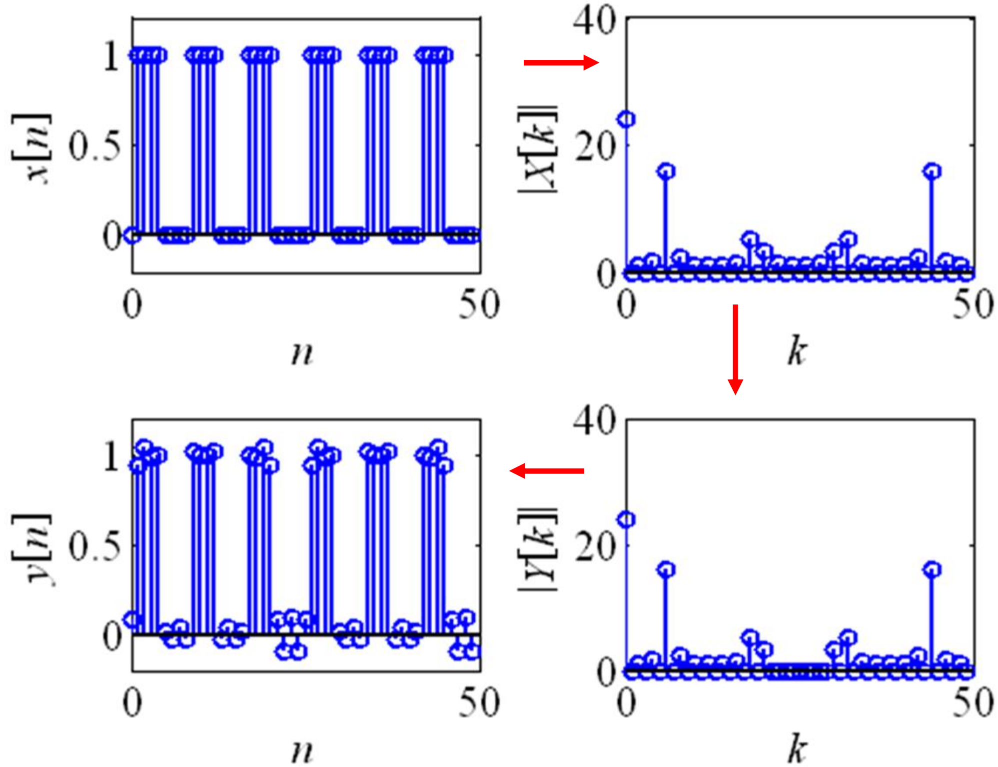
```
y =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313
```

Šiftovana DFT



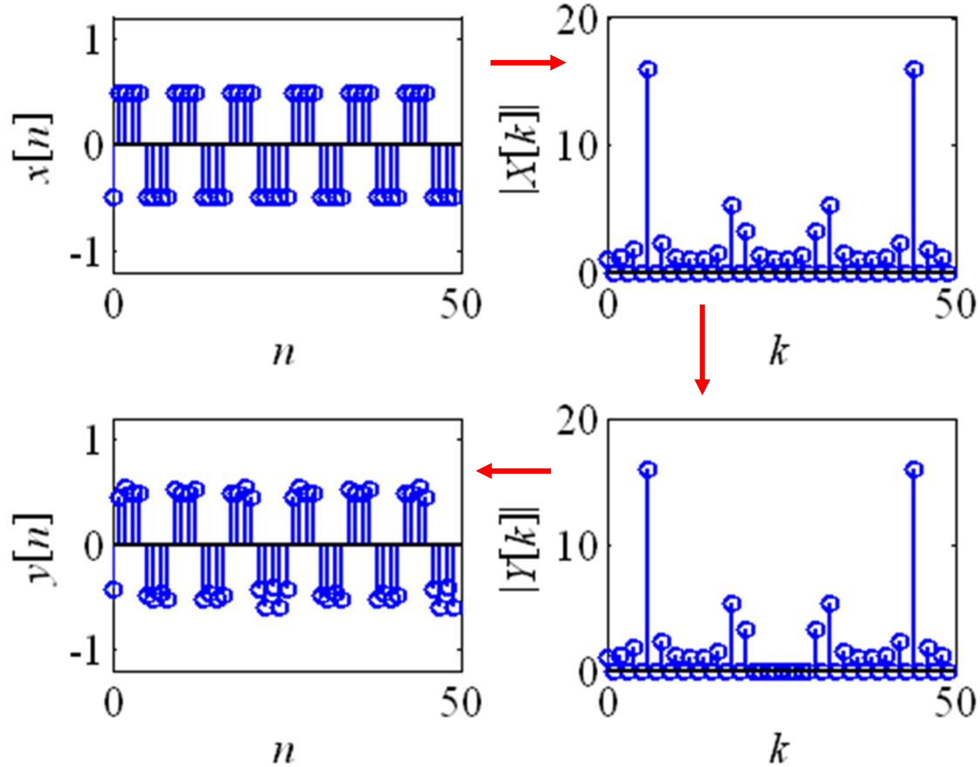

```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128 1/256 1/512]
X = fft(x)
Y = fftshift(X)
y = ifft(Y)
n = 0:length(x)-1;
```

Uticaj promene spektra



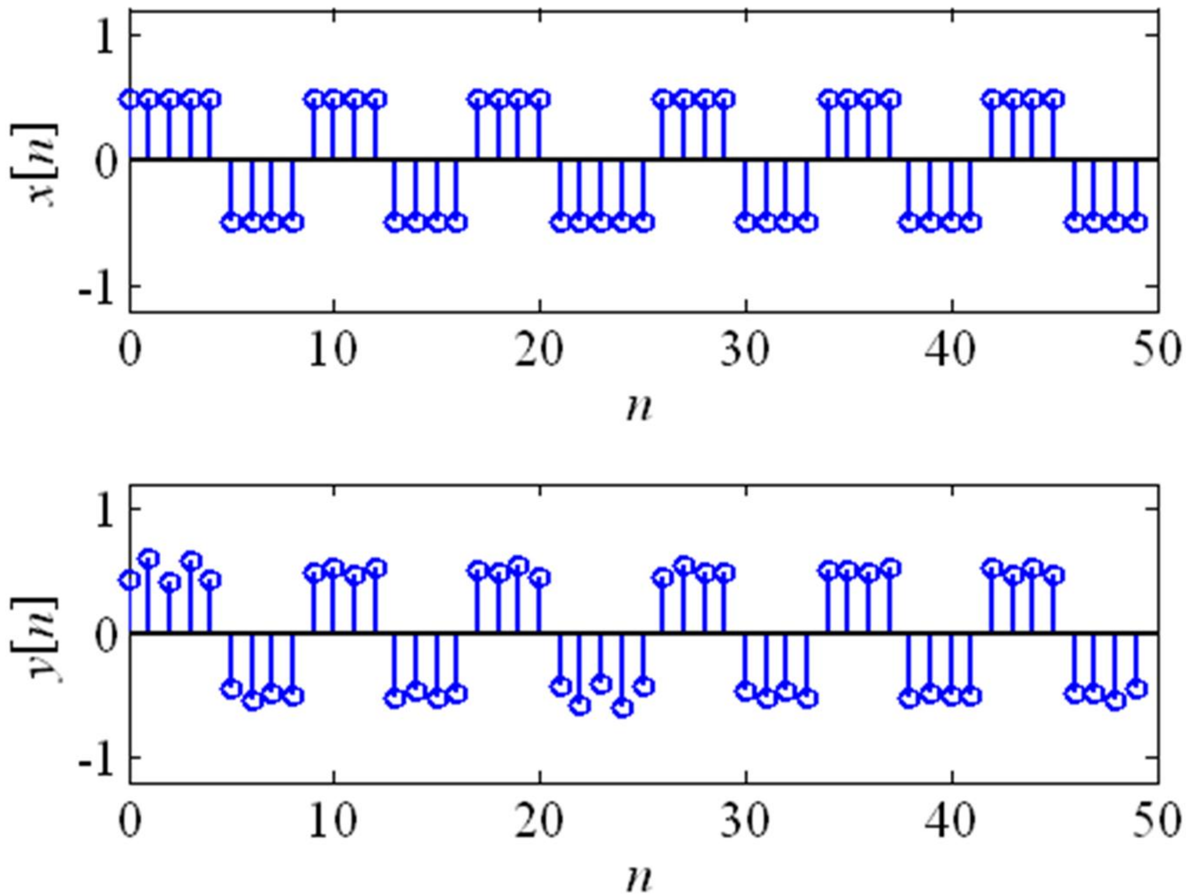

```
N=50; n=0:N-1; k=6; f=k/N;
x=2*sin(2*pi*f*n); x=x>0.25;
X=fft(x); Y=X; n2=3;
Y(N/2-n2:N/2+n2+1)=0*Y(N/2-n2:N/2+n2+1);
```

Uticaj promene spektra, DC=0




```
N=50; n=0:N-1; k=6; f=k/N;
x=2*sin(2*pi*f*n); x=(x>0)-0.5;
X=fft(x); Y=X; n2=3;
Y(N/2-n2:N/2+n2+1)=0*Y(N/2-n2:N/2+n2+1);
```

Uticaj ograničenja spektra



Odziv IIR sistema u vremenskom domenu

$$y[n] = \sum_{k=0}^{N} b_k x[n-k] - \sum_{k=1}^{M} a_k y[n-k]$$

Izlazni signal

Nerekurzivni deo

Rekurzivni deo

Diskretna Furijeova Transformacija DFT

Sekvenca u vremenskom domenu $x[n]$
preslikava se u
sekvencu u frekvencijskom domenu $X[k]$

$$\boxed{X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}, \quad 0 \le k \le N-1}$$

Inverzna DFT

Sekvenca u frekvencijskom domenu $X[k]$ preslikava se u sekvencu u vremenskom domenu $x[n]$

$$\boxed{x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn}}$$

DFT diskretnih signala

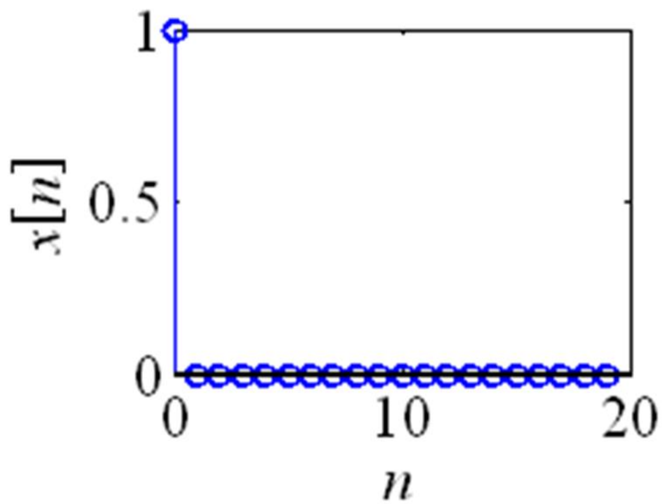
Uređeni par

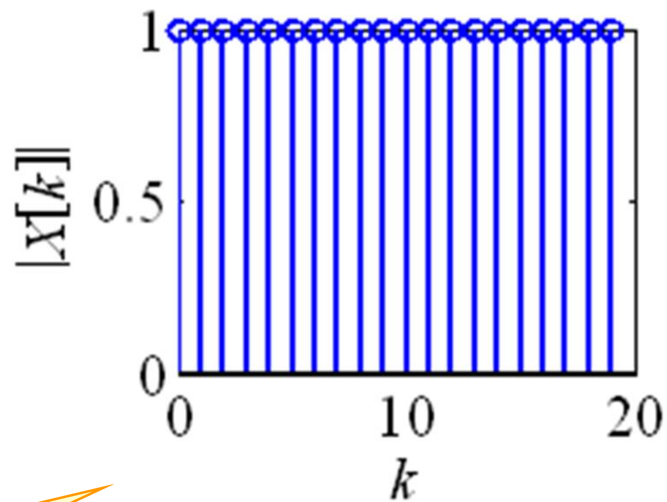
sekvenca u vremenskom domenu $x[n]$

sekvenca transformacije $X[k]$

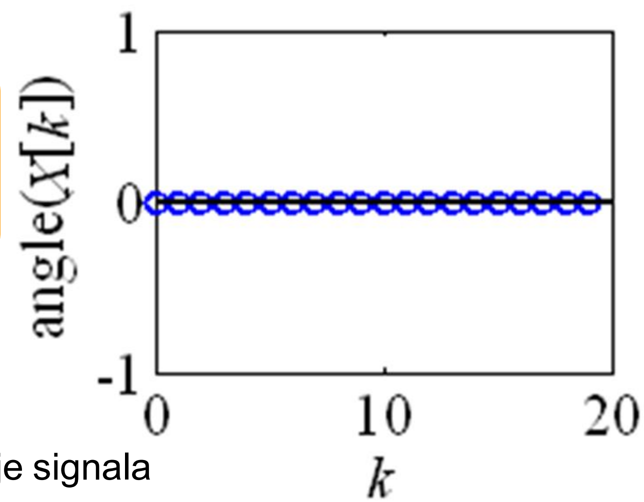
$$\boxed{\{x[n] \leftrightarrow \{X[k]\}\}} \qquad \boxed{\{y[n] \leftrightarrow \{Y[k]\}\}}$$

DFT jedinične funkcije



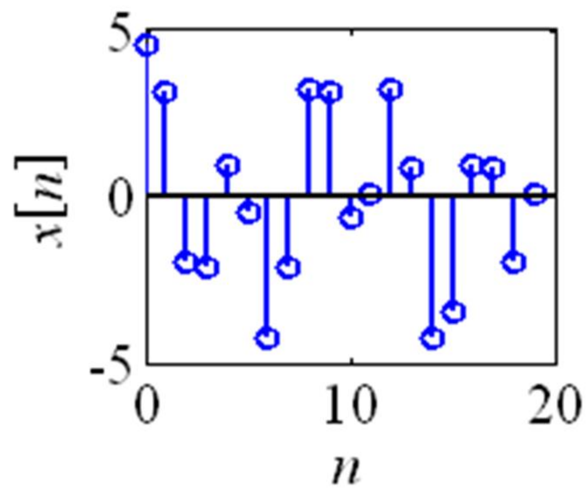


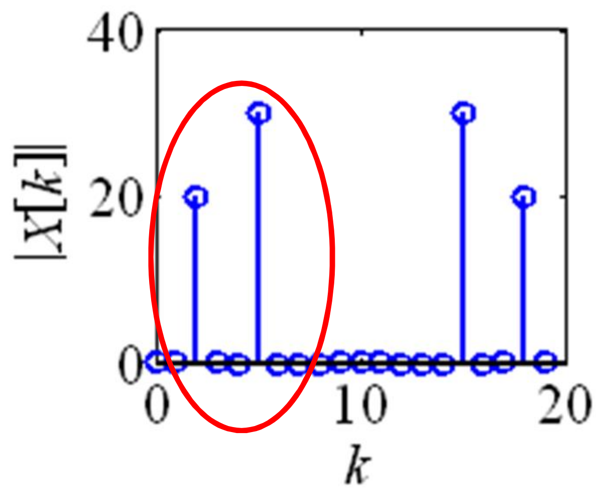
$$x[n]=\delta[n]$$
$$|X[k]|=1,\ \arg(X[k])=0$$



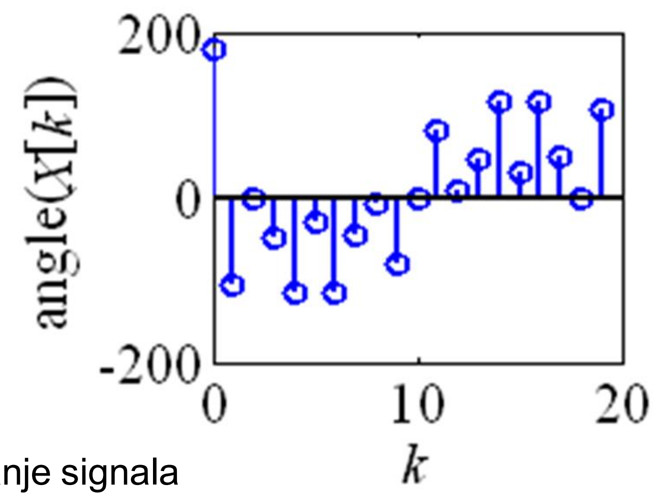

```
N=20; n=0:N-1;
x = (n==0);
X = fft(x)
```

FFT – dve sinusoide





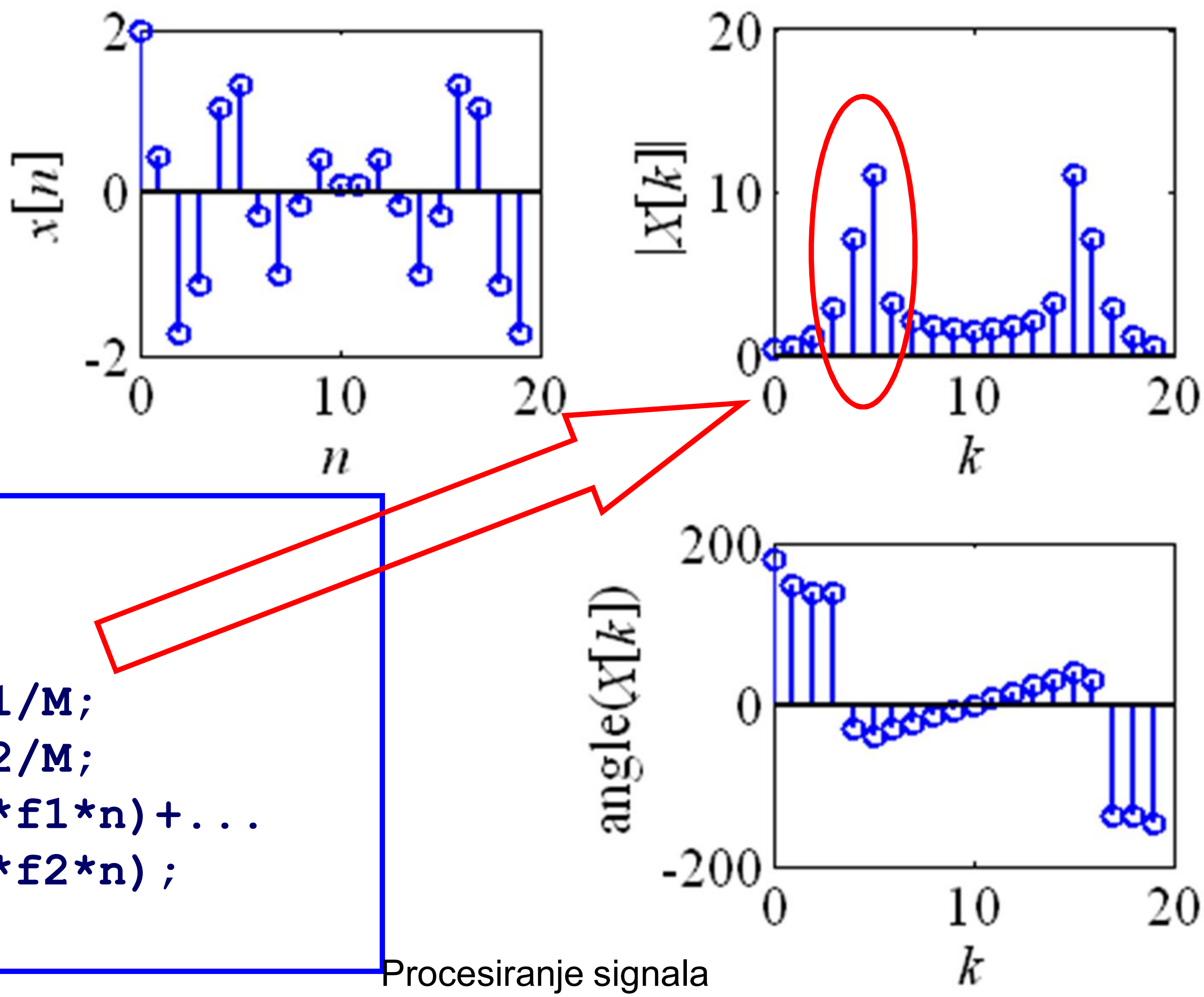

```
N=20;
n=0:N-1;
k1=2;    k2=5;
f1=k1/N; f2=k2/N;
fi2=-pi/6;
x = 2*cos(2*pi*f1*n+0)+...
    3*cos(2*pi*f2*n+fi2);
X = fft(x)
```



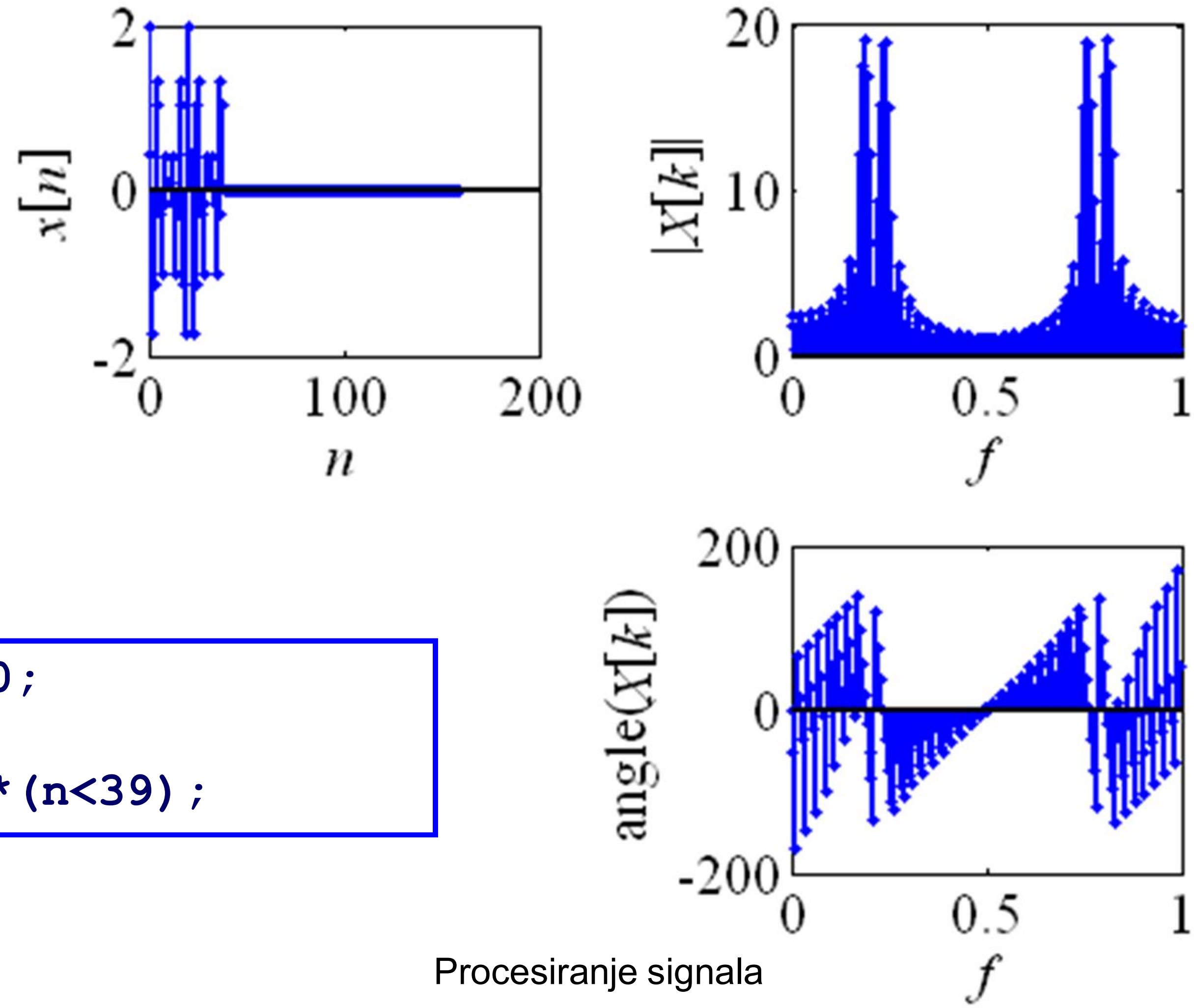
FFT – frekvencijska rezolucija

frequency resolution

```
N = 20;
M = 21;
n = 0:N-1;
k1=4; f1=k1/M;
k2=5; f2=k2/M;
x=cos(2*pi*f1*n)+...
  cos(2*pi*f2*n);
X = fft(x)
```

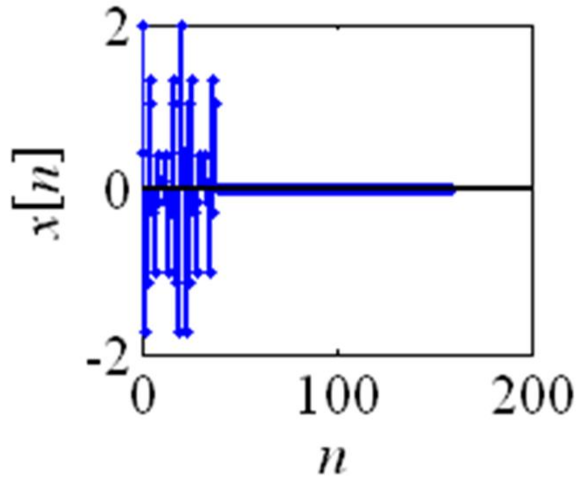


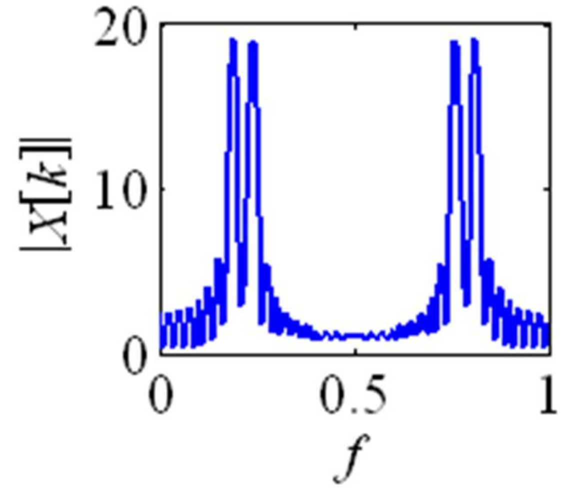
FFT – dopuna nulama



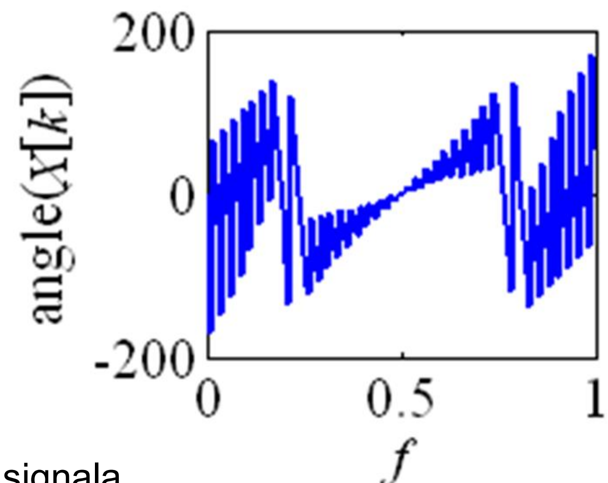

```
N = 160;
. . .
x = x.*(n<39);
```

FFT – dopuna nulama anvelopa DFT






```
plot(n/N,abs(y),'-')
. . .
plot(n/N,angle(y)*180/pi,'-')
```



Matlab **`fft, ifft`**

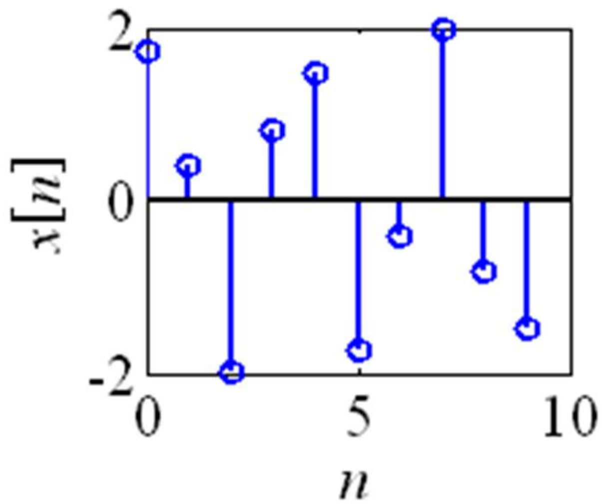
```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128]
X = fft(x,8+2)
y = ifft(X)
```

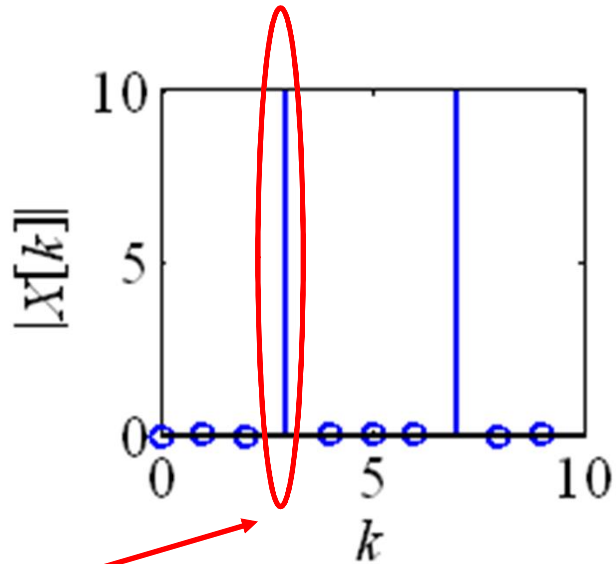
```
x =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

```
X =
  Columns 1 through 4
   1.9922  1.1861-0.6487i 0.7969-0.3984i 0.6889-0.1799i
  Columns 5 through 8
   0.6641  0.6889+0.1799i 0.7969+0.3984i 1.1861+0.6487i
```

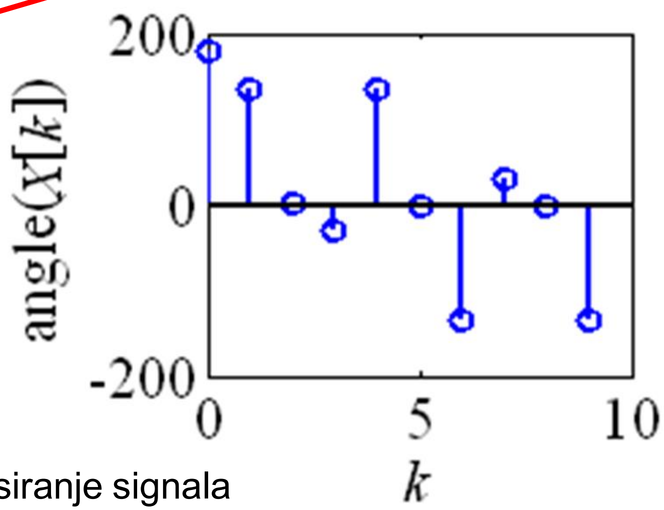
```
y =
 1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078 0 0
```

FFT






```
N = 10;
n = 0:N-1;
k = 3;
f = k/N;
fi = -pi/6;
x = 2*cos(2*pi*f*n+fi);
stem(n,x)
X = fft(x)
stem(n,abs(X))
```



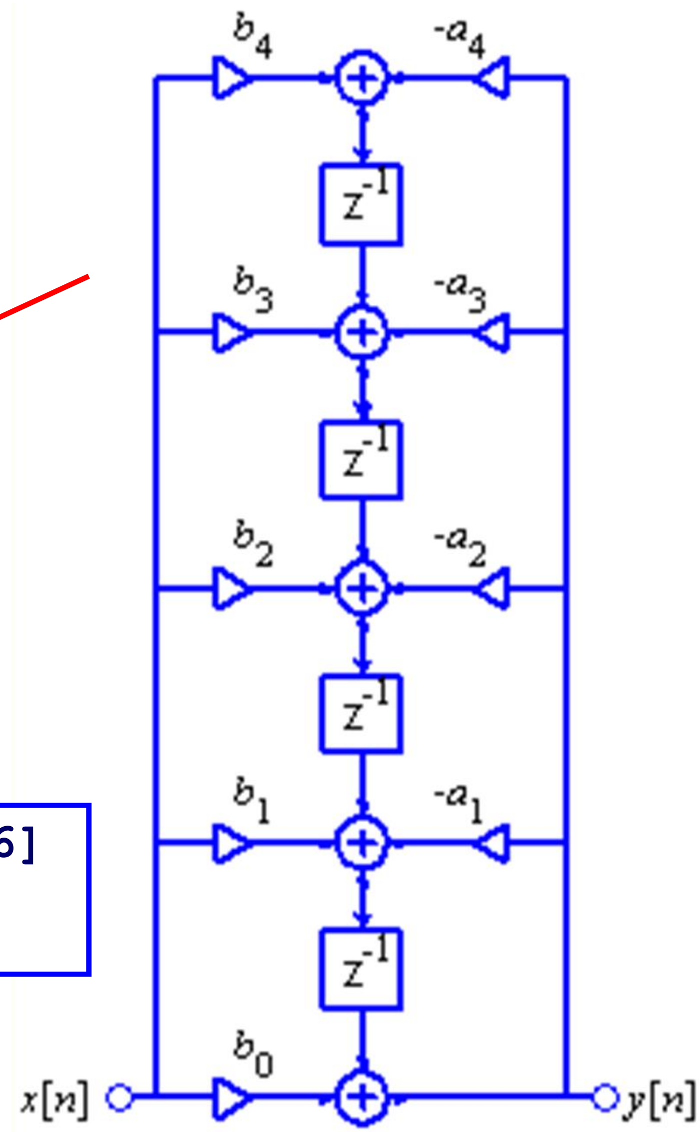
Projektovanje sistema

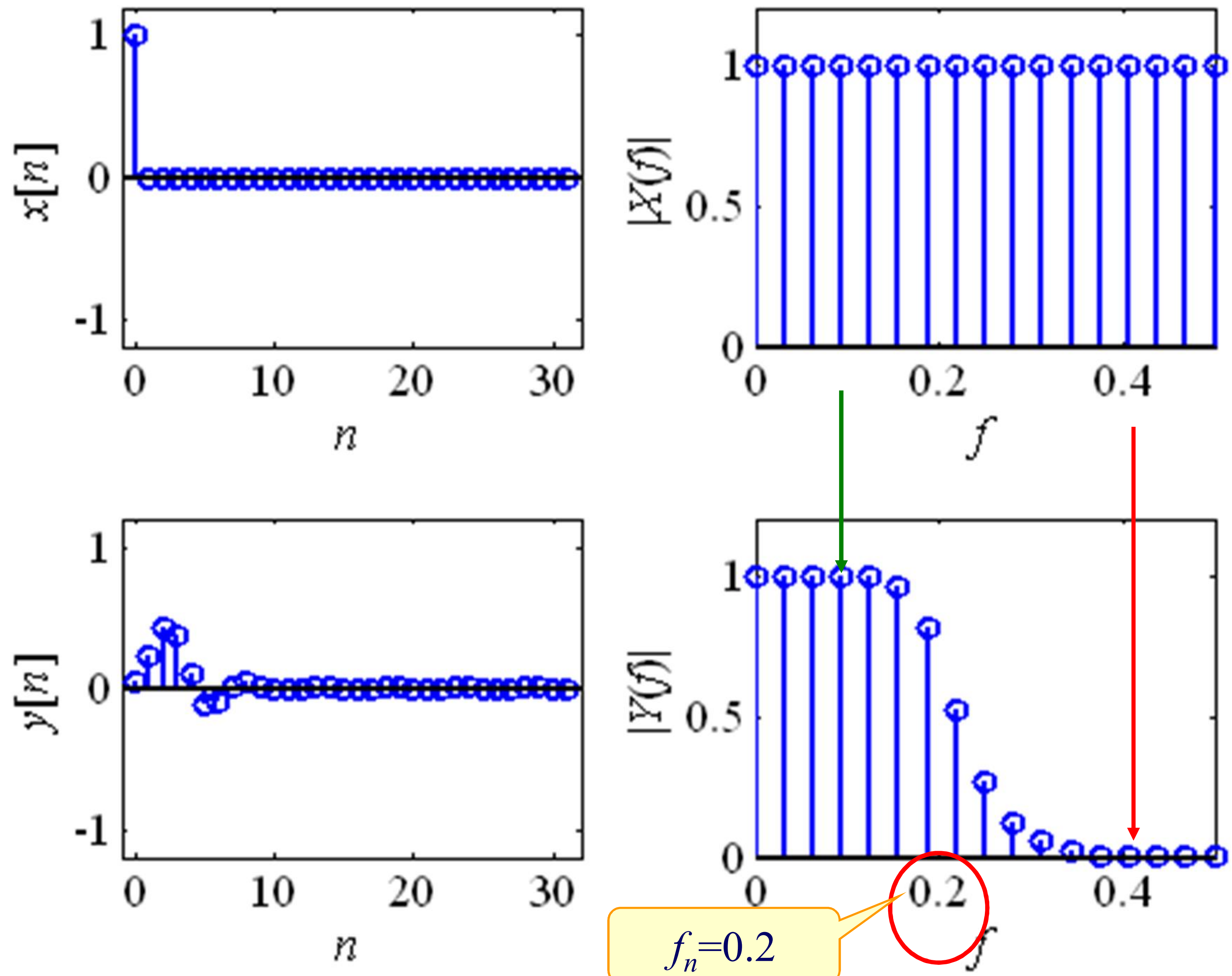
Izračunavanje koeficijenata sistema da zadovolje zadate specifikacije

```
b = [b0 b1 b2 b3 b4]
a = [1 a1 a2 a3 a4]
```

```
order = 4;
fn = 0.2;
[b,a] = butter(order,2*fn)
```

```
b = [0.0466 0.1863 0.2795 0.1863 0.0466]
a = [1 -0.7821 0.6800 -0.1827 0.0301]
```



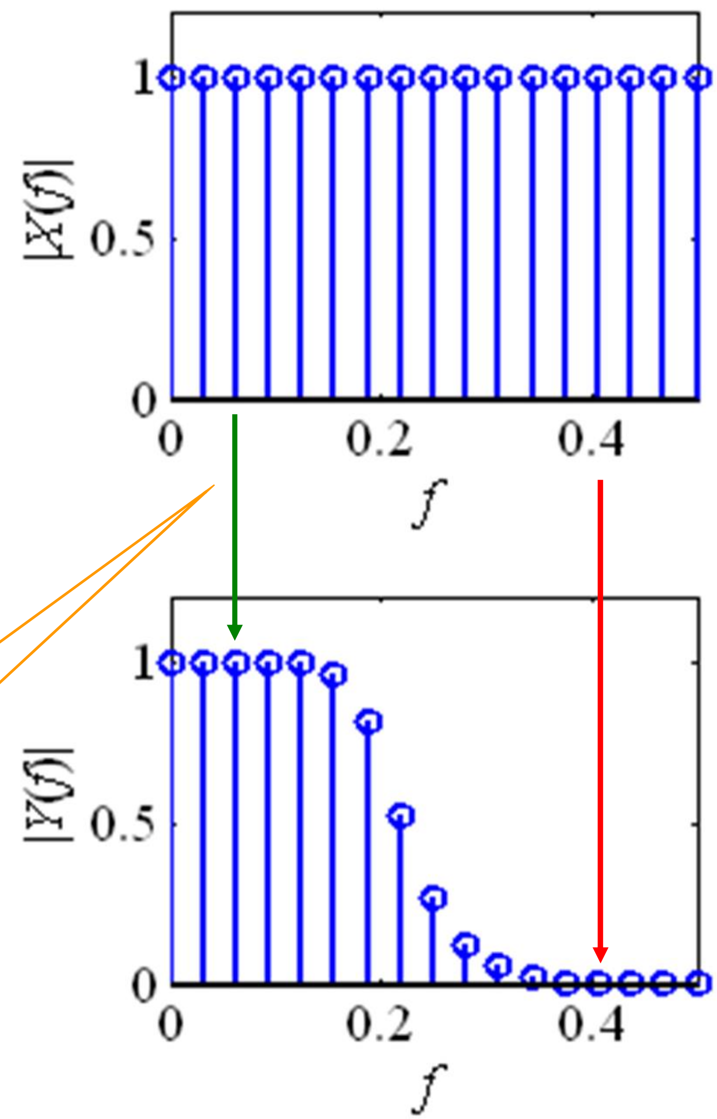


x[n]
n
|X(f)|
f
y[n]
n
|Y(f)|
f
f_n=0.2

Impulsna pobuda – frekvencijska karakteristika

Na osnovu odziva na impulsnu pobudu možemo da zaključimo da li će sinusoidalan signal da prođe kroz filtar

- ➢bez slabljenje
- ➢oslabljen
- ➢pojačan

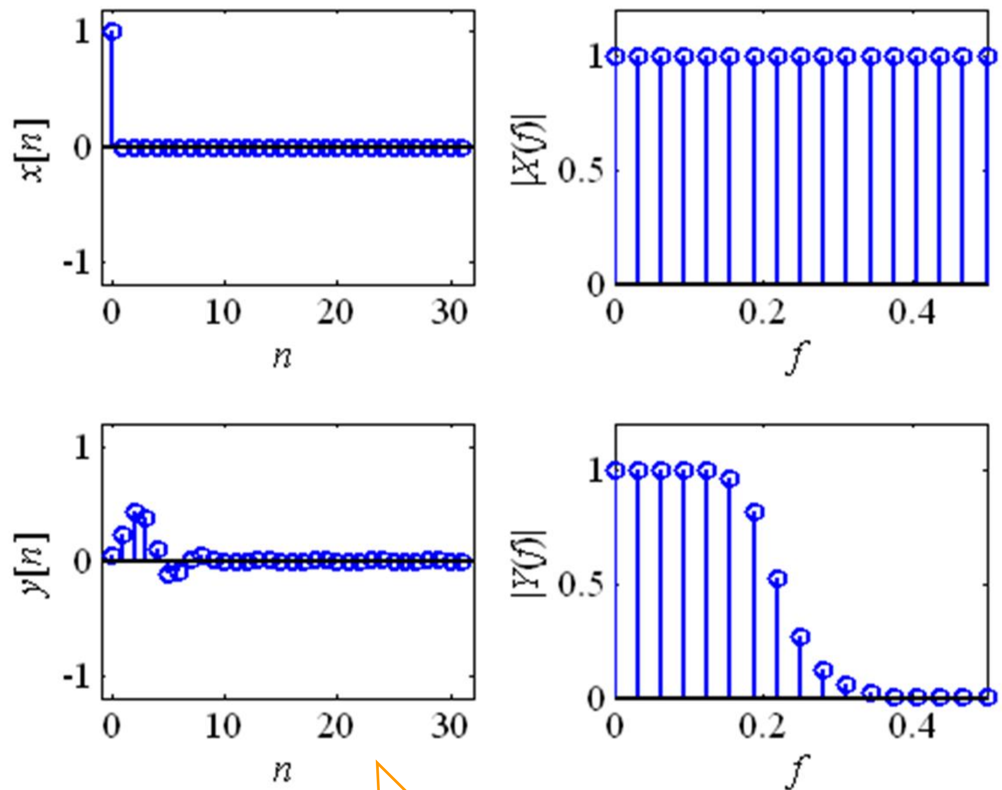


Bez slabljenja

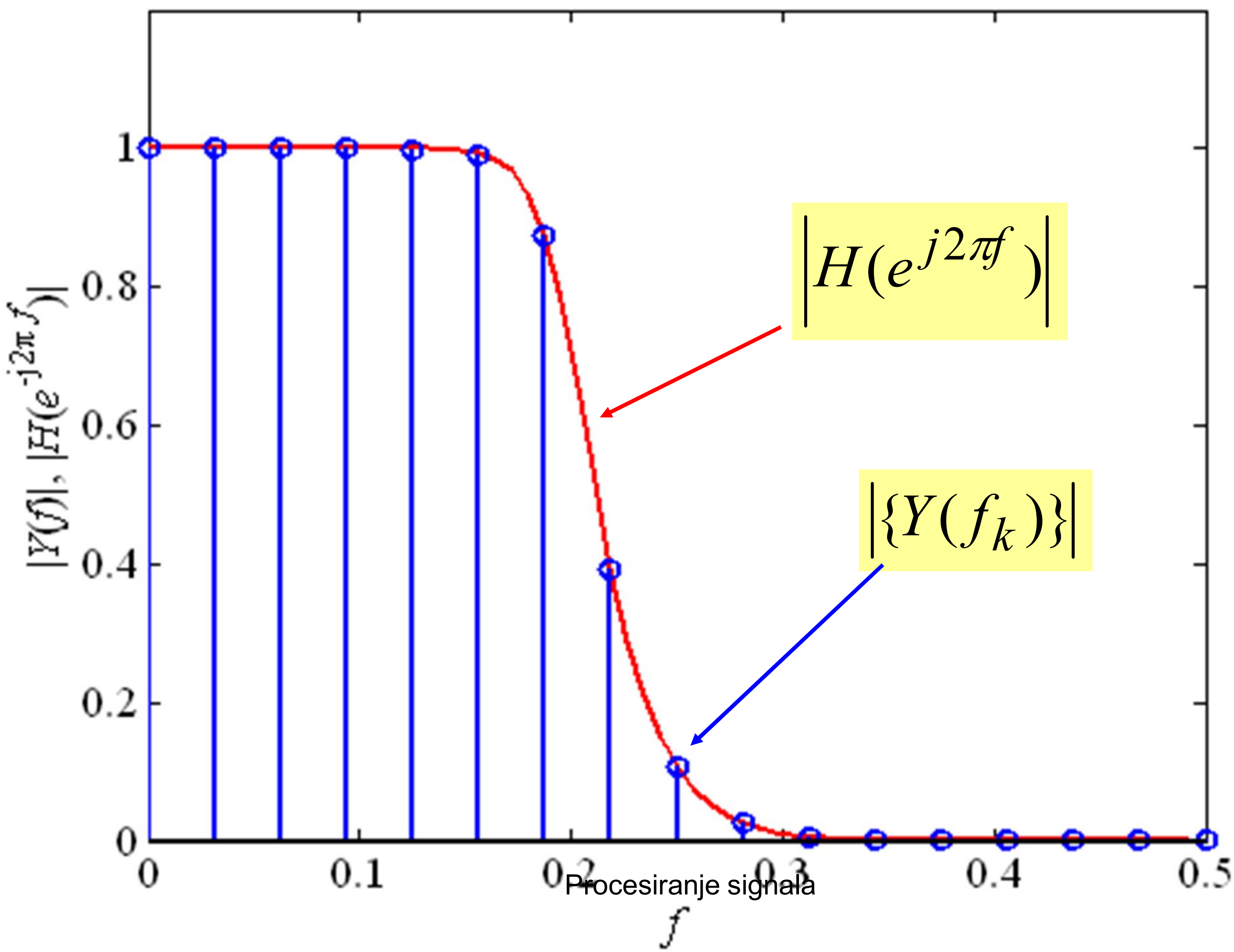
Procesiranje sistemom

Procesiranje sistemom realizuje se funkcijom **`filter`**

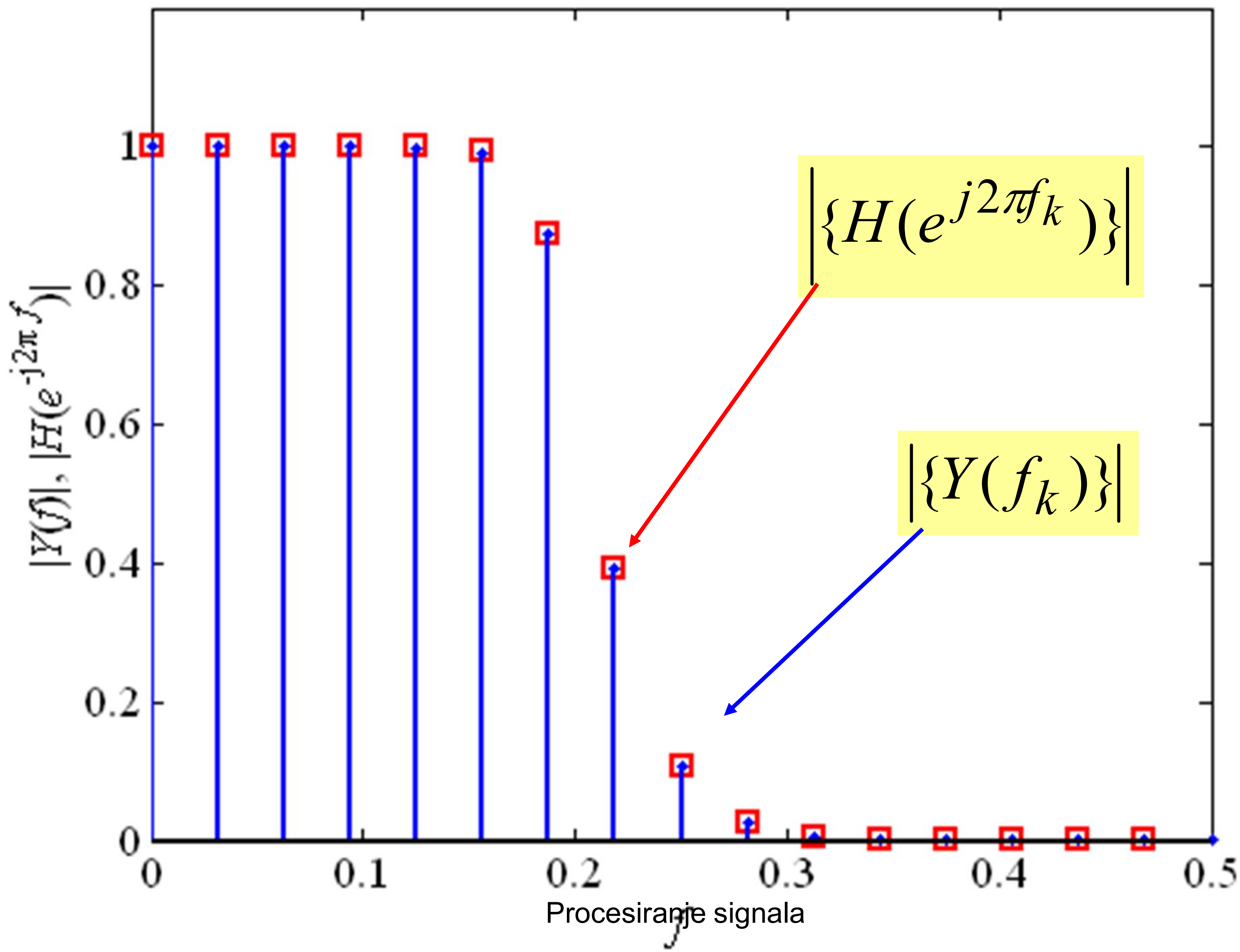
```
N = 32;
n = 0:N-1;
x = (n==0);
y = filter(b,a,x);
X = fft(x);
Y = fft(y);
stem(n,y)
stem(n/N,abs(Y))
```



subplot(2,2,3)



$|Y(f)|, |H(e^{j2\pi f})|$
f
$\left|H(e^{j2\pi f})\right|$
$\left|\{Y(f_k)\}\right|$
0
0.1
0.2
0.3
0.4
0.5
0.2
0.4
0.6
0.8
1

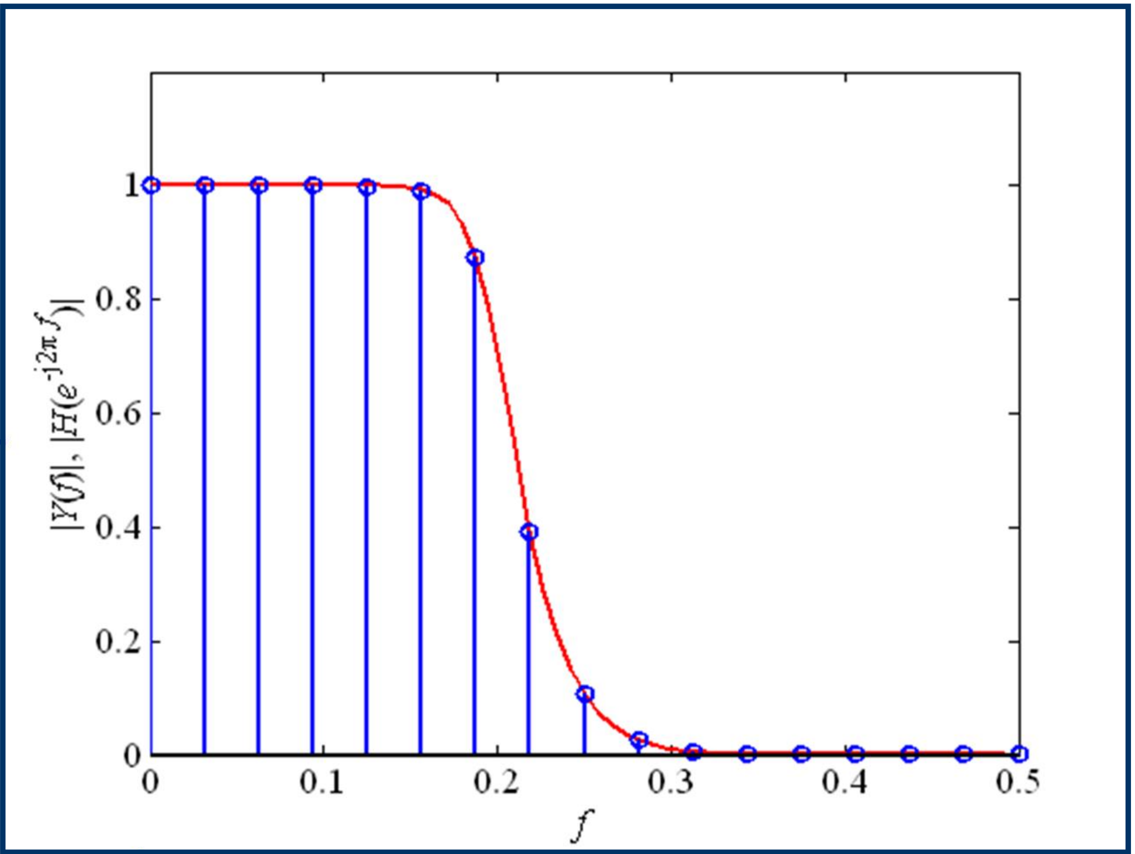


$\left|\{H(e^{j2\pi f_k})\}\right|$
$\left|\{Y(f_k)\}\right|$
$|Y(f)|, |H(e^{j2\pi f})|$
f
0
0.2
0.4
0.6
0.8
1
0.1
0.3
0.5

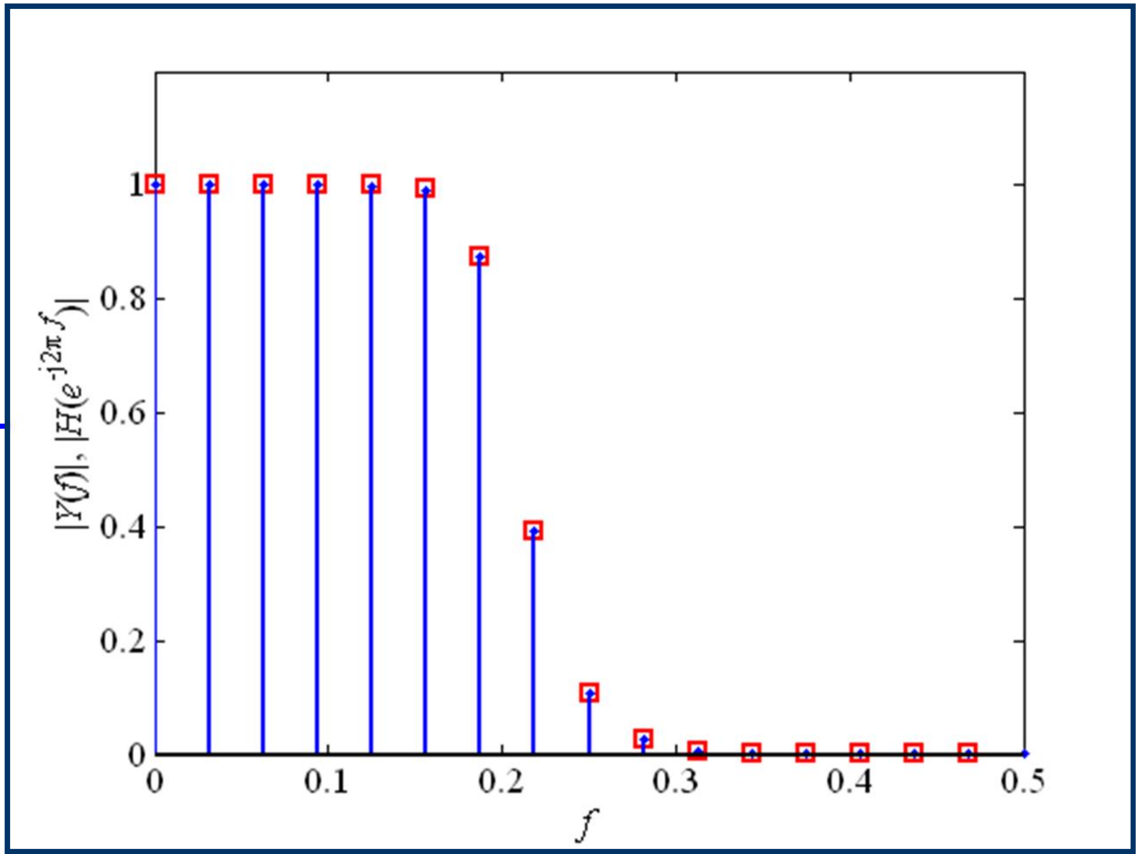
Frekvencijska karakteristika sistema

Frekvencijska karakteristika računa se funkcijom **`freqz`**

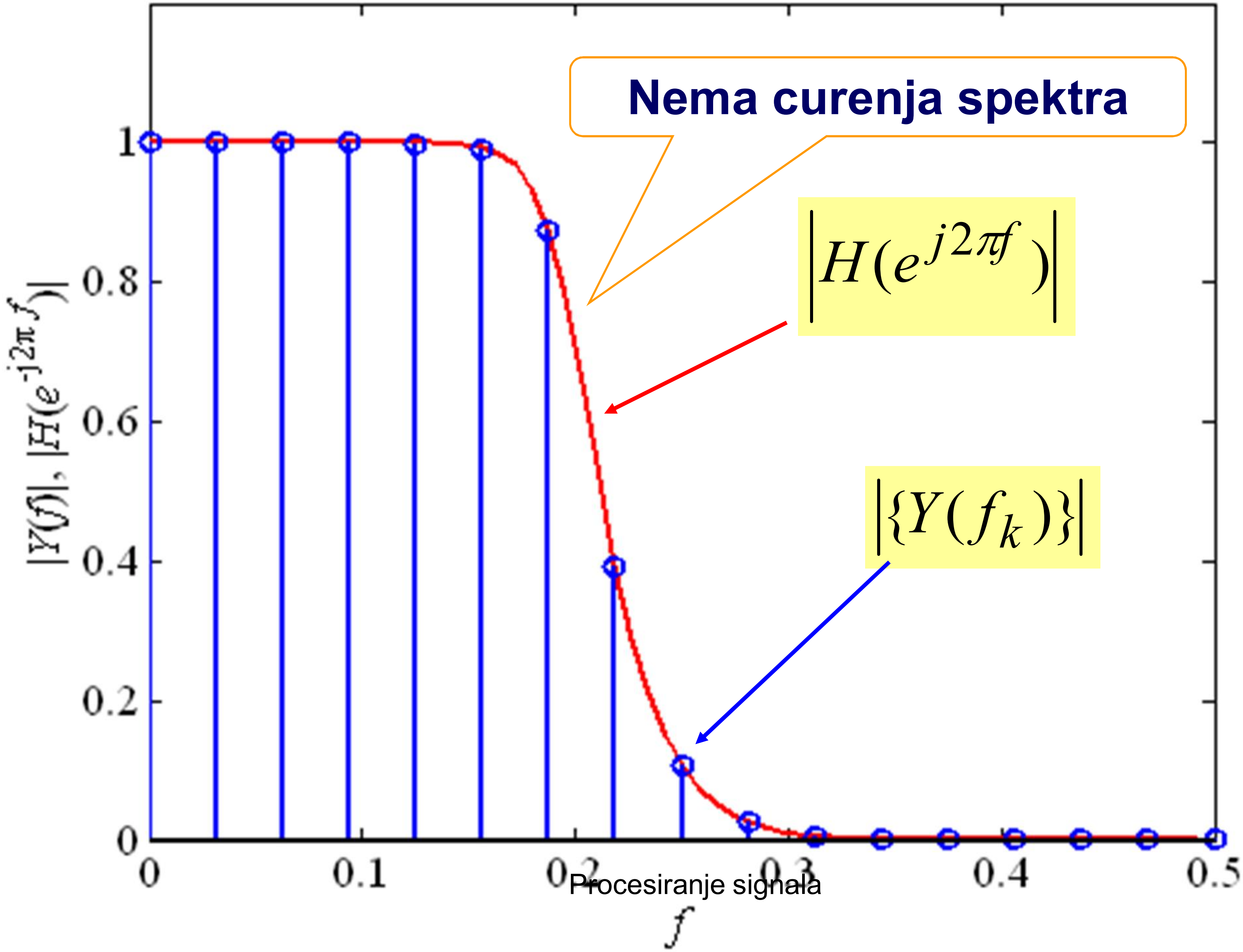
```
fn = 0.2;
[b,a] = butter(7,2*fn)
N = 32;
n = 0:N-1;
x = (n==0);
y = filter(b,a,x);
Y = fft(y);
[h,w] = freqz(b,a,2*N);
plot(w/(2*pi),abs(h),'r-')
hold on
 stem(n/N,abs(Y))
hold off
```



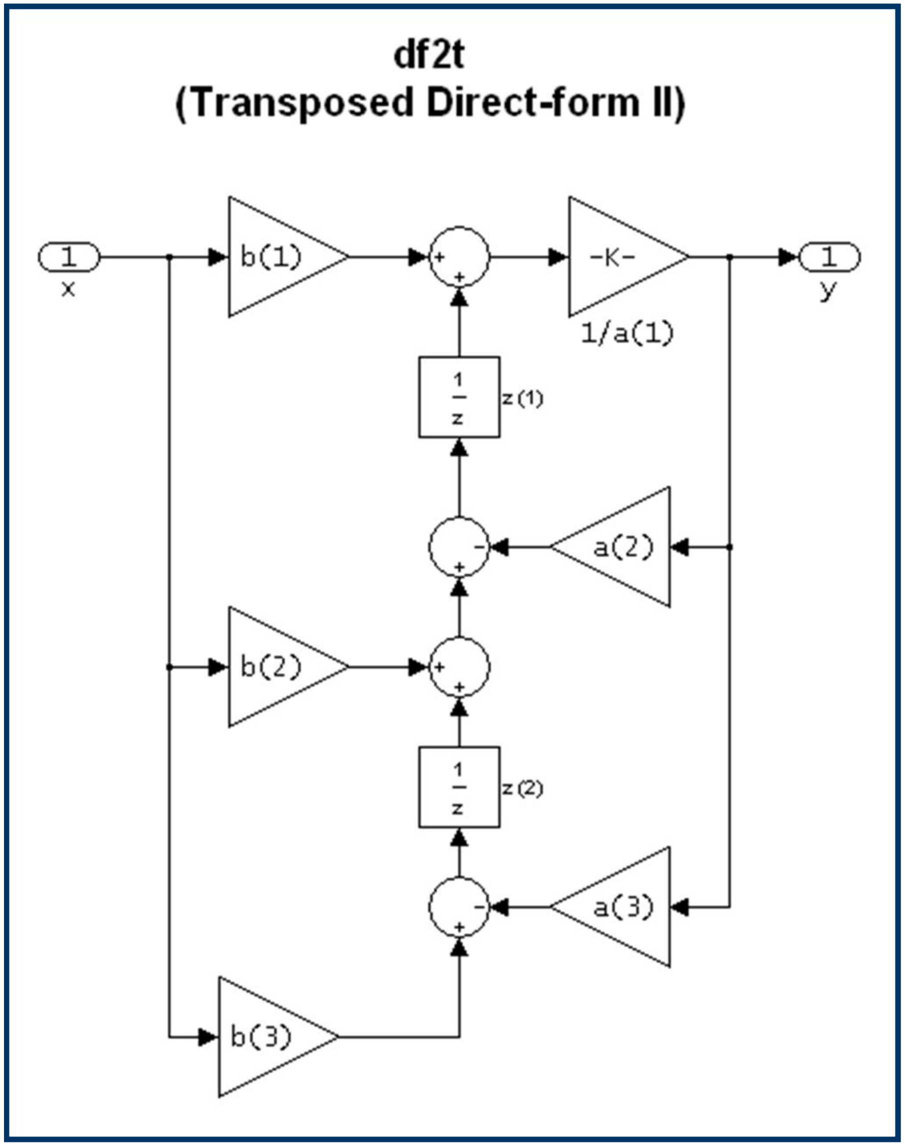
freqz i fft




```
fn = 0.2;
[b,a] = butter(7,2*fn)
N = 32;
n = 0:N-1;
x = (n==0);
y = filter(b,a,x);
Y = fft(y);
[h,w] = freqz(b,a,N/2);
plot(w/(2*pi),abs(h),'rs')
hold on
 stem(n/N,abs(Y))
hold off
```



Nema curenja spektra
$\left|H(e^{j2\pi f})\right|$
$\left|\{Y(f_k)\}\right|$
$|Y(f)|, |H(e^{-j2\pi f})|$
f
0
0.2
0.4
0.6
0.8
1
0.1
0.3
0.5



Implementacija

Hd = dfilt.df2t(b,a)

Hd = dfilt.df2t

Sistema prvog reda predstavljen diferencnom jednačinom

$x(n)$ →

$$y(n) = x(n) - x(n-1) - a\ y(n-1)$$

$$x(n) = 0, n < 0 \qquad a \neq 0$$

$$y(n) = 0, n < 0$$

→ $y(n)$

$$y(n) = -1/2^n, n > 0$$

$a = -1/2$

n	-2	-1	0	1	2	3	4	5
x	0	0	1	0	0	0	0	0
y	0	0	1	-1/2	-1/4	-1/8	-1/16	-1/32

Odziv sistema za impulsnu pobudu

$x(n)$ →

$$y(n) = x(n) - x(n-1) - a\ y(n-1)$$

$$x(n) = 0,\ n < 0 \qquad\qquad a \neq 0$$

$$y(n) = 0,\ n < 0$$

→ $y(n)$

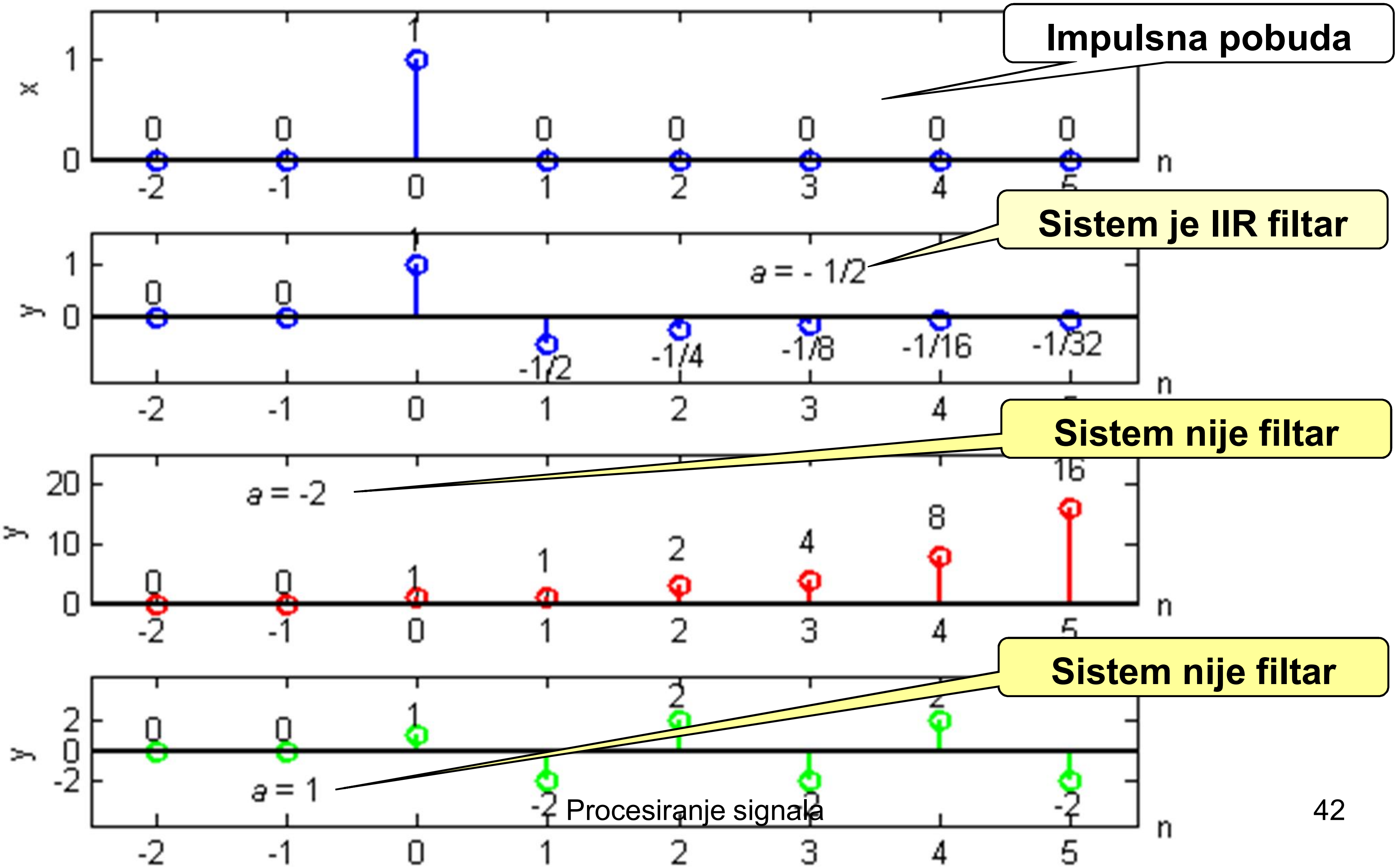
Impulsna pobuda

$$x(n) = \begin{cases} 0,\ n < 0 \\ 1,\ n = 0 \\ 0,\ n > 0 \end{cases}$$

Beskonačan odziv, $a \neq -1$

$$y(n) = \begin{cases} 0,\ n < 0 \\ 1,\ n = 0 \\ (-1)^n a^{n-1}(1+a),\ n > 0 \end{cases}$$

Kada je sistem filtar?



Kada sistem nije filtar?

$x(n)$

$$y(n) = x(n) - x(n-1) - a\ y(n-1)$$

$$x(n) = 0,\ n < 0 \qquad\qquad a \neq 0$$

$$y(n) = 0,\ n < 0$$

$y(n)$

$$|a| < 1 \rightarrow \lim_{n \to +\infty} y(n) = 0$$

$$|a| > 1 \rightarrow \lim_{n \to +\infty} y(n) = \pm\infty$$

$$a = 1 \rightarrow y(n) = \begin{cases} +2 \\ -2 \end{cases}$$

Sistem nije filtar

$$a = -1 \rightarrow y(n) = x(n)$$

z transformacija

$$y(n) = x(n) - x(n-1) - a\,y(n-1)$$

$$Y(z) = X(z) - z^{-1}X(z) - a\,z^{-1}Y(z)$$

$$(1 + a\,z^{-1})Y(z) = (1 - z^{-1})X(z)$$

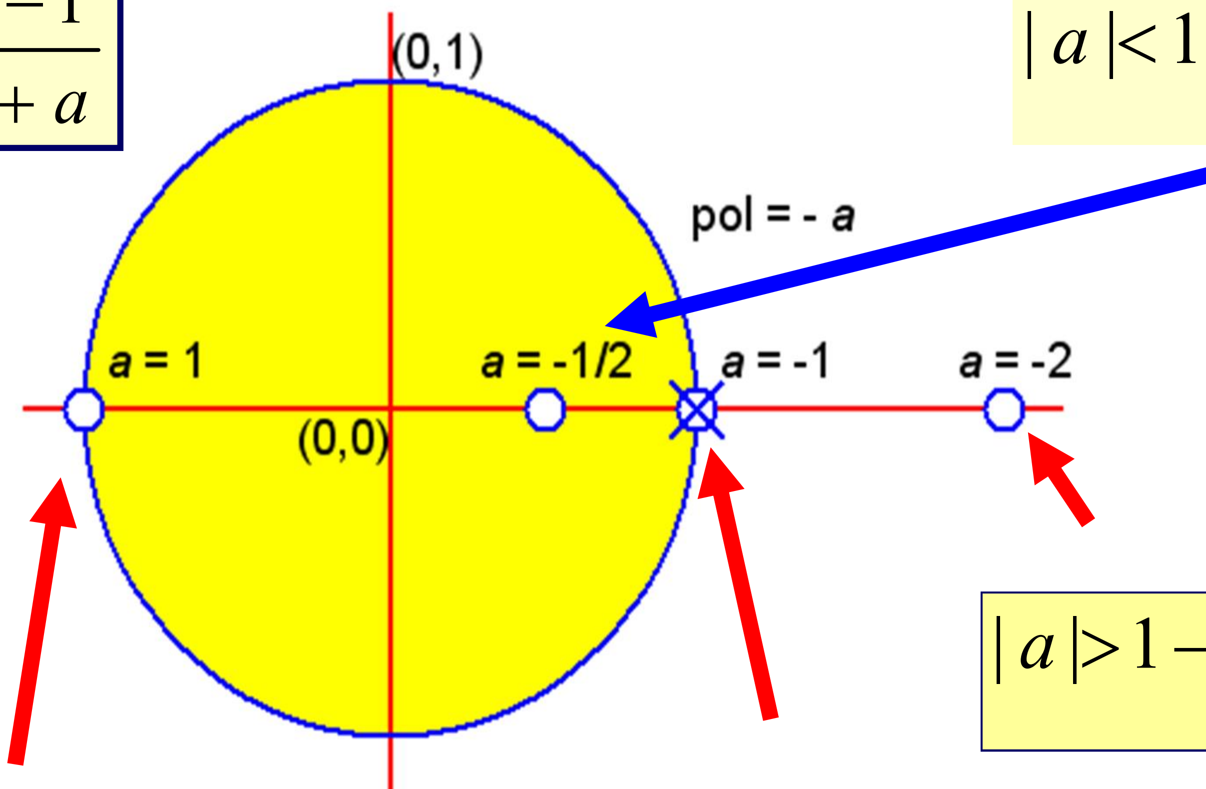
Funkcija prenosa

$$\frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 + a\,z^{-1}}$$

$$\frac{Y(z)}{X(z)} = \frac{z - 1}{z + a}$$

Polovi funkcije prenosa u *z* ravni

$$\boxed{\frac{z-1}{z+a}}$$



$$|a|<1 \rightarrow \lim_{n\rightarrow+\infty} y(n)=0$$

$$|a|>1 \rightarrow \lim_{n\rightarrow+\infty} y(n)=\pm\infty$$

$$a=1 \rightarrow y(n)=\begin{cases}+2\\-2\end{cases}$$

$$a=-1 \rightarrow y(n)=x(n)$$

Osnovni pojmovi o IIR filtrima

- **IIR** (Infinite Impulse Response) beskonačni impulsni odziv
- Digitalni IIR filtar – funkcija prenosa mora da ima sve **polove unutar jediničnog kruga**
- Ulazno-izlazne relacije se izražavaju **rekurzivnom diferencnom jednačinom**
  - rekurzivan znači da se za izračunavanje tekućeg izlaznog odbirka koriste prethodni izlazni odbirci

Prednosti i nedostaci IIR filtara

- Prednost:
želјeni oblik amplitudske karakteristike
dobro se aproksimira
funkcijom niskog reda

- Nedostatak:
ne može se realizovati
linearna fazna karakteristika

Sinteza funkcije prenosa

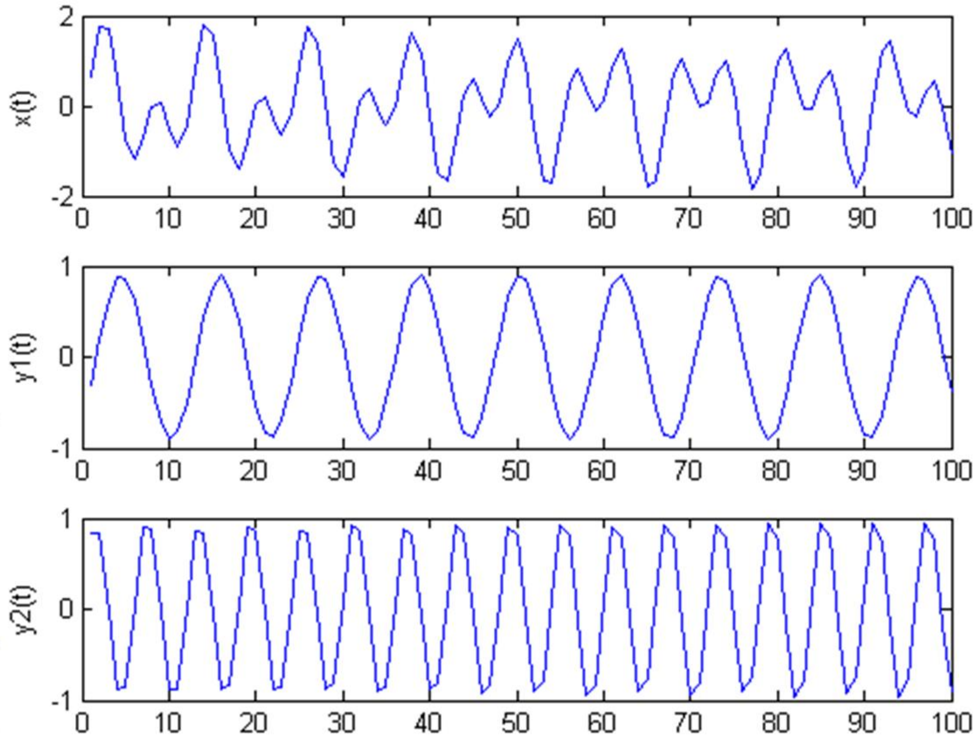
1. Transformacijom funkcije prenosa referentnog analognog prototip filtra

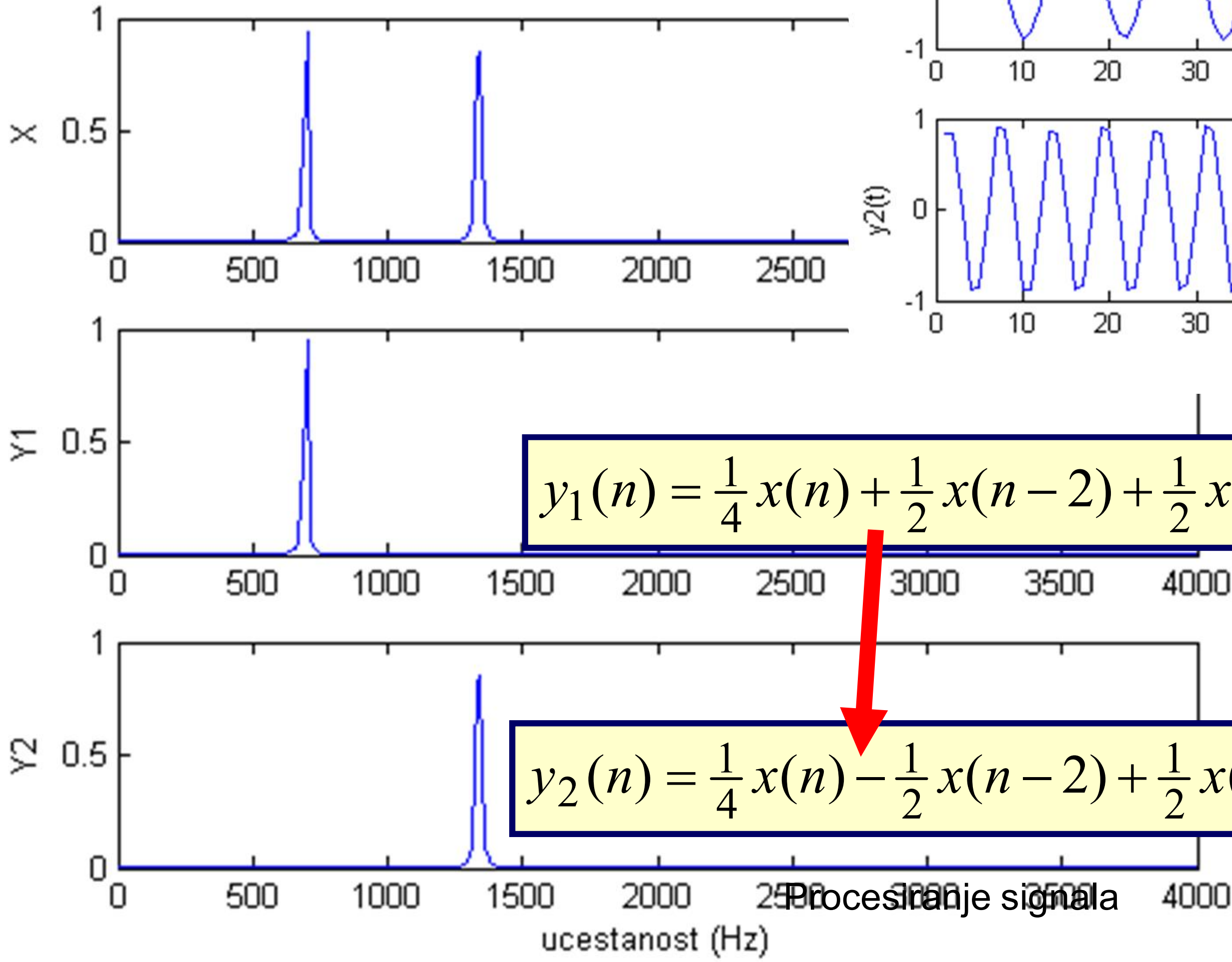
2. Direktna sinteza u *z* ravni

3. Na osnovu zadatog ponašanja u vremenskom domenu

Telefon #2

697Hz + 1336 Hz





$$y_1(n) = \frac{1}{4}x(n) + \frac{1}{2}x(n-2) + \frac{1}{2}x(n-4) + \frac{1}{4}x(n-6) - \frac{1}{2}y(n-4)$$

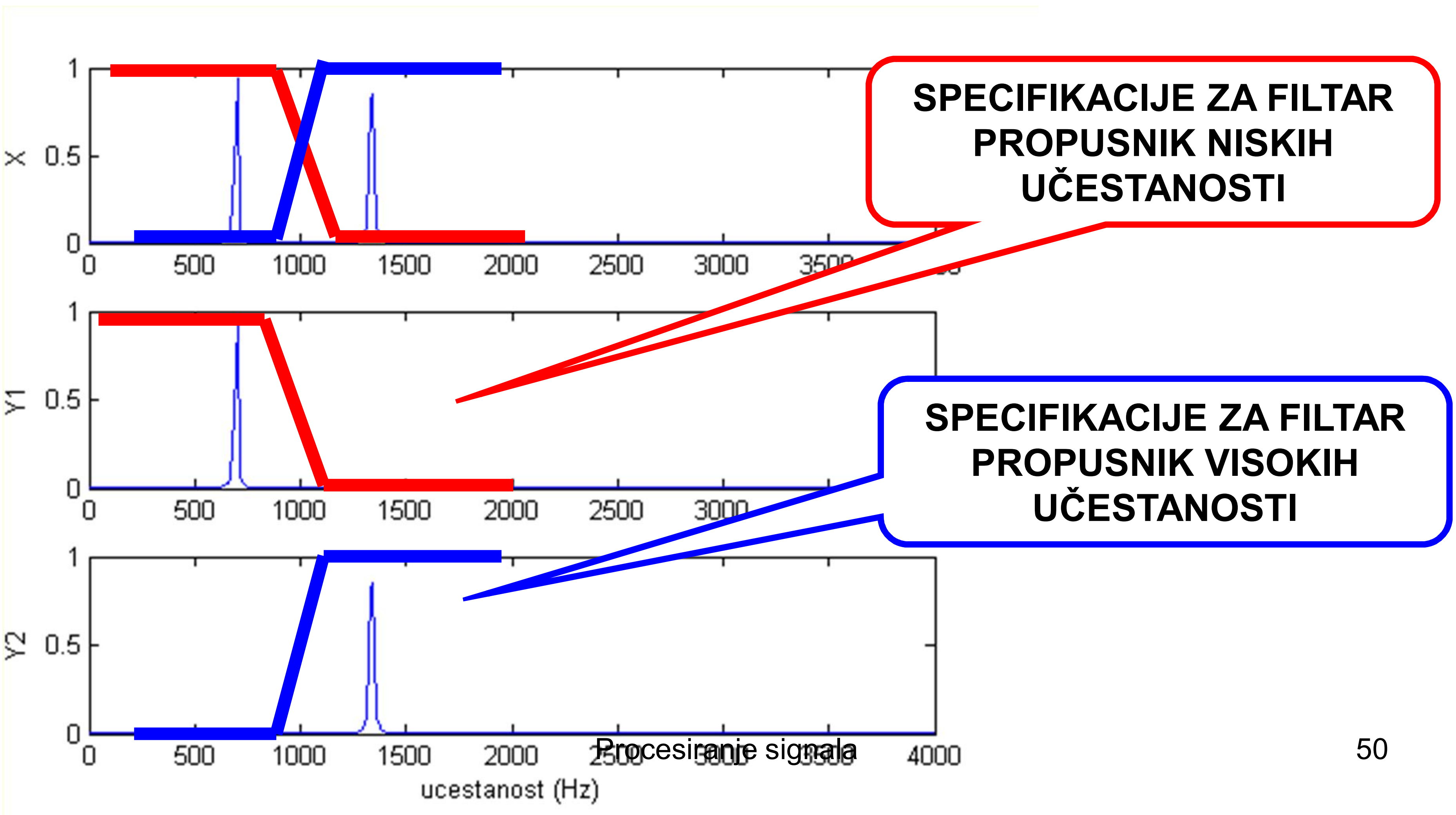
$$y_2(n) = \frac{1}{4}x(n) - \frac{1}{2}x(n-2) + \frac{1}{2}x(n-4) - \frac{1}{4}x(n-6) - \frac{1}{2}y(n-4)$$

Telefon #2

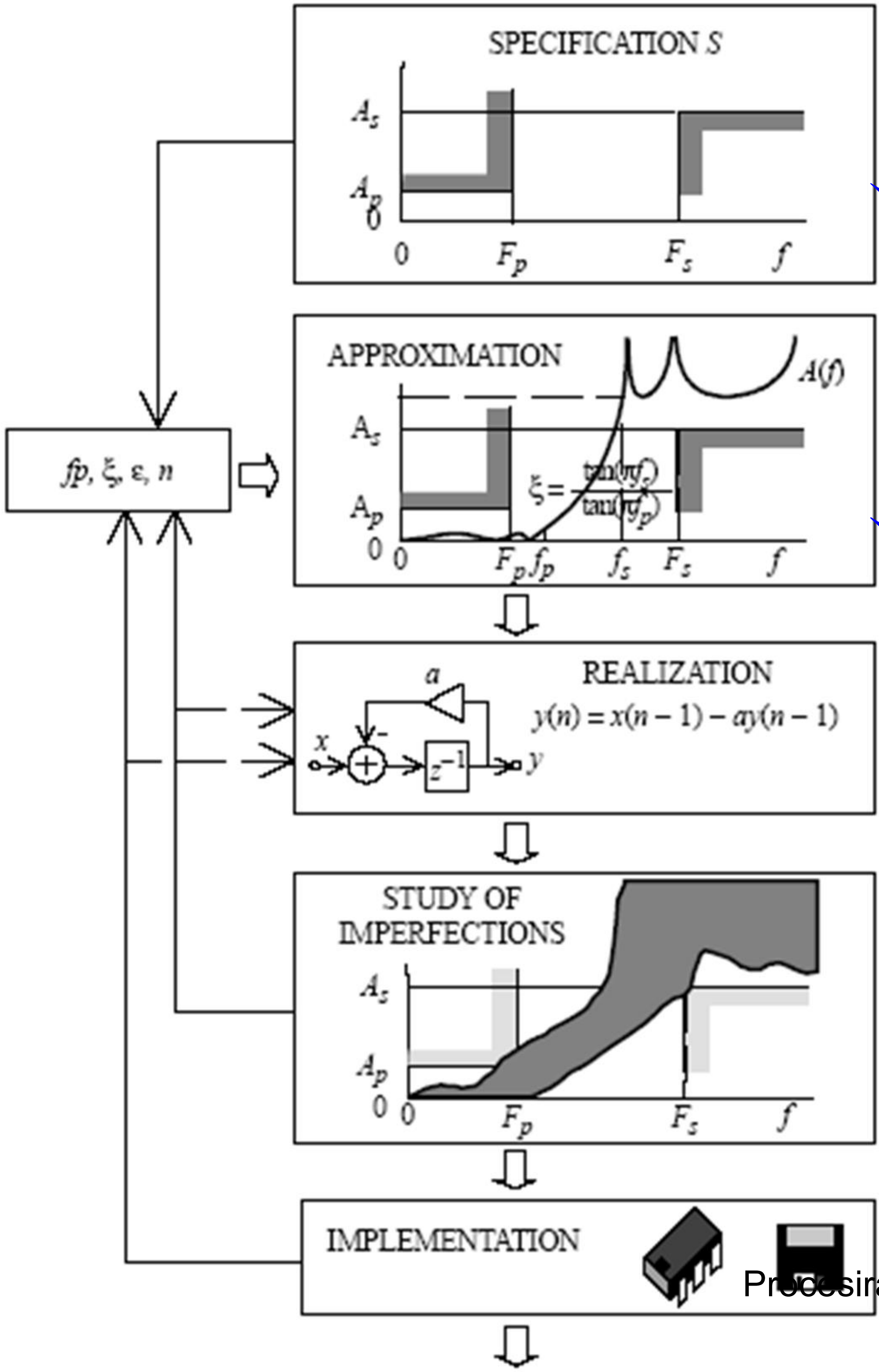
697Hz + 1336 Hz

697, 770, 852, 941 Hz

1209, 1336, 1477, 1633 Hz



Od specifikacija do realizacije



Specifikacije

Sinteza

Specifikacije IIR filtara

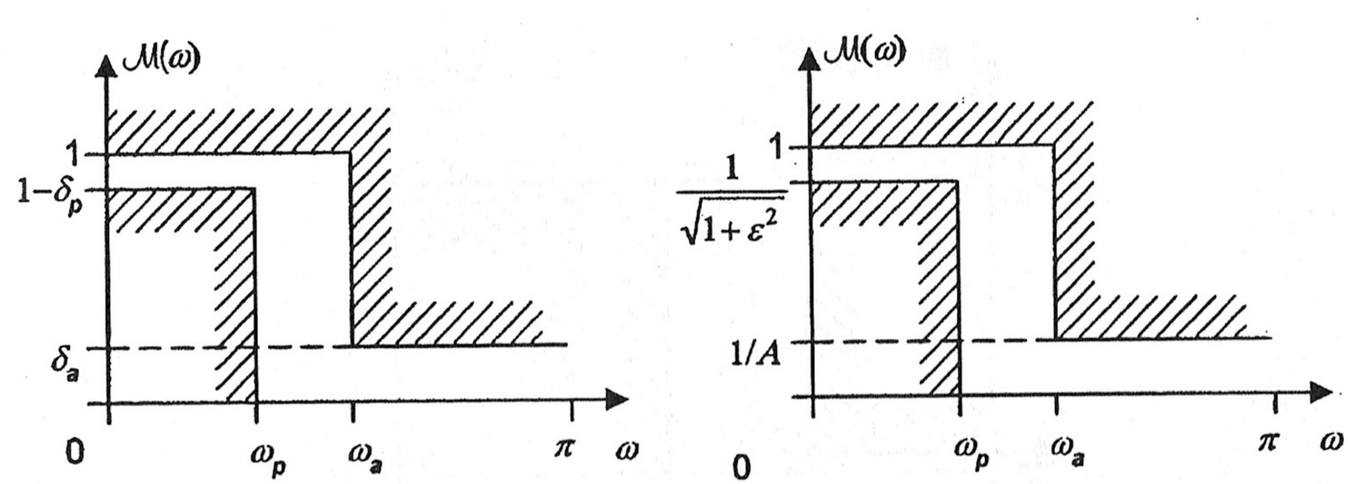
- Definišu se dozvoljene **tolerancije** u određenim frekvencijskim opsezima

- **Sinteza** – naći funkciju prenosa takvu da je racionalna funkcija po *z* što nižeg reda i da amplitudska karakteristika zadovoljava specifikacije (da se koristi minimalan broj operacija u diferencnoj jednačini)

Amplitudska karakteristika

- **Amplitudska karakteristika filtra**, $\boldsymbol{M}(\omega)$, definiše se kao apsolutna vrednost funkcije prenosa na jediničnom krugu z ravni, $z=e^{j\omega}$.
- $\boldsymbol{M}(\omega)$ je **parna periodična** funkcija po ω sa periodom 2π
- Posmatra se opseg kružnih frekvencija $0 \leq \omega \leq \pi$
- $\boldsymbol{M}(\omega)=|H(e^{j\omega})|$

Tolerancije amplitudske karakteristike



a) $1-\delta_p \leq \boldsymbol{M}(\omega) \leq 1$ za $0 \leq \omega \leq \omega_p$

$0 \leq \boldsymbol{M}(\omega) \leq \delta_a$ za $\omega_a \leq \omega \leq \pi$

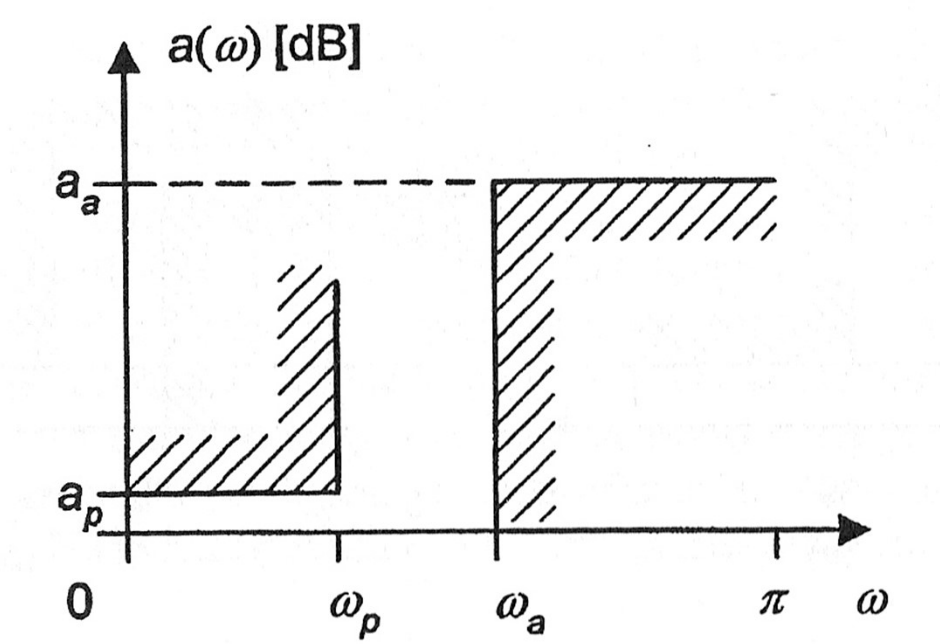
Propusni opseg

Nepropusni opseg

b) $1/\sqrt{(1+\varepsilon^2)} \leq \boldsymbol{M}(\omega) \leq 1$ za $0 \leq \omega \leq \omega_p$

$0 \leq \boldsymbol{M}(\omega) \leq 1/A$ za $\omega_a \leq \omega \leq \pi$

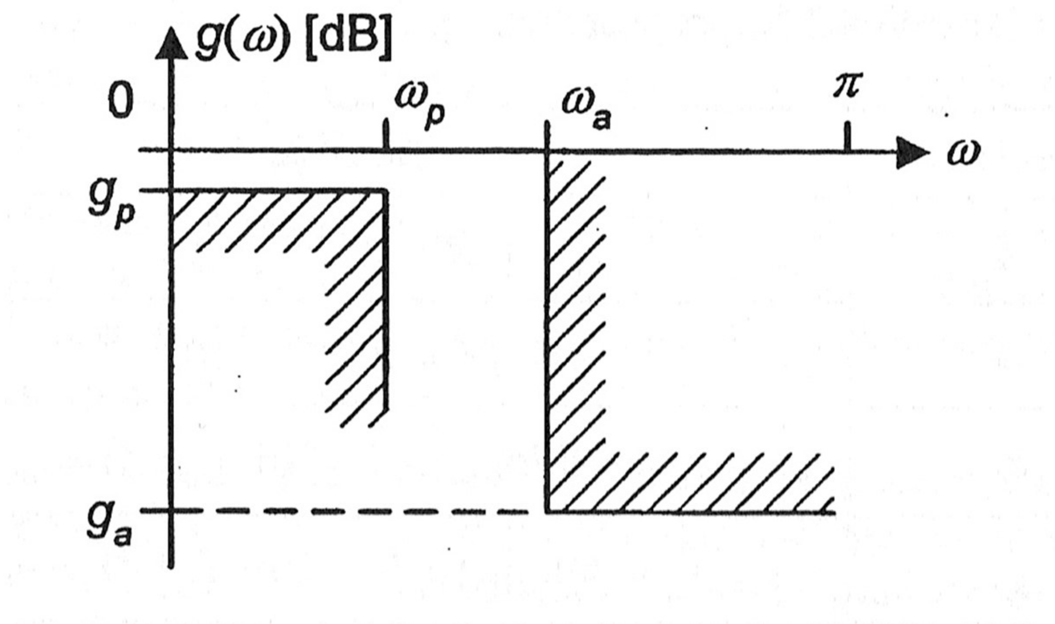
Tolerancije karakteristike slabljenja



$0 \le a(\omega) \le a_p$ za $0 \le \omega \le \omega_p$ — **Propusni opseg**

$a_a \le a(\omega) \le +\infty$ za $\omega_a \le \omega \le \pi$ — **Nepropusni opseg**

Tolerancije karakteristike pojačanja



$g_p \leq g(\omega) \leq 0$ za $0 \leq \omega \leq \omega_p$

Propusni opseg

$-\infty \leq g(\omega) \leq g_a$ za $\omega_a \leq \omega \leq \pi$

Nepropusni opseg

Propusni opseg – odnosi parametara

Tabela 8.1 Propusni opseg.

Veličina	Relacije
δ_p	$\delta_p = 1 - \dfrac{1}{\sqrt{1+\varepsilon^2}} = 1 - 10^{-a_p/20} = 1 - 10^{g_p/20}$
ε	$\varepsilon = \dfrac{\sqrt{\delta_p(2-\delta_p)}}{1-\delta_p} = \dfrac{\sqrt{1-10^{-a_p/20}}}{10^{-a_p/20}} = \dfrac{\sqrt{1-10^{g_p/20}}}{10^{g_p/20}}$
a_p	$a_p = -20\log_{10}(1-\delta_p) = -10\log_{10}(1+\varepsilon^2) = -g_p$
g_p	$g_p = 20\log_{10}(1-\delta_p) = 10\log_{10}(1+\varepsilon^2) = -a_p$

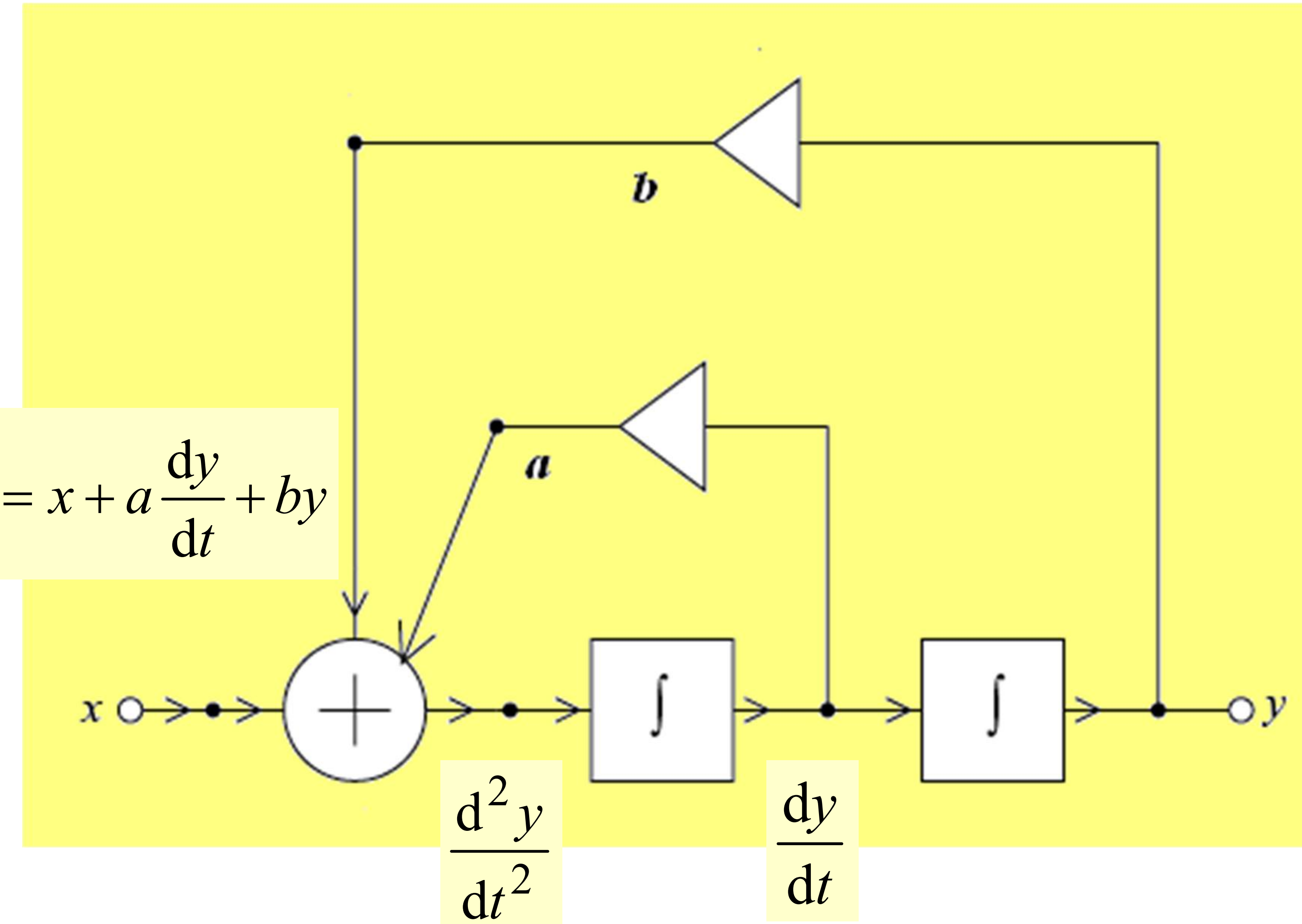
Nepropusni opseg - odnosi parametara

Tabela 8.2 Nepropusni opseg.

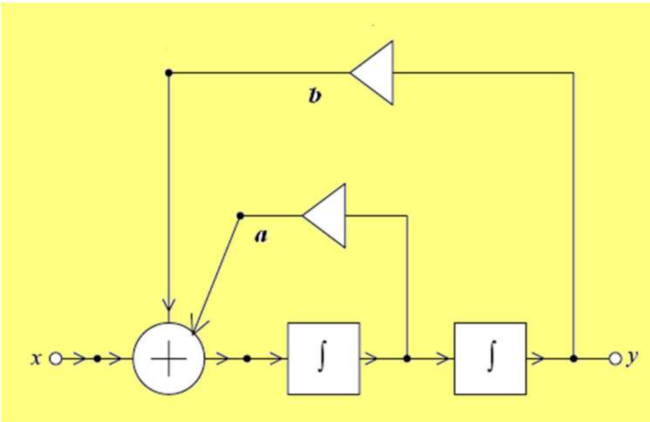
Veličina	Relacije
δ_a	$\delta_a = 1/A = 10^{-a_a/20} = 10^{g_a/20}$
A	$A = 1/\delta = 10^{a_a/20} = 10^{-g_a/20}$
a_a	$a_a = -20\log_{10}(\delta_a) = 20\log_{10}(A) = -g_a$
g_a	$g_a = 20\log_{10}(\delta_a) = -20\log_{10}(A) = -a_a$

Kontinualni sistemi

$$\frac{d^2y}{dt^2} = x + a\frac{dy}{dt} + by$$



Primena Laplasove transformacije



$$\frac{d^2 y}{dt^2} = x + a\frac{dy}{dt} + by$$

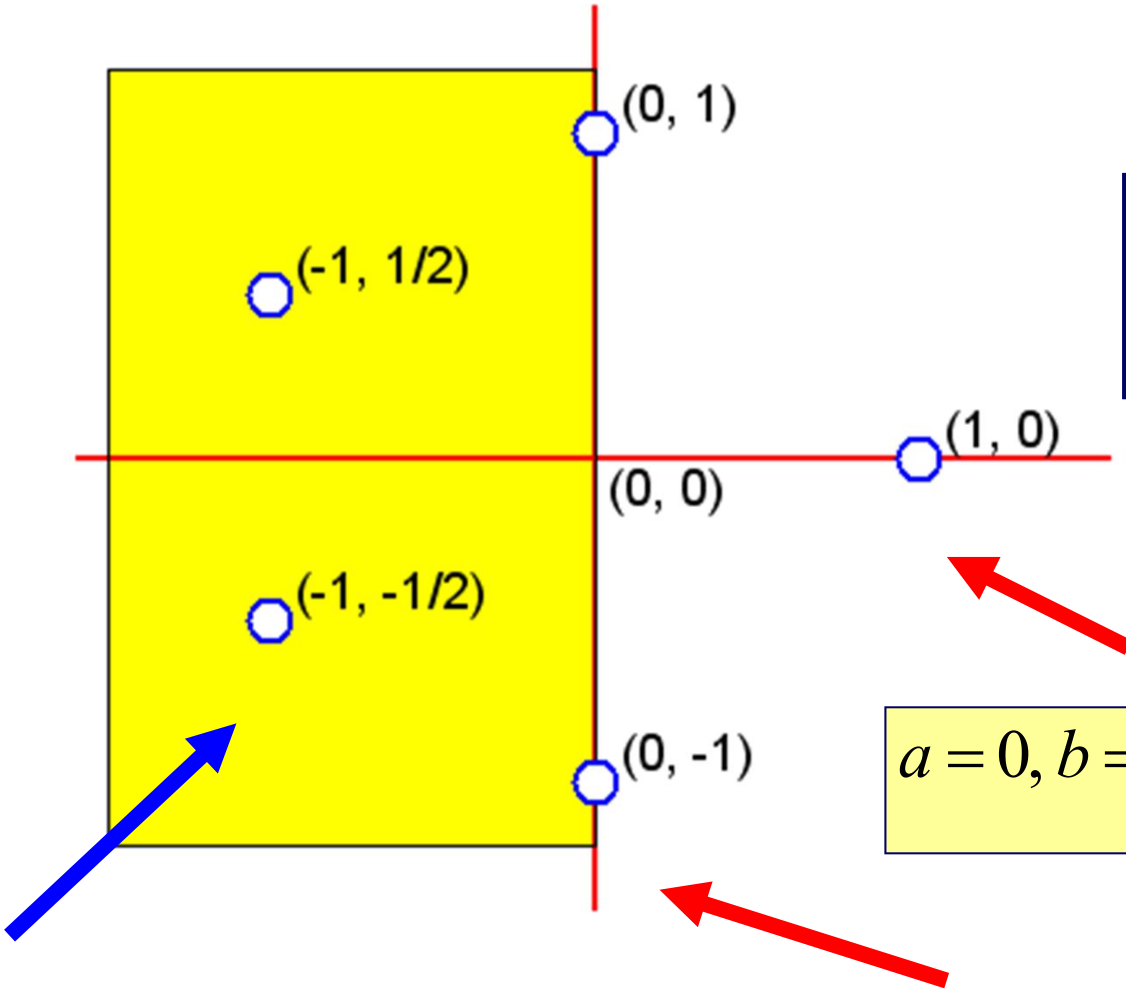
$$s^2\, Y(s) = X(s) + a\, s\, Y(s) + b\, Y(s)$$

$$(s^2 - a\, s - b)Y(s) = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2 - a\, s - b}$$

Funkcija prenosa

Polovi funkcije prenosa u *s* ravni



$$\frac{1}{s^2 - as - b}$$

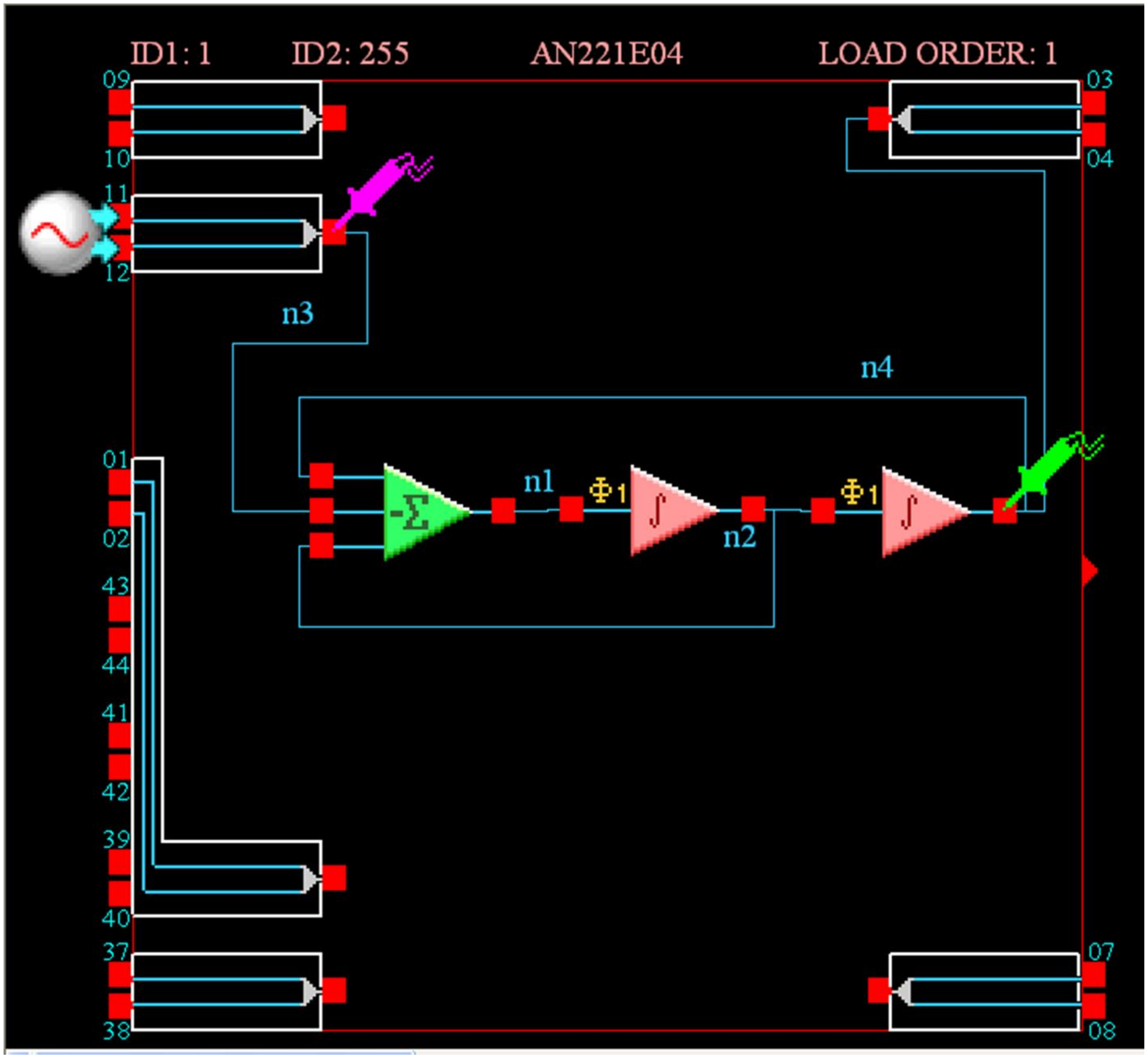
$$a = 0, b = 1 \rightarrow \lim_{t \to +\infty} y(t) = \pm\infty$$

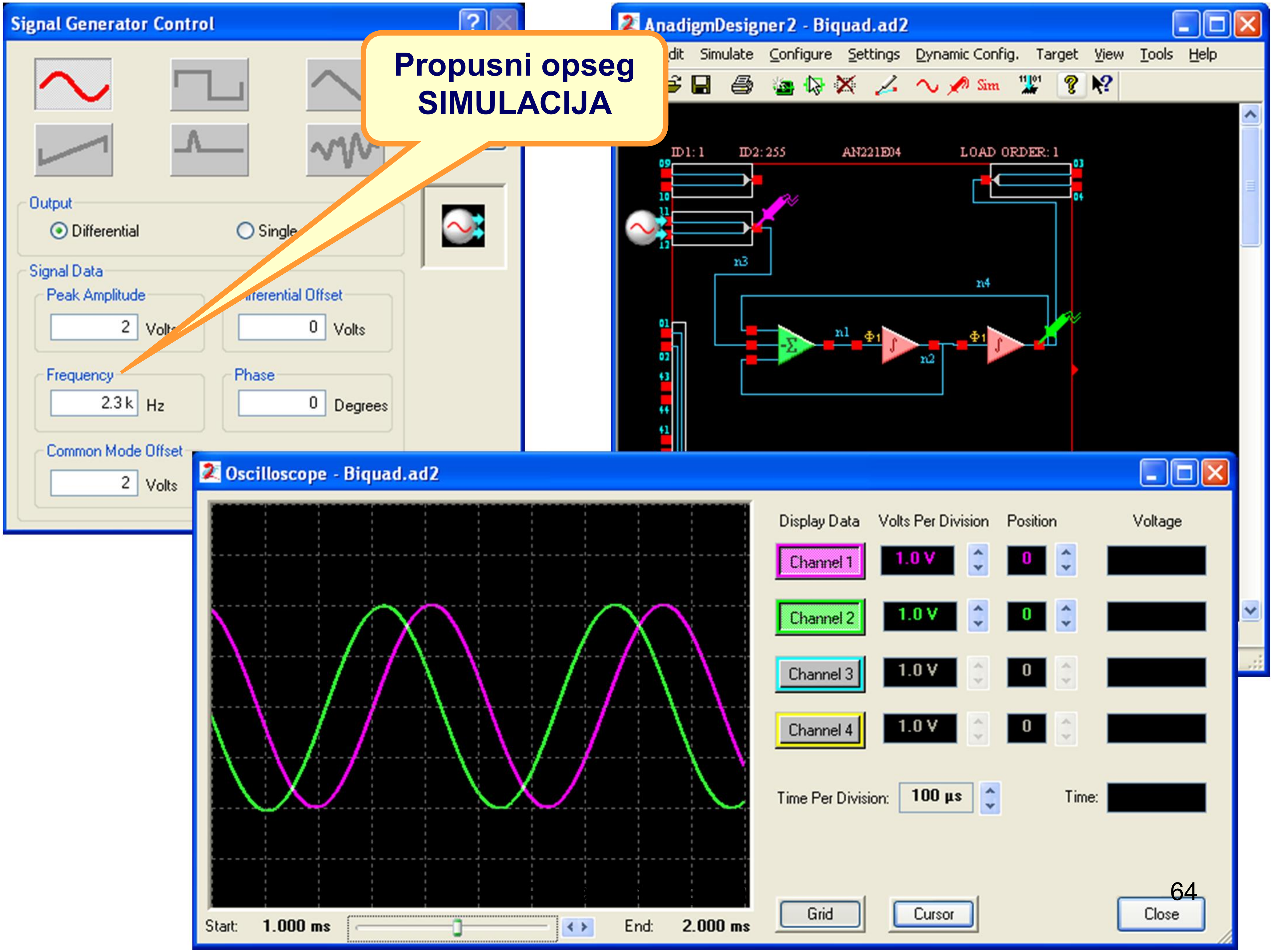
$$\mathrm{Re}(\mathrm{pol}) < 0 \rightarrow \lim_{t \to +\infty} y(t) = 0$$

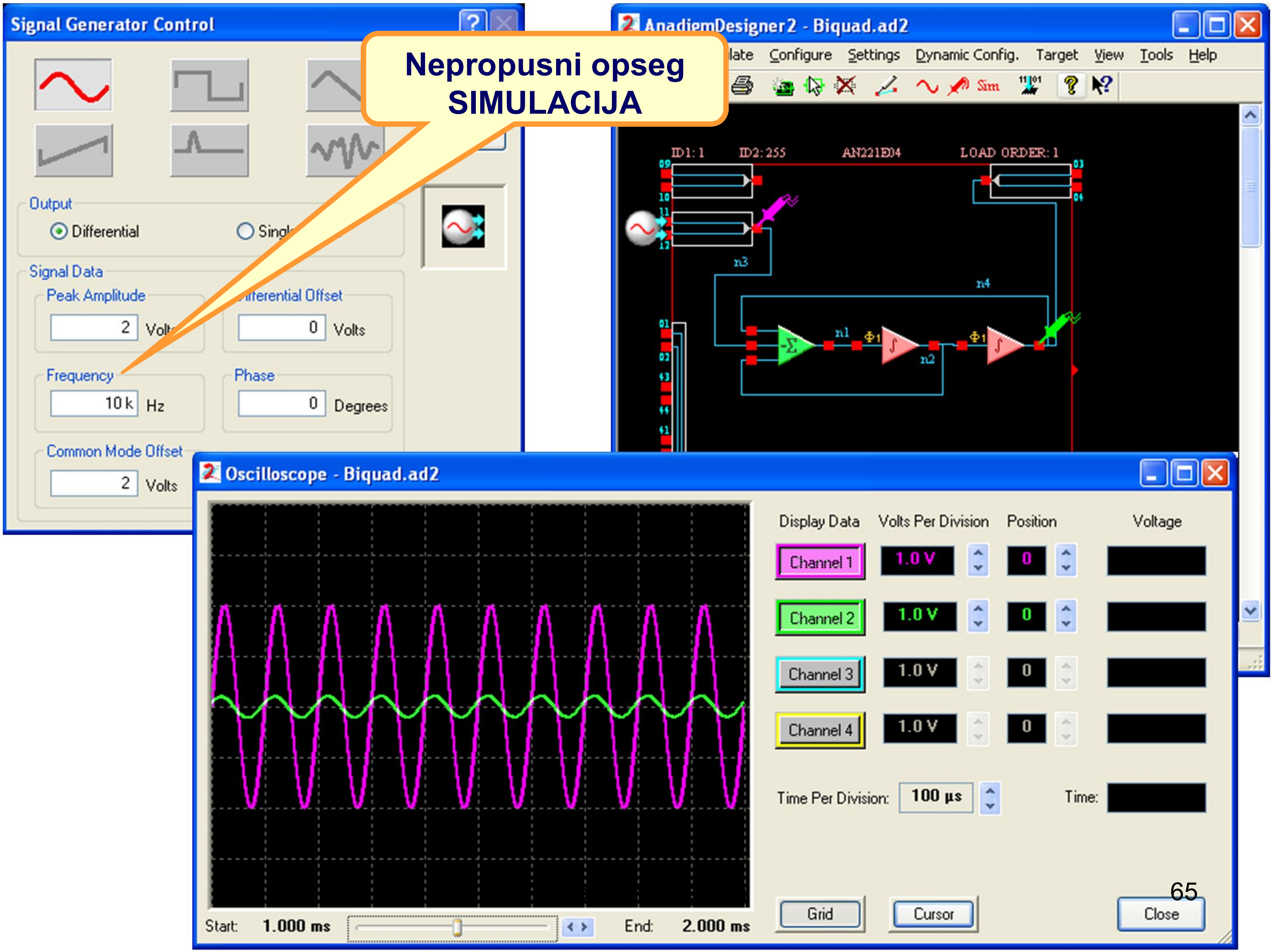
$$a = 0, b = -1 \rightarrow y(t) = \sin(t)$$

Hardverska implementacija

Sistem sa integratorima




Signal Generator Control
Propusni opseg
SIMULACIJA
Output
Differential
Single
Signal Data
Peak Amplitude
2 Volts
Differential Offset
0 Volts
Frequency
2.3 k Hz
Phase
0 Degrees
Common Mode Offset
2 Volts
AnadigmDesigner2 - Biquad.ad2
Simulate Configure Settings Dynamic Config. Target View Tools Help
ID1: 1 ID2: 255 AN221E04 LOAD ORDER: 1
n1 n2 n3 n4
Oscilloscope - Biquad.ad2
Display Data
Volts Per Division
Position
Voltage
Channel 1 1.0 V 0
Channel 2 1.0 V 0
Channel 3 1.0 V 0
Channel 4 1.0 V 0
Time Per Division: 100 µs
Time:
Start: 1.000 ms
End: 2.000 ms
Grid
Cursor
Close


Nepropusni opseg
SIMULACIJA
Signal Generator Control
Output
Differential
Signal Data
Peak Amplitude
2 Volts
Differential Offset
0 Volts
Frequency
10 k Hz
Phase
0 Degrees
Common Mode Offset
2 Volts
AnadigmDesigner2 - Biquad.ad2
Configure Settings Dynamic Config. Target View Tools Help
ID1: 1 ID2: 255 AN221E04 LOAD ORDER: 1
Oscilloscope - Biquad.ad2
Display Data Volts Per Division Position Voltage
Channel 1 1.0 V 0
Channel 2 1.0 V 0
Channel 3 1.0 V 0
Channel 4 1.0 V 0
Time Per Division: 100 µs
Time:
Start: 1.000 ms
End: 2.000 ms
Grid
Cursor
Close

Transformacije

- Laplasova transformacija impulsnog odziva
- Z transformacija impulsnog odziva

$$H_a(s) = \int_{-\infty}^{\infty} h(t) e^{-st} \mathrm{d}t$$

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

Funkcije prenosa

- Racionalna funkcija kompleksne frekvencije $s=\delta+j\Omega$
- Racionalna funkcija kompleksne frekvencije z

$$H_a(s)=\frac{\sum_{k=0}^{M} c_k s^k}{\sum_{k=0}^{N} d_k s^k}=\frac{C(s)}{D(s)}$$

$$H(z)=\frac{\sum_{k=0}^{M} a_k z^{-k}}{1+\sum_{k=1}^{N} b_k z^{-k}}=\frac{Q(z^{-1})}{P(z^{-1})}$$

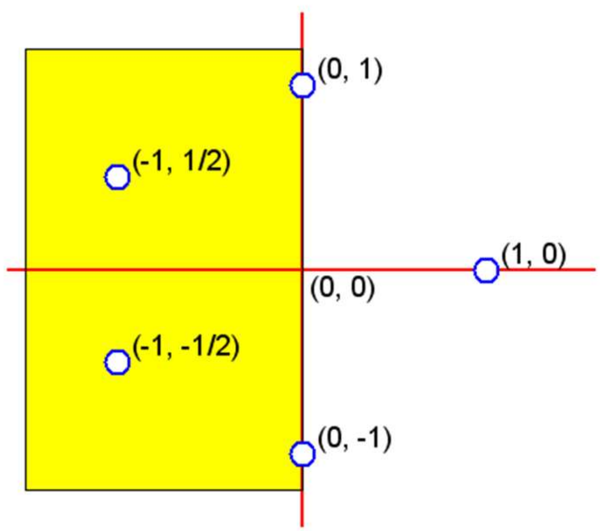
Polovi funkcije prenosa

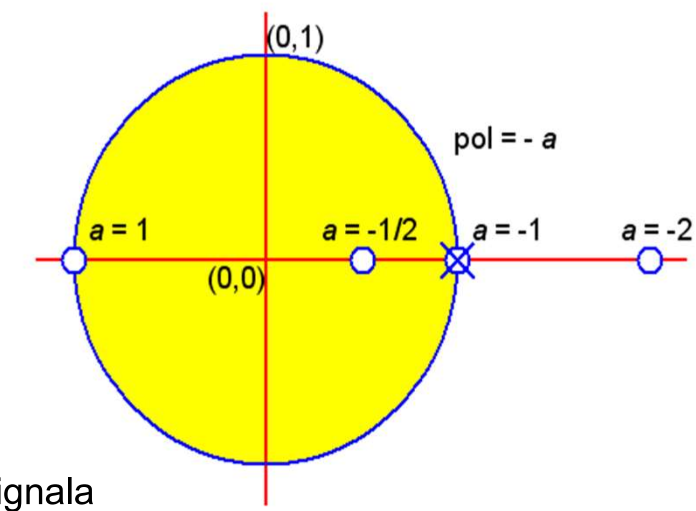
- Leva polovina kompleksne ***s*** ravni

$$H_a(s) = \frac{C(s)}{D(s)}$$

- Unutar jediničnog kruga kompleksne ***z*** ravni

$$H(z) = \frac{Q(z^{-1})}{P(z^{-1})}$$





Frekvencijski odziv

$s = j\Omega$

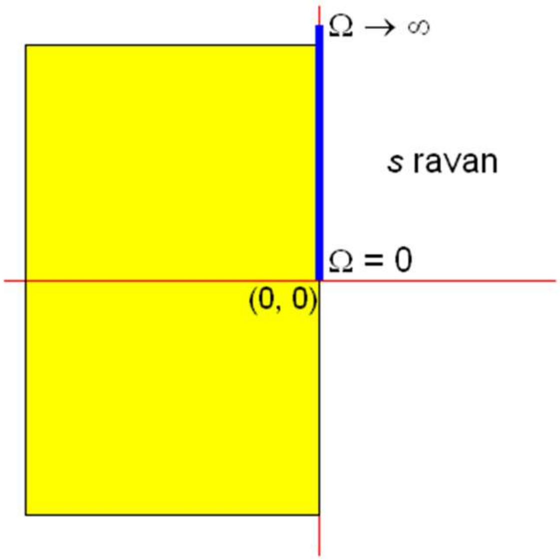
$z = e^{j\omega}$

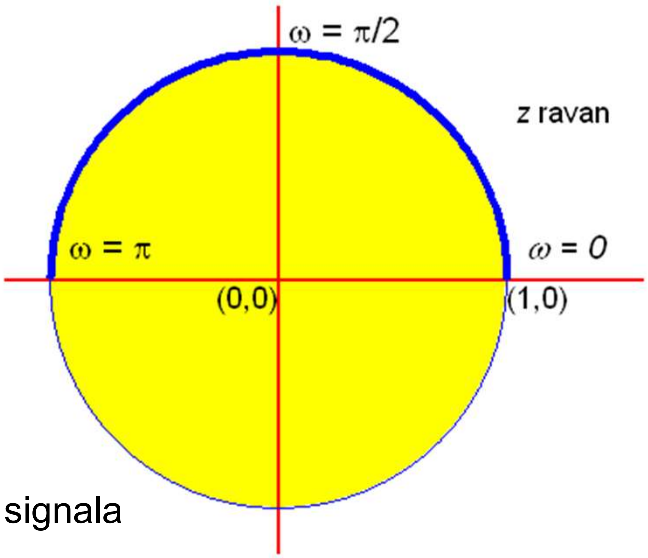
$\boxed{H_a(j\Omega)}$

$\boxed{H(e^{j\omega})}$

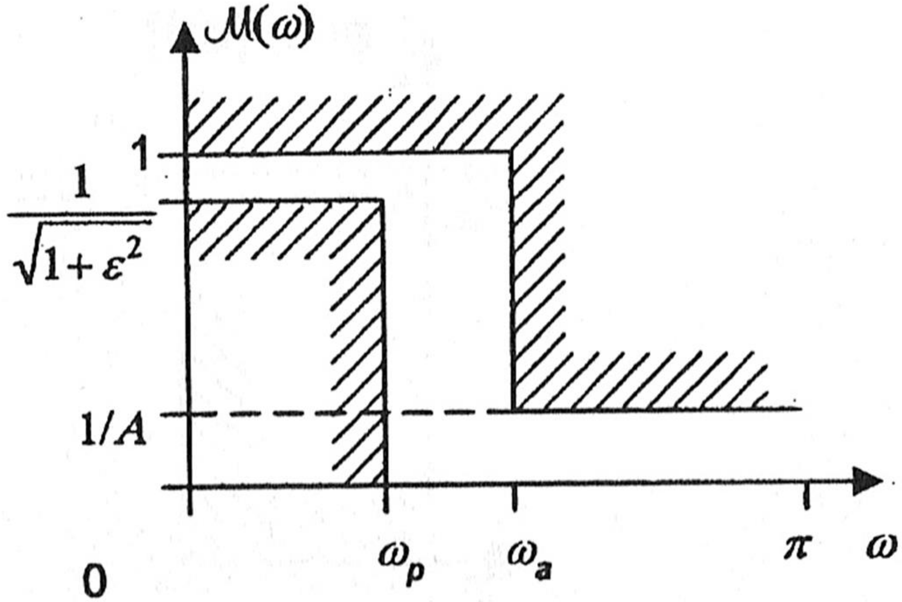
$\boxed{0 \le \Omega \le \infty}$

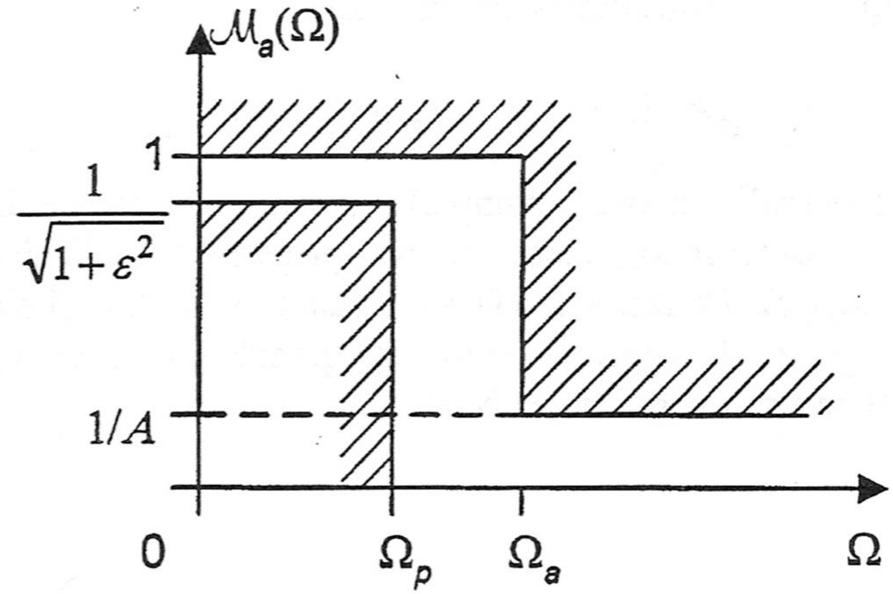
$\boxed{0 \le \omega \le \pi}$





Specifikacije





$$\omega_p \to \Omega_p$$
$$\omega_a \to \Omega_a$$
$$\pi \to \infty$$

Profesor dr Miroslav Lutovac
mlutovac@viser.edu.rs