



Procesiranje signala

Profesor dr Miroslav Lutovac

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Sadržaj predmeta, Teorijska nastava

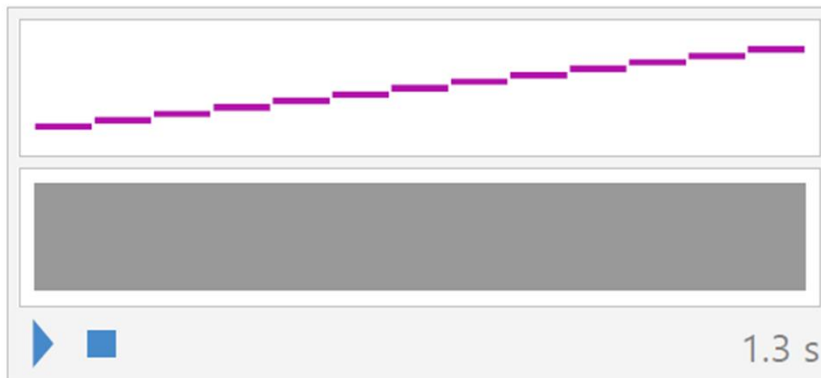
1. Uvodno predavanje. Upoznavanje sa planom i programom, ciljevima, ishodom i metodama.
2. Šta je procesiranje signala, istorijski pregled obrade signala, primeri primene.
3. Vizuelizacija signala (Python, Excel).
4. Kompleksni ekponencijalni diskretni signali. Primer sinteze muzičkog signala.
5. Furijeova analiza: Diskretna Furijeova transformacija (DFT) i serija (DFS). Brza Furijeova transformacija (Fast Fourier transform, FFT) i primena za spektralne analizatore i osciloskope.
6. Linearani filtri: konvolucija, idealni i realni filtri, dizajn filtra. Primena konvolucije u GPS sistemima.

...

Sinteza muzičkog signala

- SoundNote[0] represents a music-like sound note with the specified pitch.
- 0, middle C
- SoundNote[n] , n semitones from middle C
- "C", "C#", "D", ... notes in the middle C octave

```
sound01 = Sound[Table[SoundNote[i, 0.1, "Violin"], {i, 0, 12}]]  
EmitSound[sound01]  
Export["sound01.WAV", sound01]
```



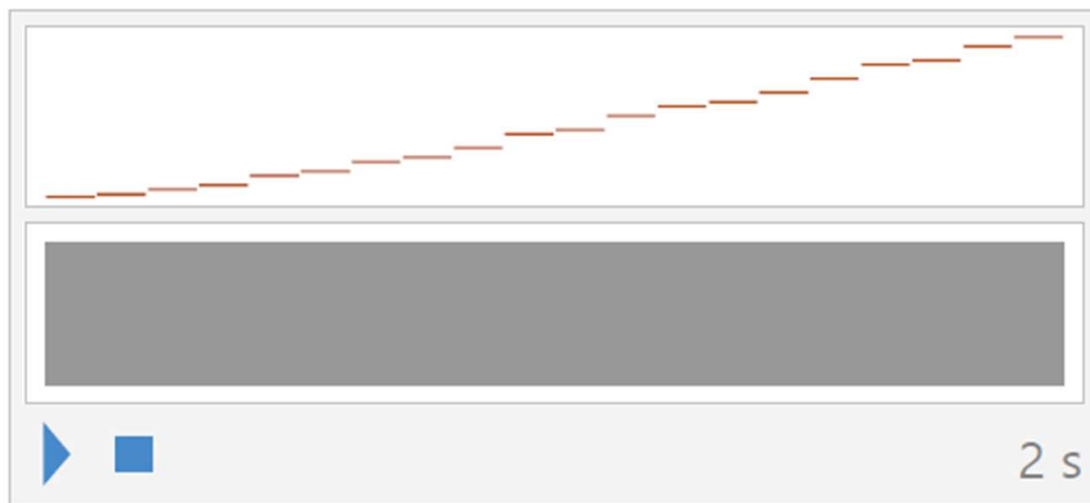
sound01.WAV

Procesiranje signala

Sinteza muzičkog signala

- SoundNote[0,t] takes the note to have duration t
- SoundNote[0,t,"Violin"] takes the note to be in the specified style

```
sound02 = Sound[Table[SoundNote[Prime[i] - 20, 0.1, "Woodblock"], {i, 20}]]  
EmitSound[sound02]  
Export["sound02.WAV", sound02]
```

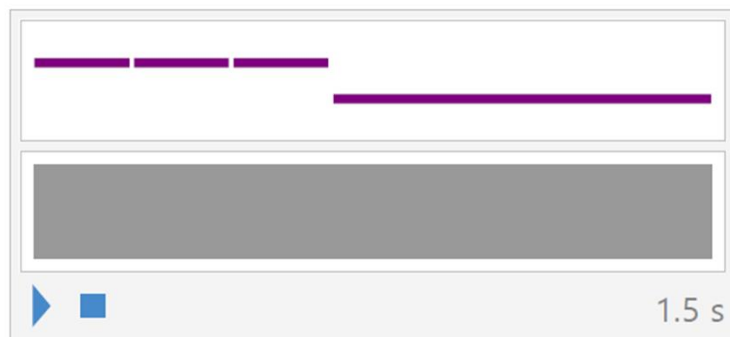


sound02.WAV

Sinteza muzičkog signala

- SoundNote[0] represents a music-like sound note with the specified pitch.

```
sound03 = Sound[{SoundNote["G"], SoundNote["G"], SoundNote["G"], SoundNote["Eb", 4]}, 1.5]  
EmitSound[sound03]  
Export["sound03.WAV", sound03]
```



sound03.WAV

Sinteza muzičkog signala

```
sound04 = Sound[SoundNote[#, 0.5, "Polysynth"] & /@ {2, 4, 0, -12, -5}]  
EmitSound[sound04]  
Export["sound04.WAV", sound04]
```



sound04.WAV

Sinteza muzičkog signala

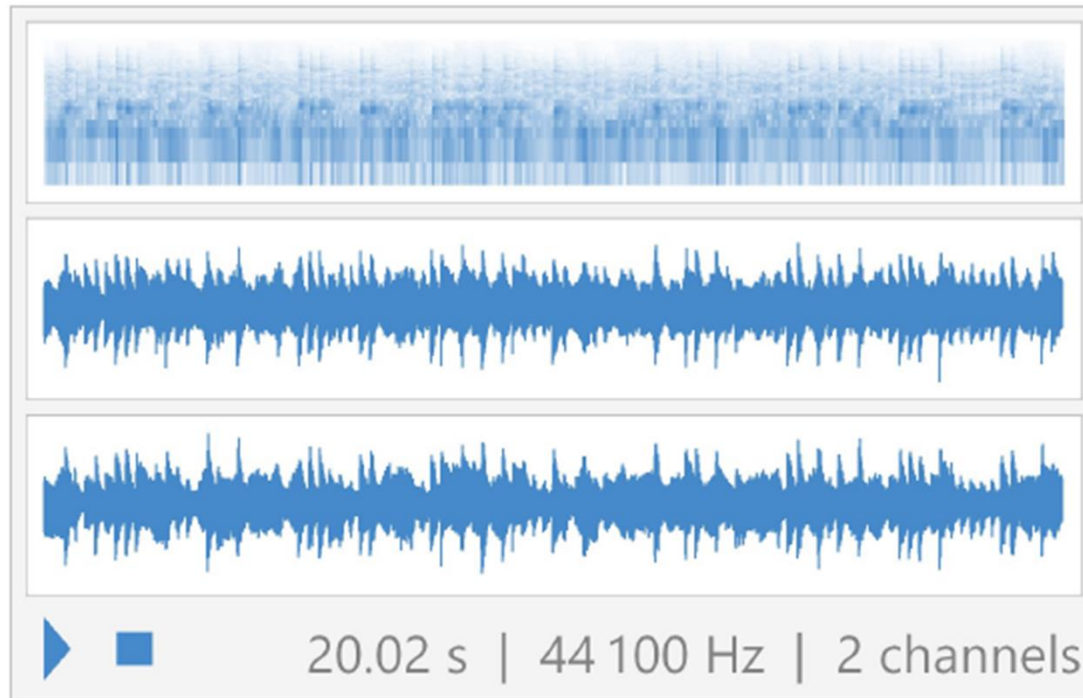
```
sound05 = Sound[Table[  
    SoundNote[RandomInteger[7], .2, RandomChoice[{"Piano", "Guitar", "Violin"}]], 100]]  
EmitSound[sound05]  
Export["sound05.WAV", sound05]
```



sound05.WAV

Sinteza muzičkog signala

```
snd05 = Import["sound05.WAV", "Sound"]
```



Styles

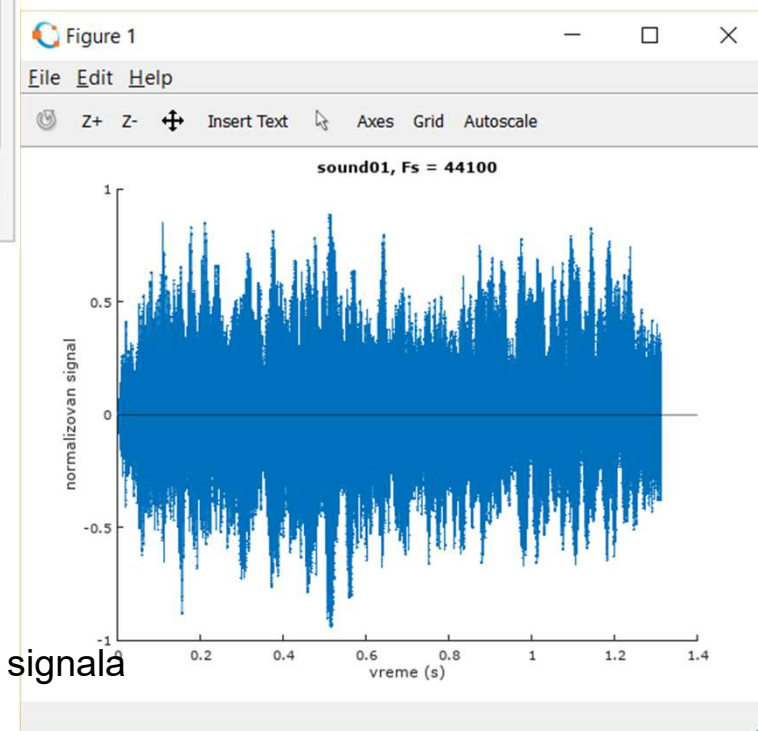
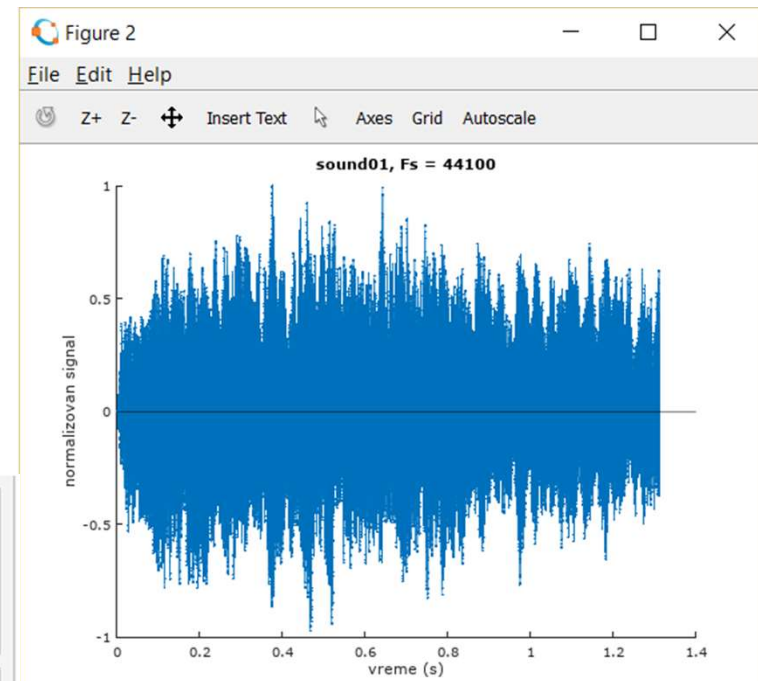
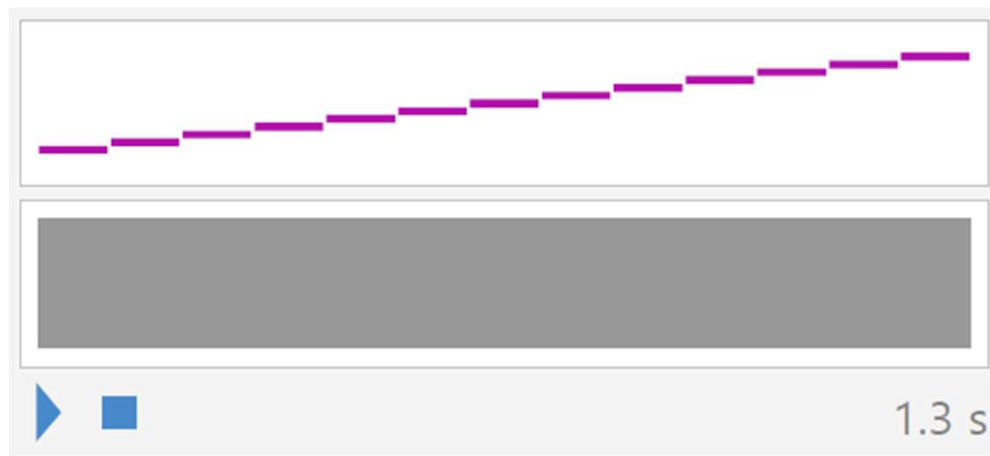
"Accordion"	"Agogo"	"AltoSax"	"Applause"
"Atmosphere"	"Bagpipe"	"Bandoneon"	"Banjo"
"BaritoneSax"	"Bass"	"BassAndLead"	"Bassoon"
"Bird"	"BlownBottle"	"Bowed"	"BrassSection"
"Breath"	"Brightness"	"BrightPiano"	"Calliope"
"Celesta"	"Cello"	"Charang"	"Chiff"
"Choir"	"Clarinet"	"Clavi"	"Contrabass"
"Crystal"	"DrawbarOrgan"	"Dulcimer"	"Echoes"
"ElectricBass"	"ElectricGrandPiano"	"ElectricGuitar"	"ElectricPiano"
"ElectricPiano2"	"EnglishHorn"	"Fiddle"	"Fifths"
"Flute"	"FrenchHorn"	"FretlessBass"	"FretNoise"

"Glockenspiel"	"Goblins"	"Guitar"	"GuitarDistorted"
"GuitarHarmonics"	"GuitarMuted"	"GuitarOverdriven"	"Gunshot"
"Halo"	"Harmonica"	"Harp"	"Harpsichord"
"Helicopter"	"HonkyTonkPiano"	"JazzGuitar"	"Kalimba"
"Koto"	"Marimba"	"MelodicTom"	"Metallic"
"MusicBox"	"MutedTrumpet"	"NewAge"	"Oboe"
"Ocarina"	"OrchestraHit"	"Organ"	"PanFlute"
"PercussiveOrgan"	"Piano"	"Piccolo"	"PickedBass"
"PizzicatoStrings"	"Polysynth"	"Rain"	"Recorder"
"ReedOrgan"	"ReverseCymbal"	"RockOrgan"	"Sawtooth"
"SciFi"	"Seashore"	"Shakuhachi"	"Shamisen"
"Shanai"	"Sitar"	"SlapBass"	"SlapBass2"
"SopranoSax"	"Soundtrack"	"Square"	"Steeldrums"
"SteelGuitar"	"Strings"	"Strings2"	"Sweep"
"SynthBass"	"SynthBass2"	"SynthBrass"	"SynthBrass2"
"SynthDrum"	"SynthStrings"	"SynthStrings2"	"SynthVoice"
"Taiko"	"Telephone"	"TenorSax"	"Timpani"
"Tinklebell"	"TremoloStrings"	"Trombone"	"Trumpet"
"Tuba"	"TubularBells"	"Vibraphone"	"Viola"
"Violin"	"Voice"	"VoiceAahs"	"VoiceOohs"
"Warm"	"Whistle"	"Woodblock"	"Xylophone"

"BassDrum"	"BassDrum2"	"BellTree"	"Cabasa"
"Castanets"	"ChineseCymbal"	"Clap"	"Claves"
"Cowbell"	"CrashCymbal"	"CrashCymbal2"	"ElectricSnare"
"GuiroLong"	"GuiroShort"	"HighAgogo"	"HighBongo"
"HighCongaMute"	"HighCongaOpen"	"HighFloorTom"	"HighTimbale"
"HighTom"	"HighWoodblock"	"HiHatClosed"	"HiHatOpen"
"HiHatPedal"	"JingleBell"	"LowAgogo"	"LowBongo"
"LowConga"	"LowFloorTom"	"LowTimbale"	"LowTom"
"LowWoodblock"	"Maracas"	"MetronomeBell"	"MetronomeClick"
"MidTom"	"MidTom2"	"MuteCuica"	"MuteSurdo"
"MuteTriangle"	"OpenCuica"	"OpenSurdo"	"OpenTriangle"
"RideBell"	"RideCymbal"	"RideCymbal2"	"ScratchPull"
"ScratchPush"	"Shaker"	"SideStick"	"Slap"
"Snare"	"SplashCymbal"	"SquareClick"	"Sticks"
"Tambourine"	"Vibraslap"	"WhistleLong"	"WhistleShort"

sound0x

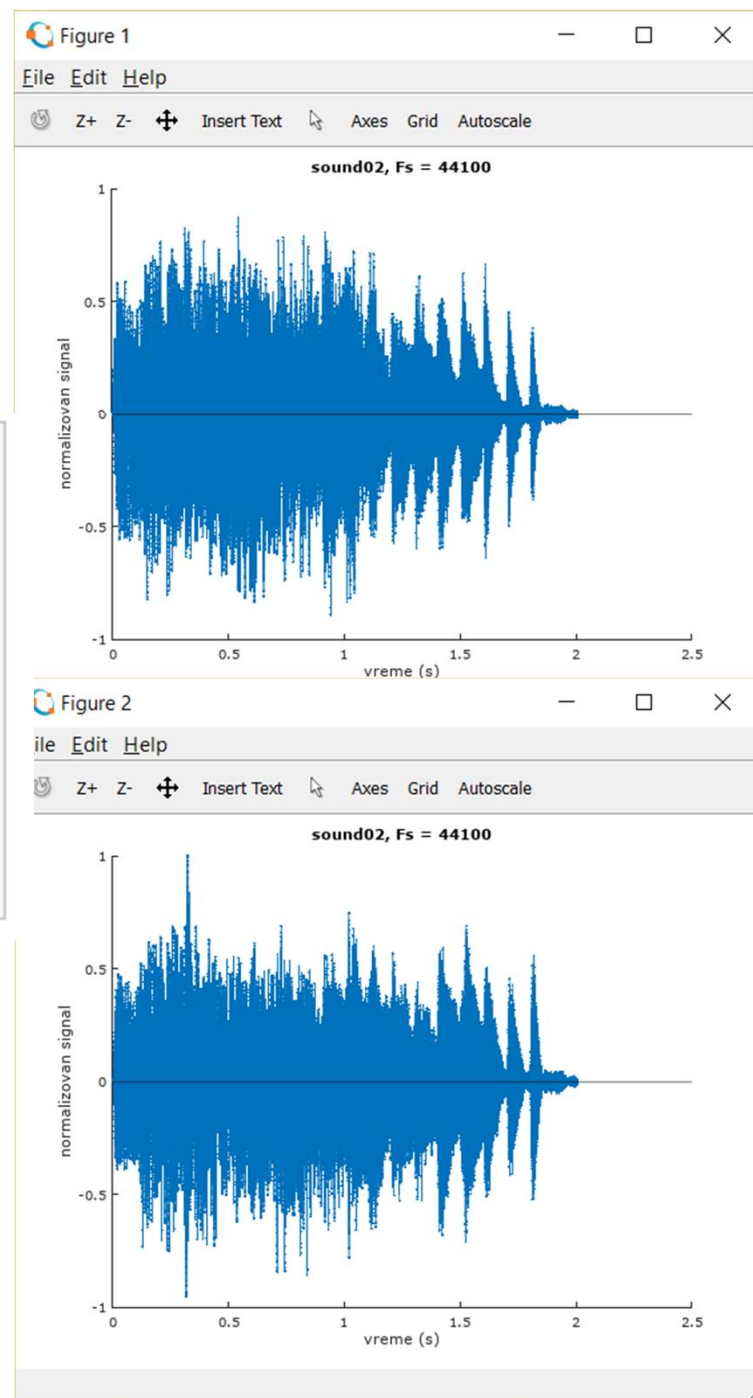
```
clear all, close all, clc
[y,fs]=audioread ("wav/sound05.wav");
y1=y(:,1);y2=y(:,2);
sound(y1,fs); pause(.2);sound(y2,fs)
t=1:length(y1);
t=t/fs;
stem(t,y1,'.')
xlabel('vreme (s)');
ylabel('normalizovan signal');
title(['sound05, Fs = ' num2str(fs)]);figure
stem(t,y2,'.')
xlabel('vreme (s)');
ylabel('normalizovan signal');
title(['sound05, Fs = ' num2str(fs)]);
```



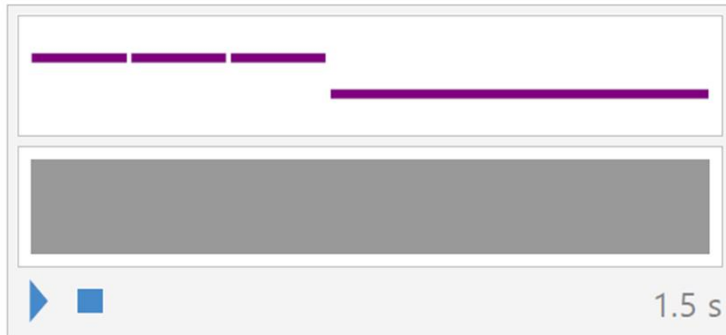
Procesiranje signala



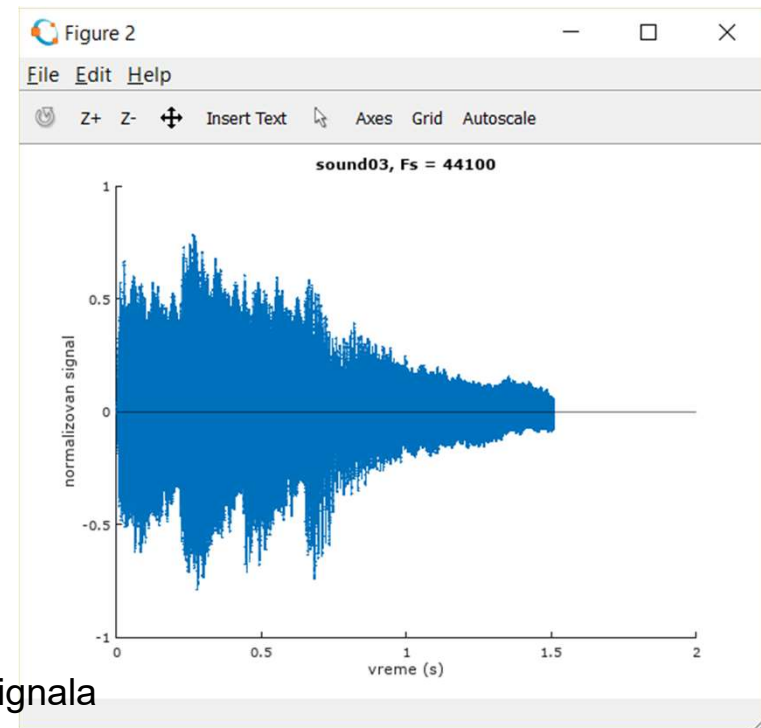
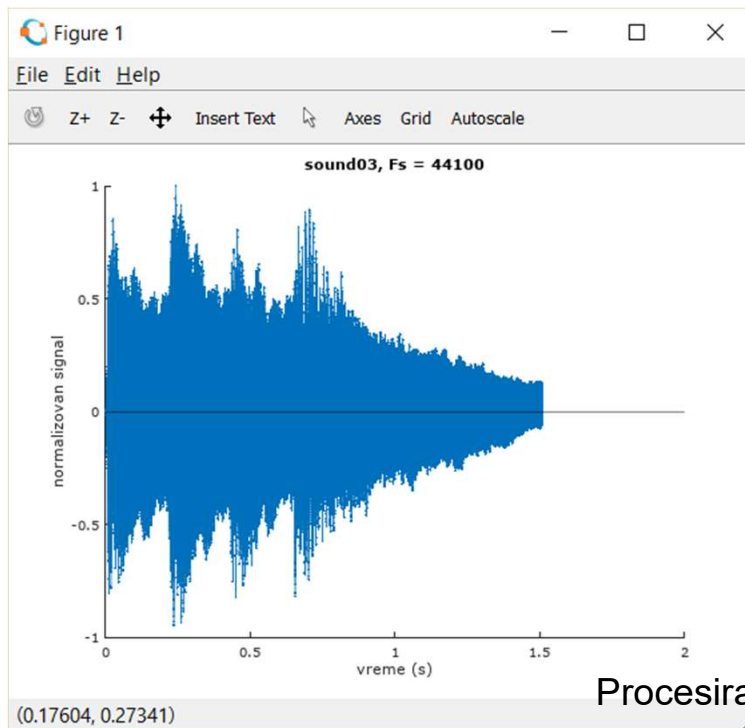
Procesiranje signala



```
sound03 = Sound[{SoundNote["G"], SoundNote["G"], SoundNote["G"], SoundNote["Eb", 4]}, 1.5]  
EmitSound[sound03]  
Export["sound03.WAV", sound03]
```



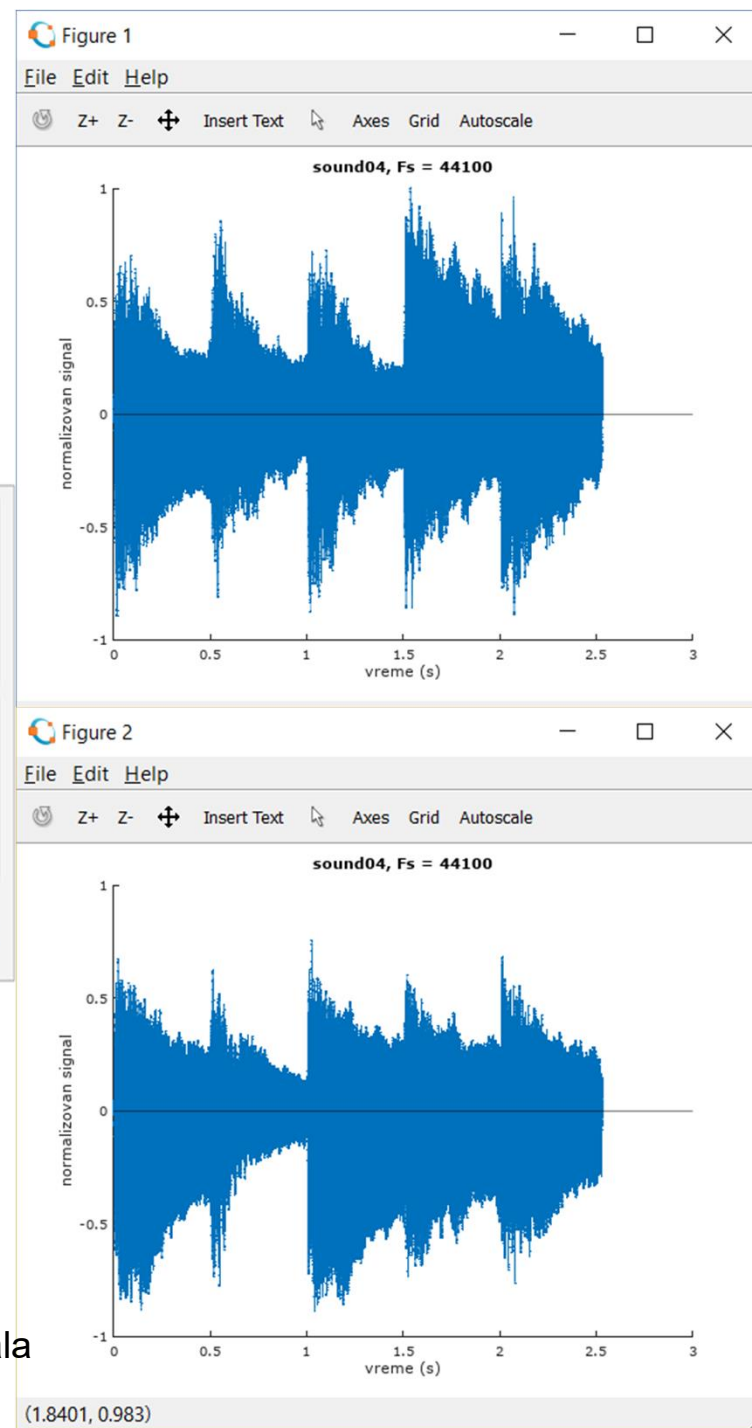
sound03.WAV

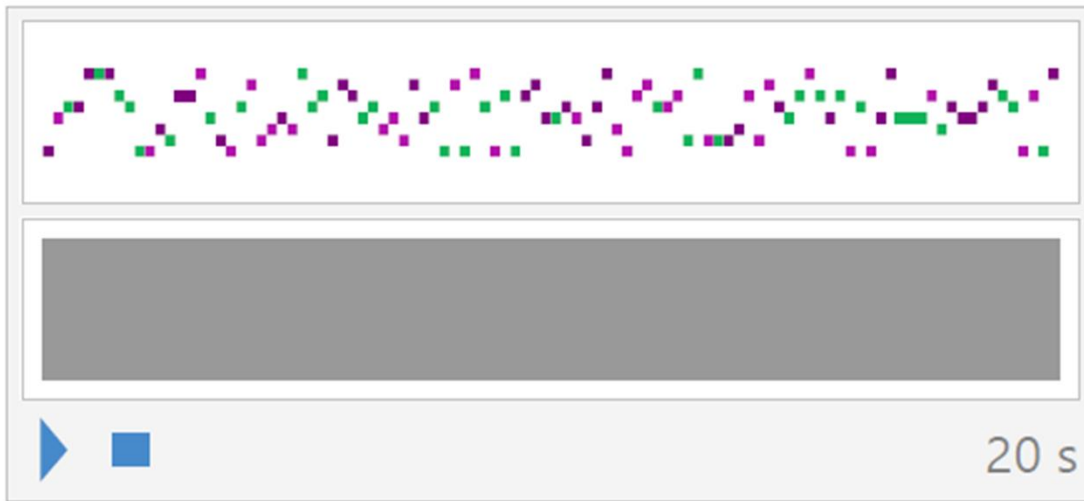


Procesiranje signala



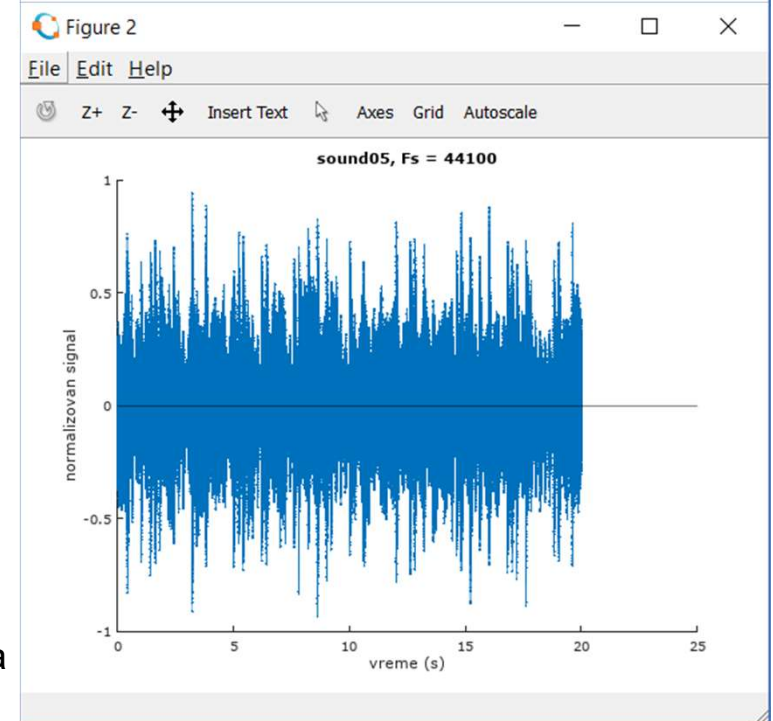
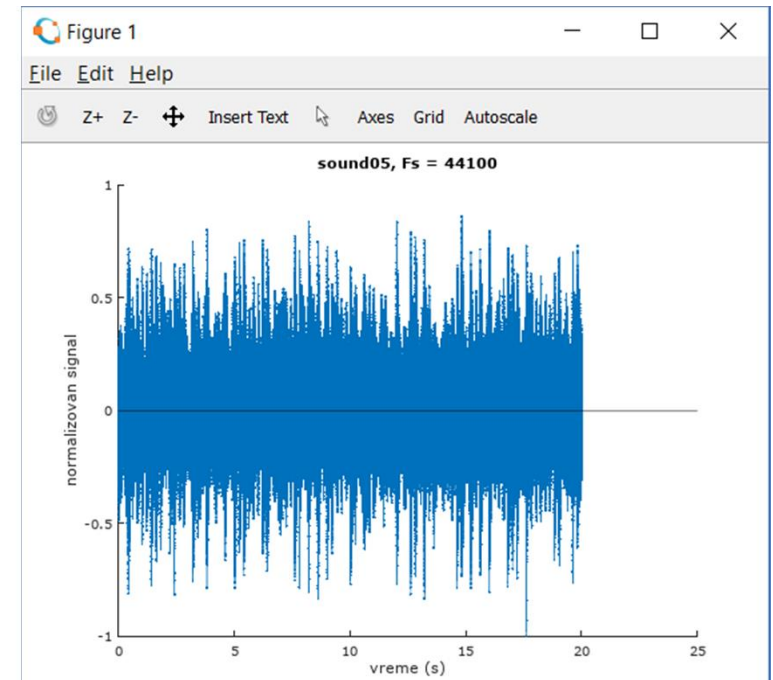
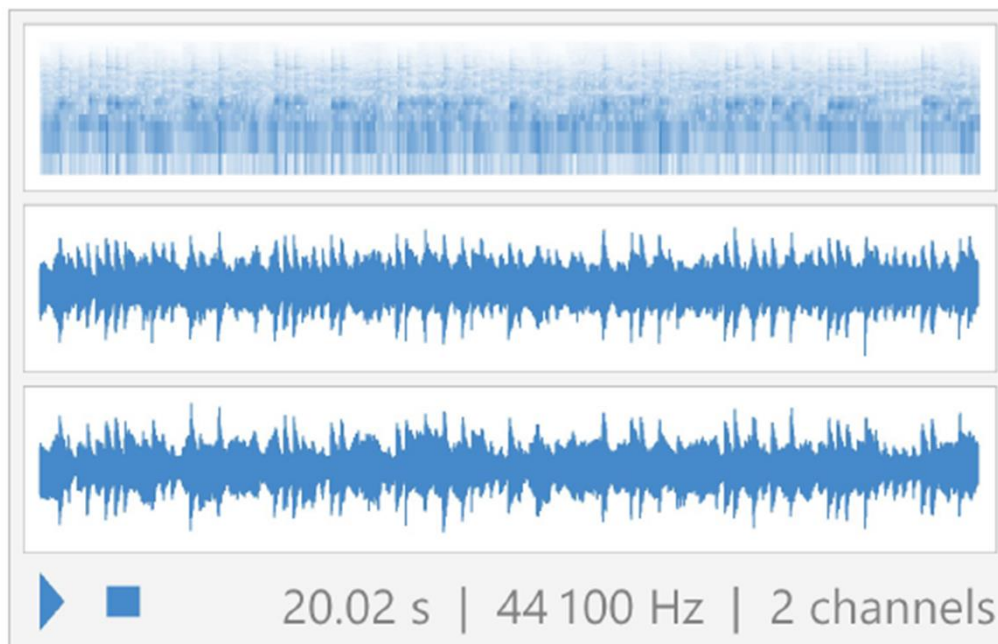
Procesiranje signala





sound05.WAV

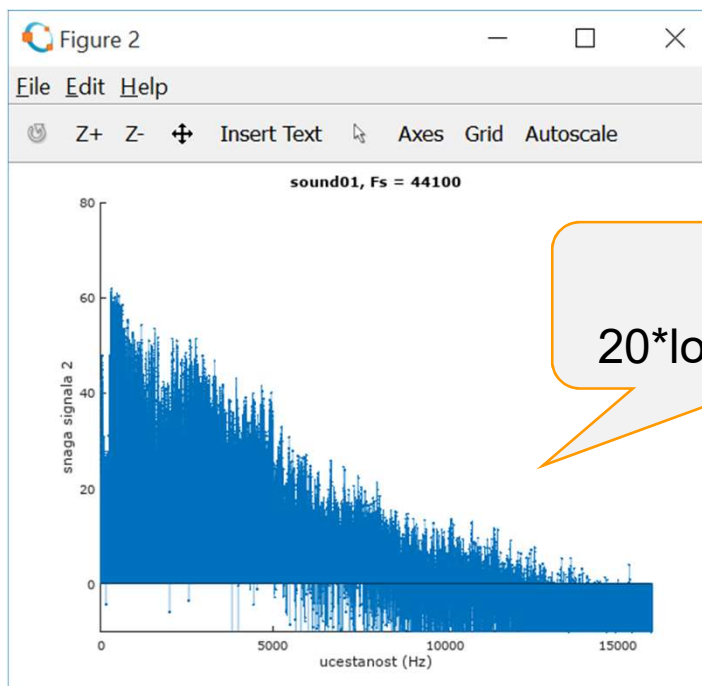
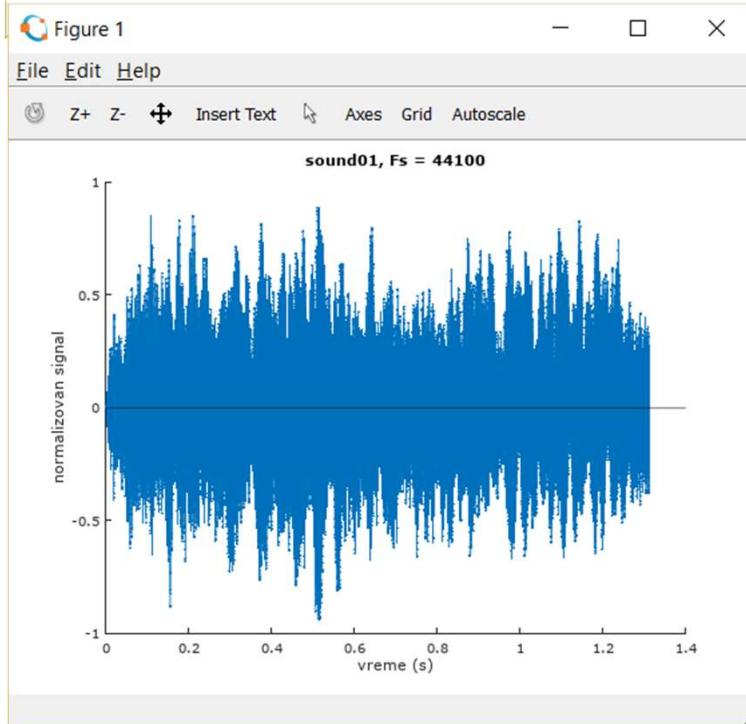
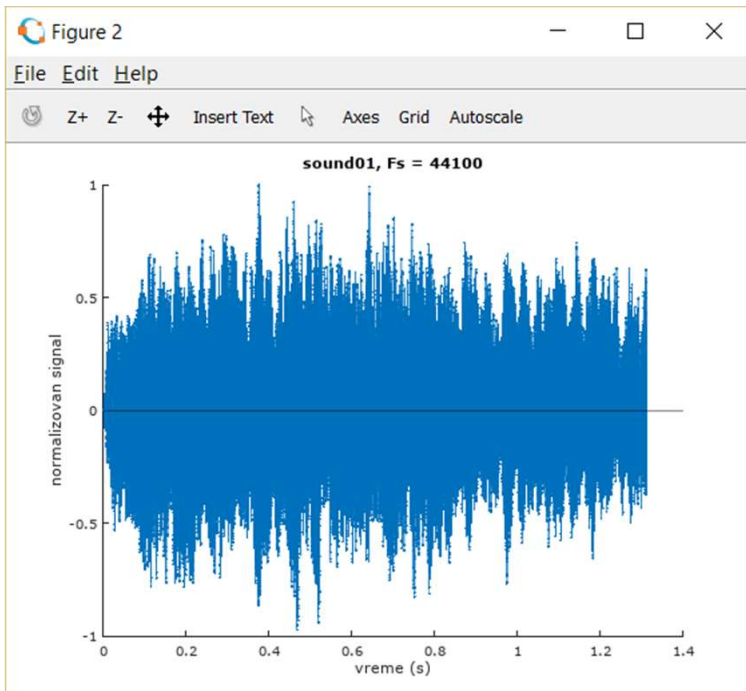
```
snd05 = Import["sound05.WAV", "Sound"]
```



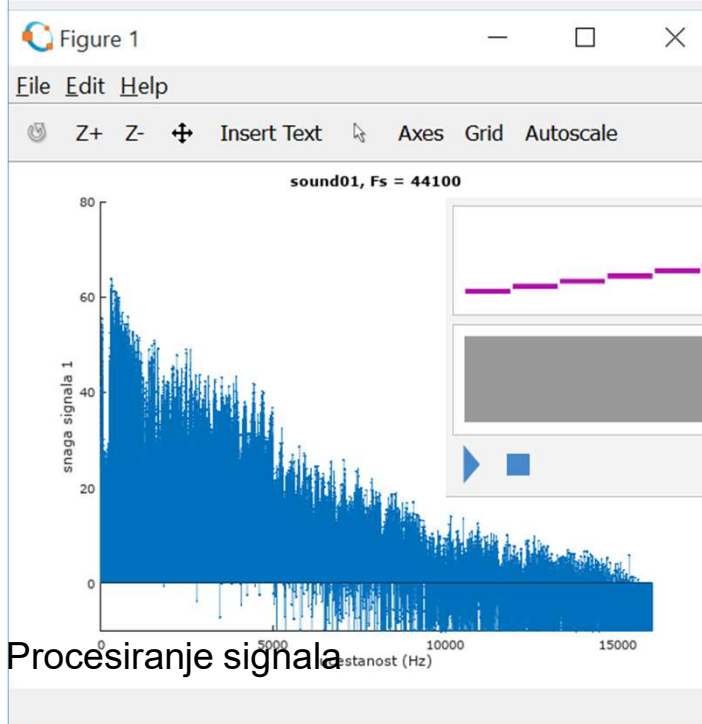
```

t=1:length(y1);
t=t/fs;
x1=fft(y1);
N1= length(y1)-1;
n= 0:N1;
a1 = 20*log10(abs(x1));
stem(fs*n/N1,a1,'.')
xlabel('ucestanost (Hz)');
ylabel('snaga signala 1');
title(['sound01, Fs = ' num2str(fs)]);
ax = axis; ax(2)=16000; ax(3)=ax(4)-90; axis(ax)
figure
x2=fft(y2);
N2= length(y2)-1;
n= 0:N2;
a2 = 20*log10(abs(x2));
stem(fs*n/N1,a2,'.')
xlabel('ucestanost (Hz)');
ylabel('snaga signala 2');
title(['sound01, Fs = ' num2str(fs)]);
ax = axis; ax(2)=16000; ax(3)=ax(4)-90; axis(ax)

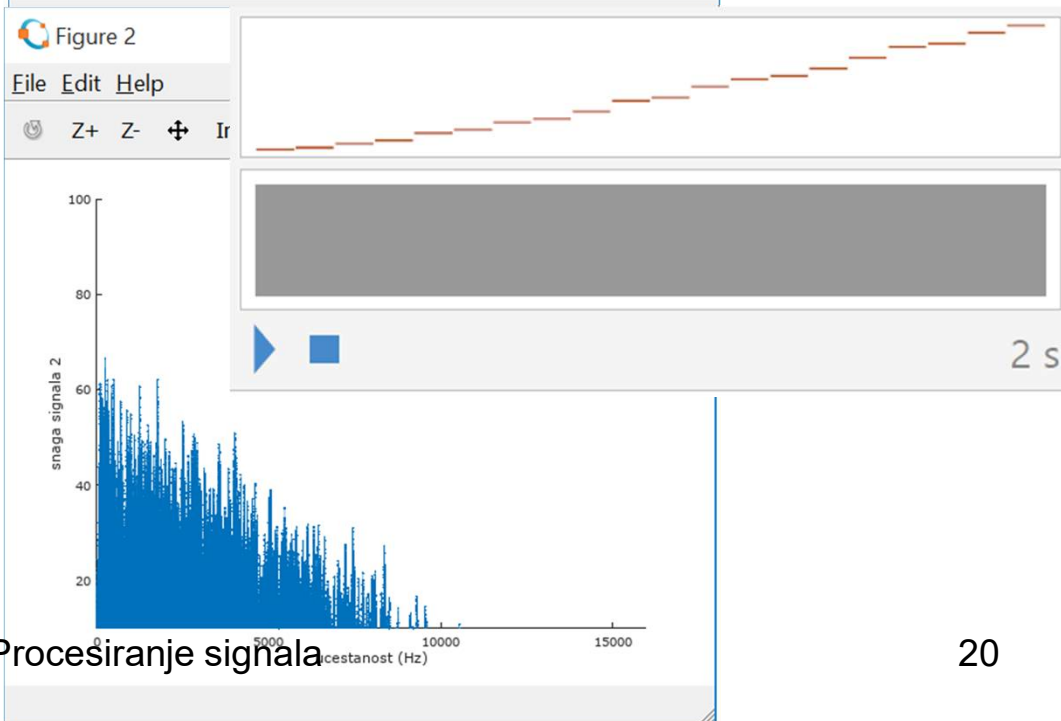
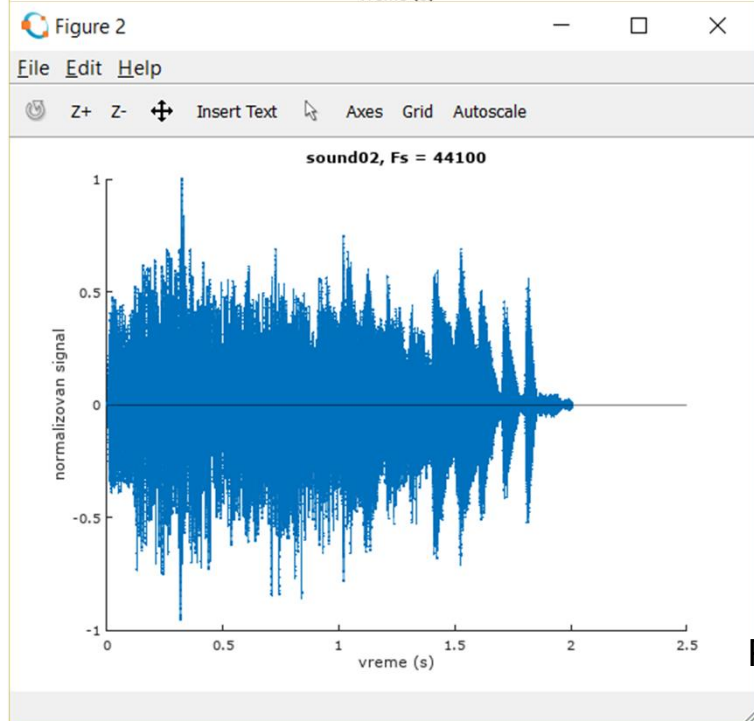
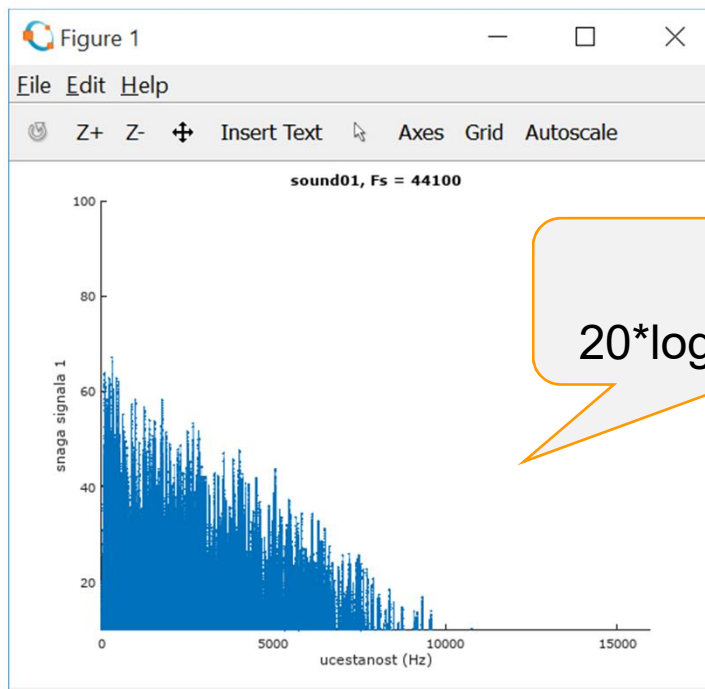
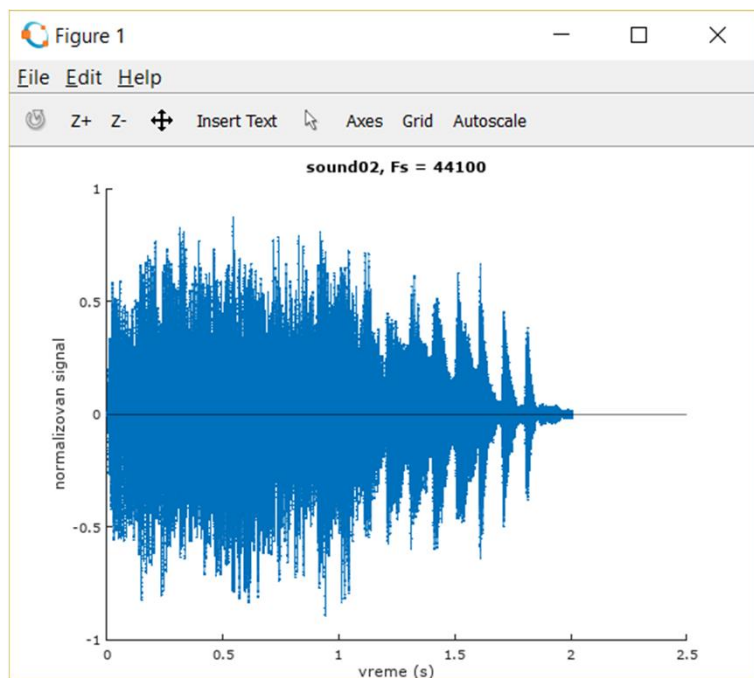
```



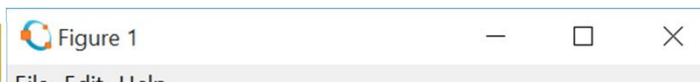
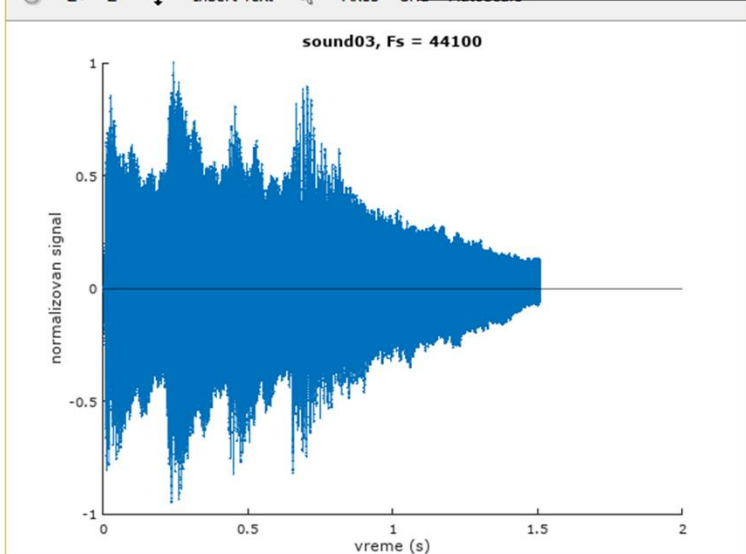
fft
 $20 \cdot \log_{10}(\text{abs}(x1));$



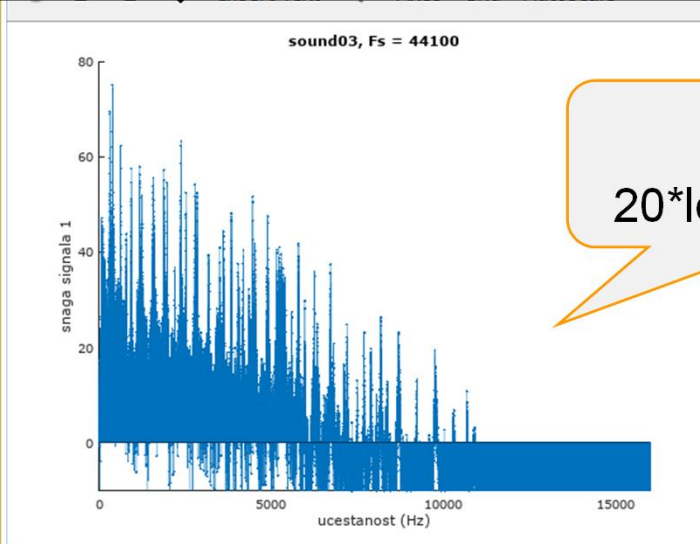
Procesiranje signala



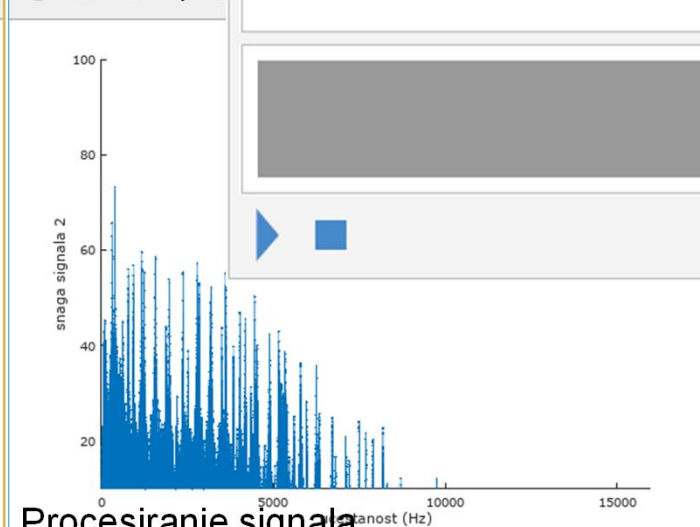
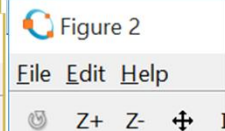
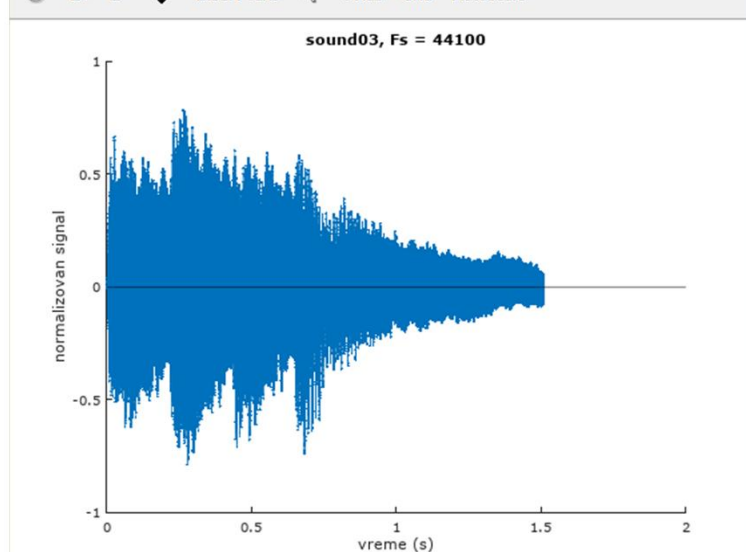
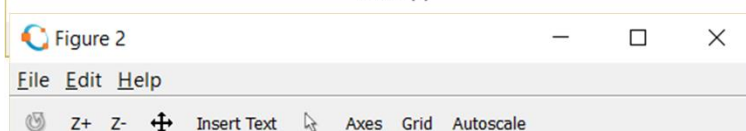
Procesiranje signala



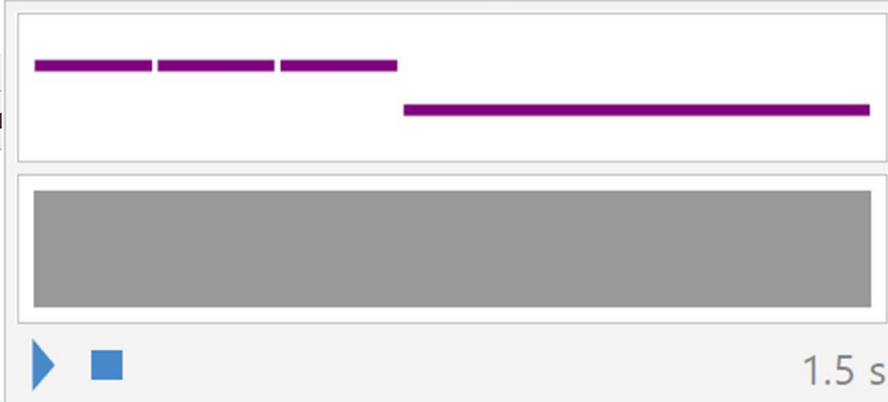
```
{ SoundNote ["G"], SoundNote ["G"], SoundNote ["G"], SoundNote ["Eb", 4] }
```

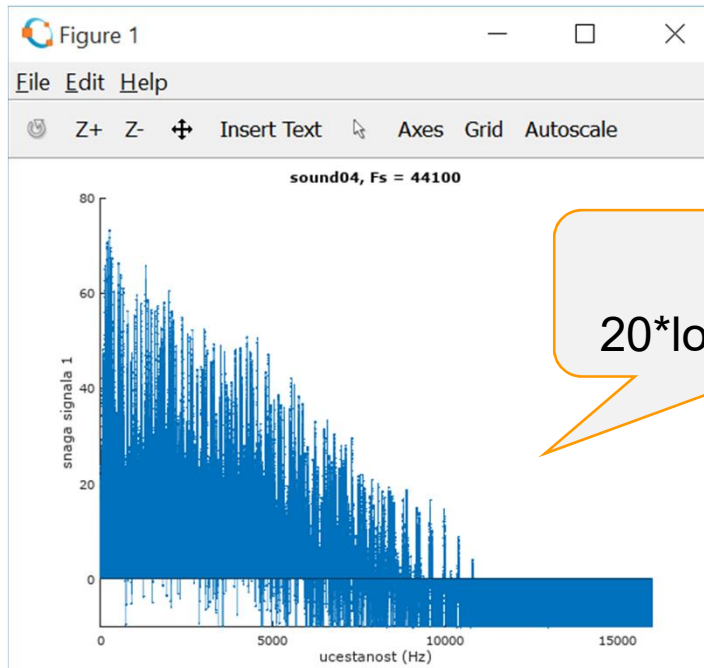
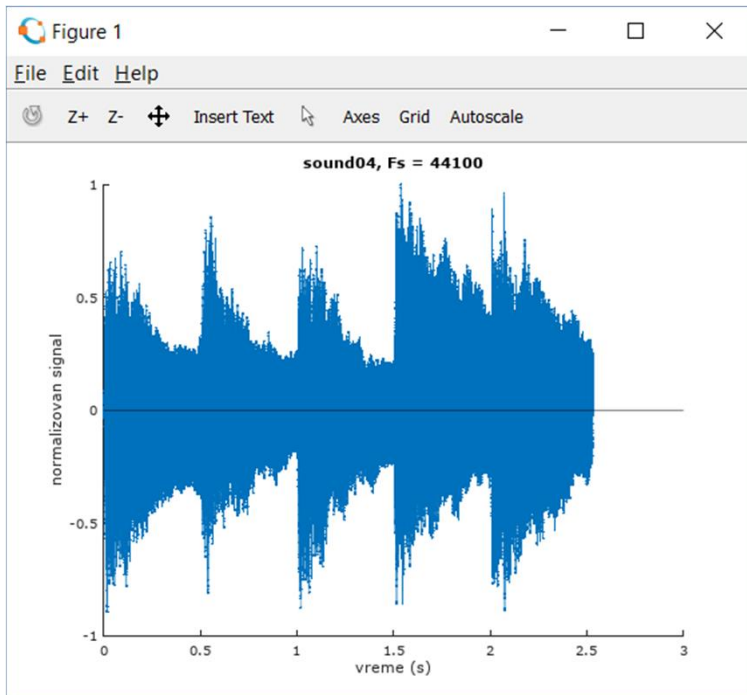


fft
 $20 \cdot \log_{10}(\text{abs}(x1));$

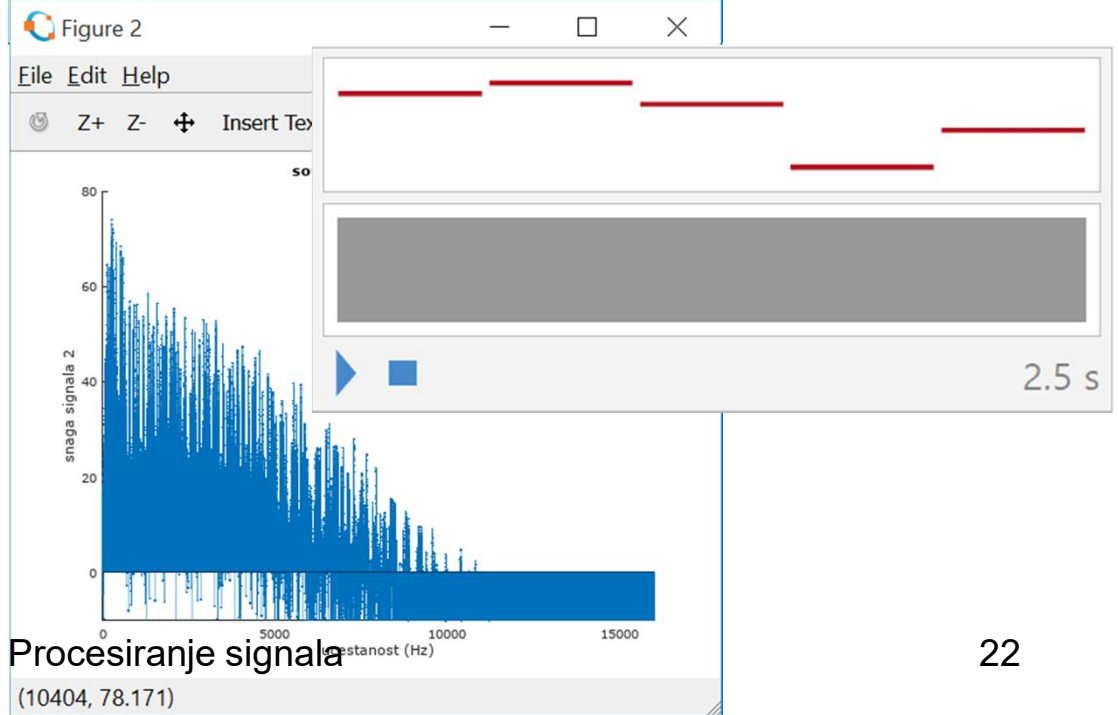
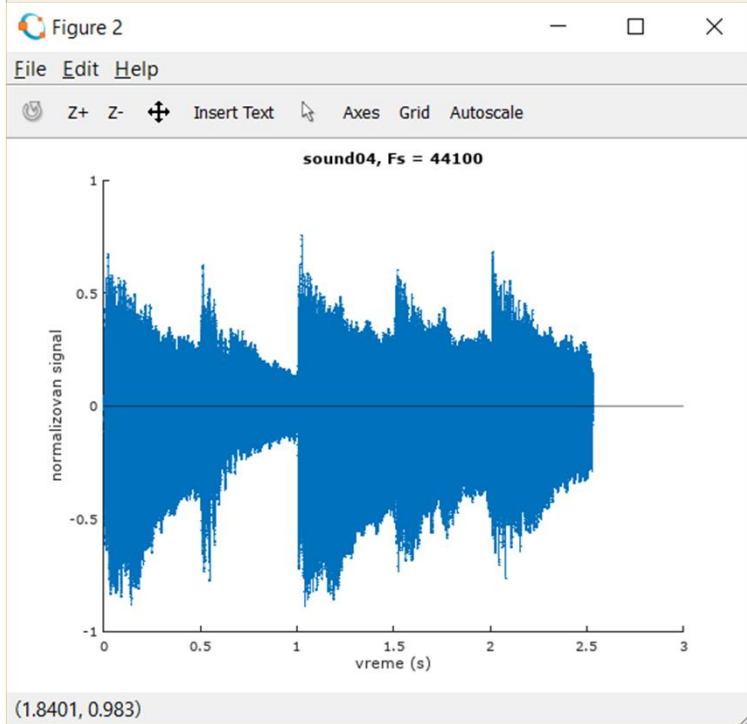


Procesiranje signala

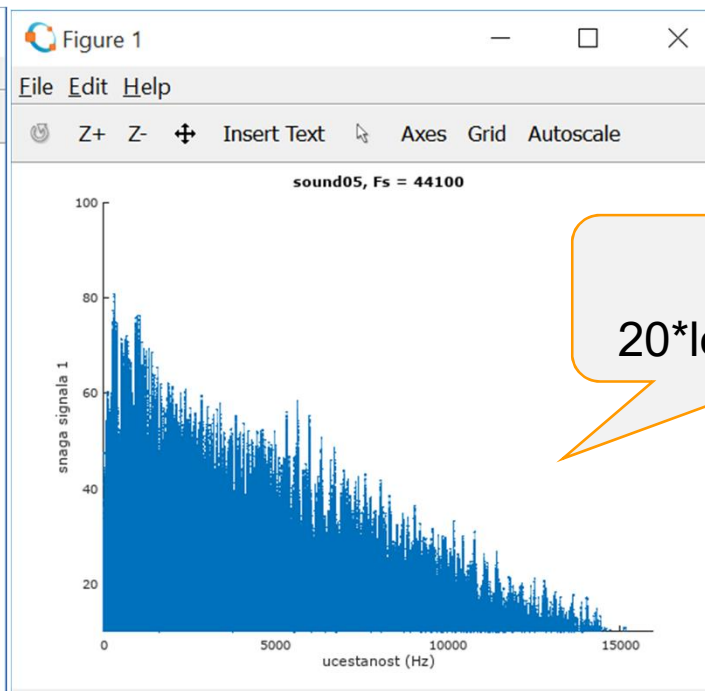
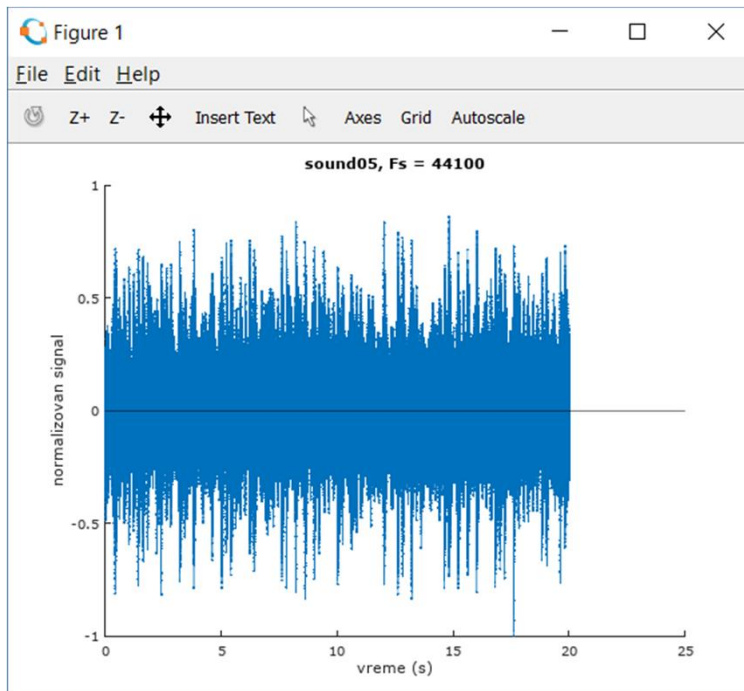




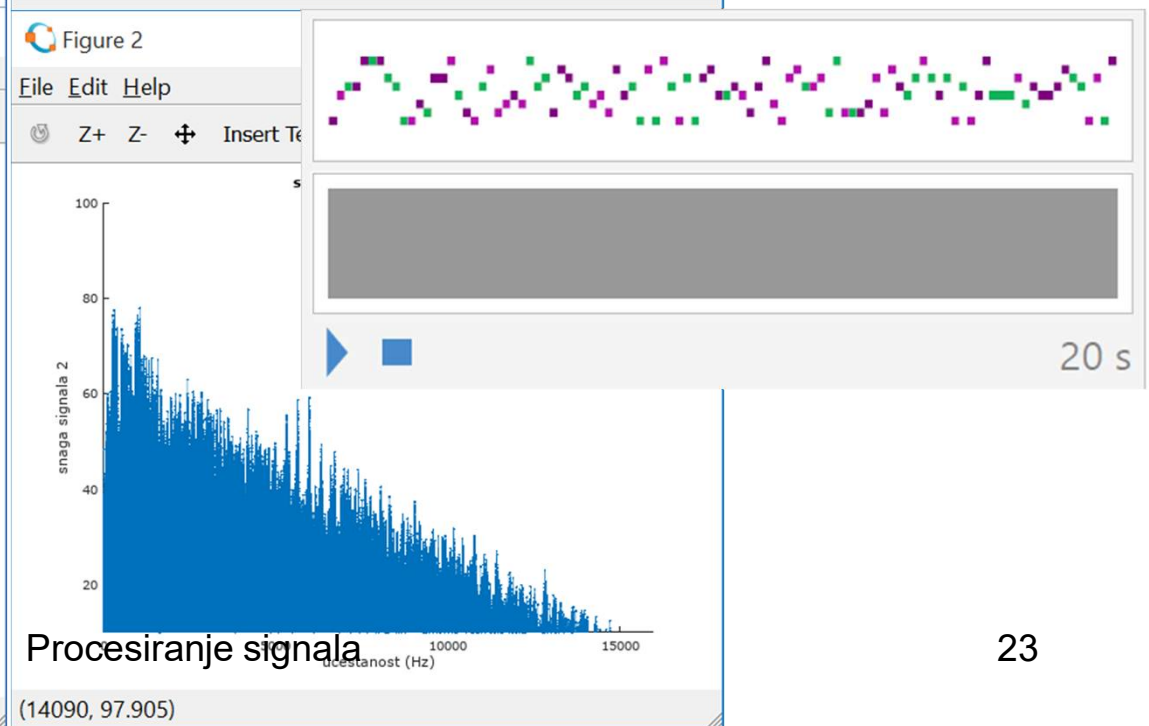
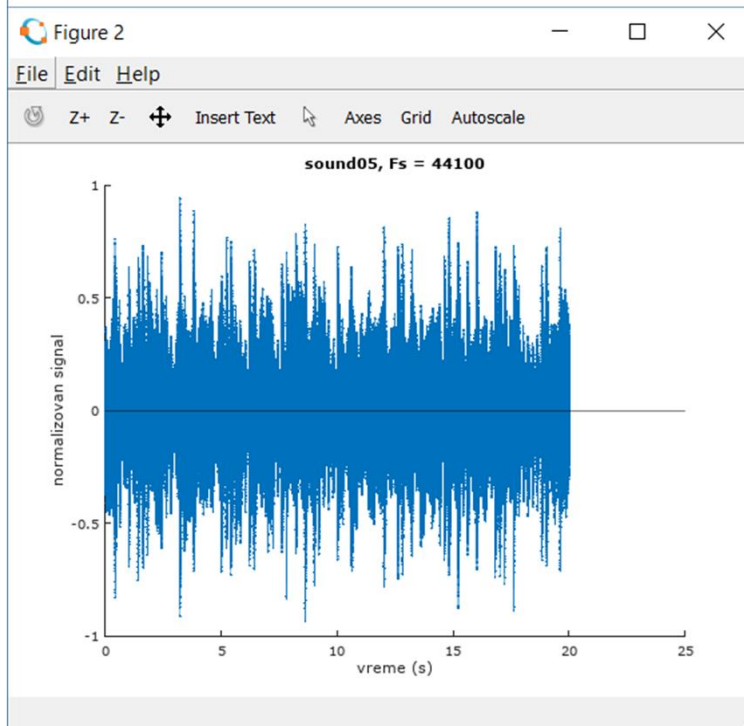
fft
 $20 \cdot \log_{10}(\text{abs}(x1));$



Procesiranje signala
 (10404, 78.171)



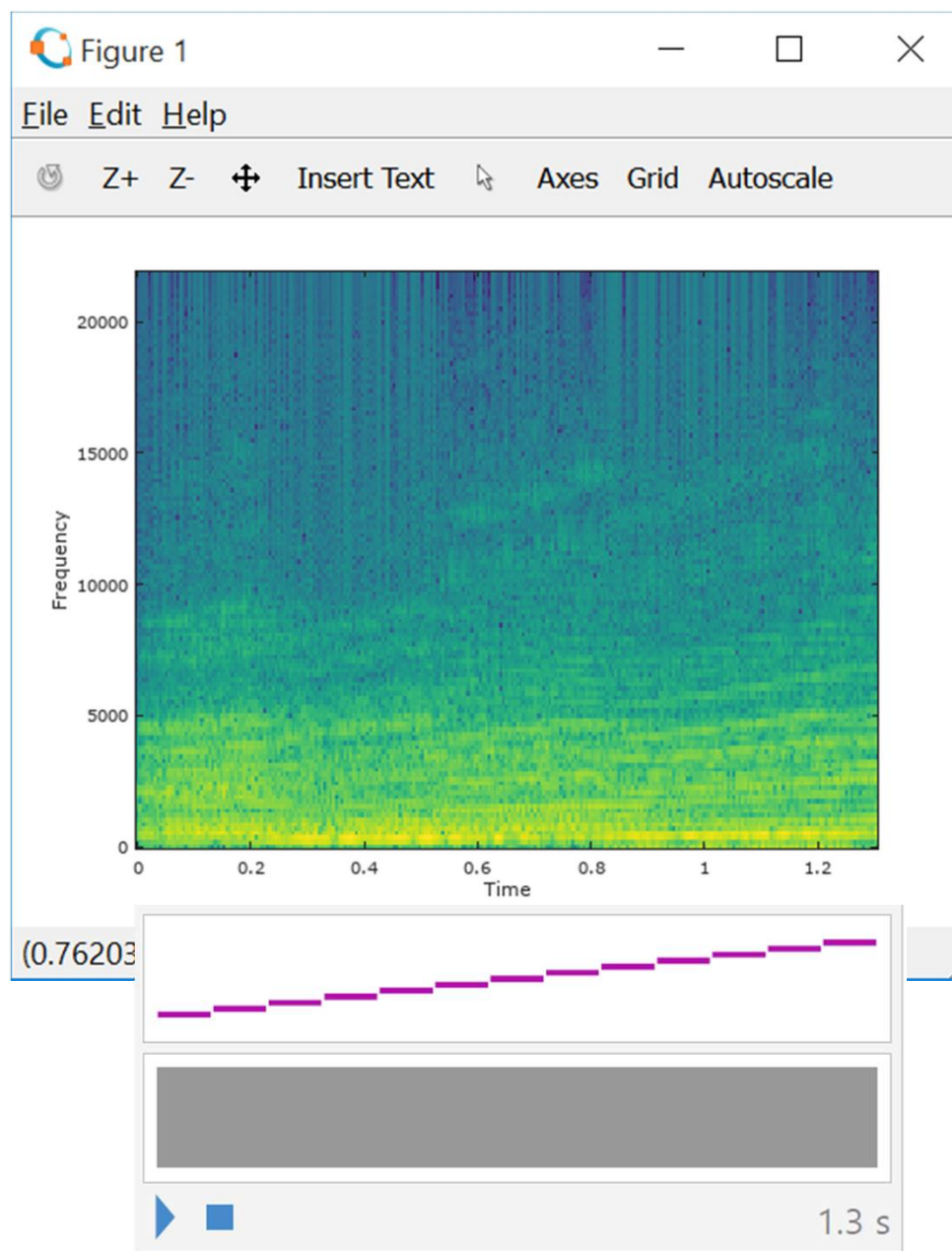
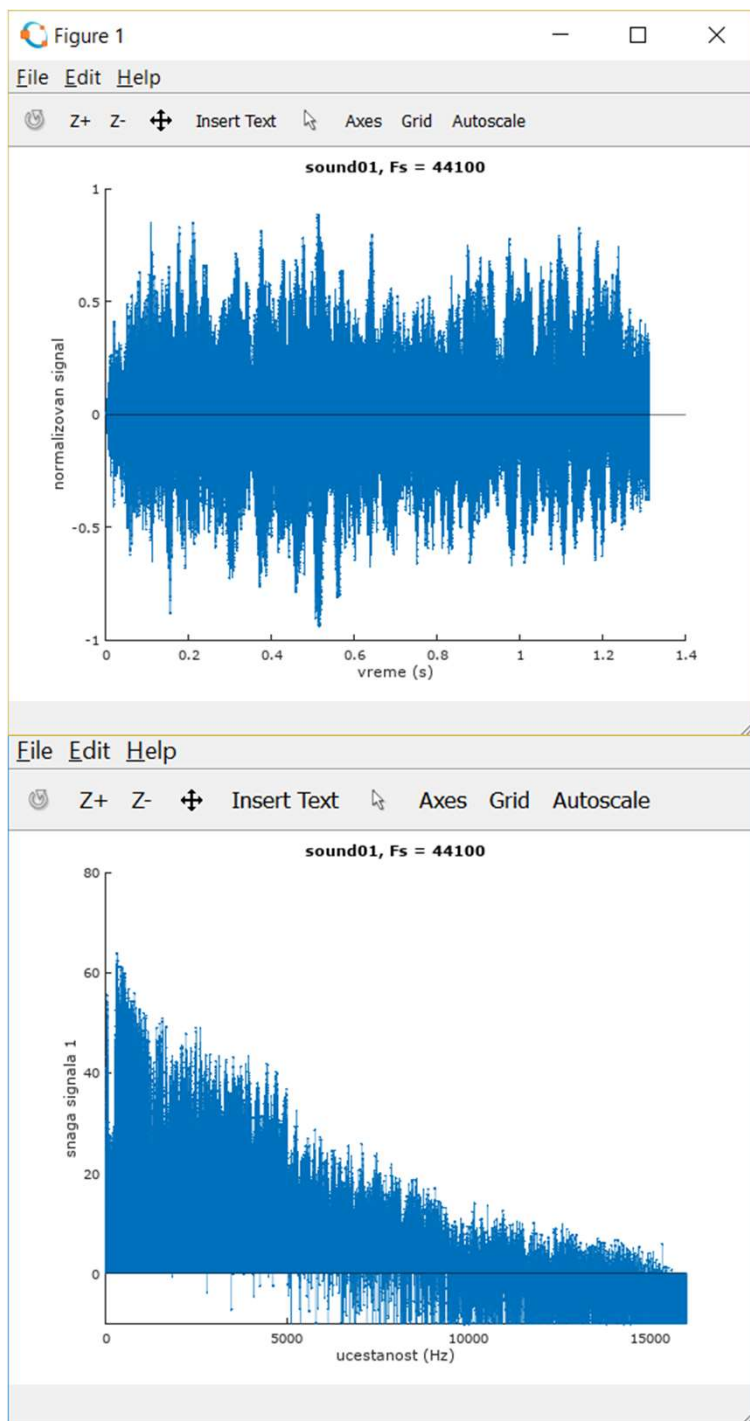
fft
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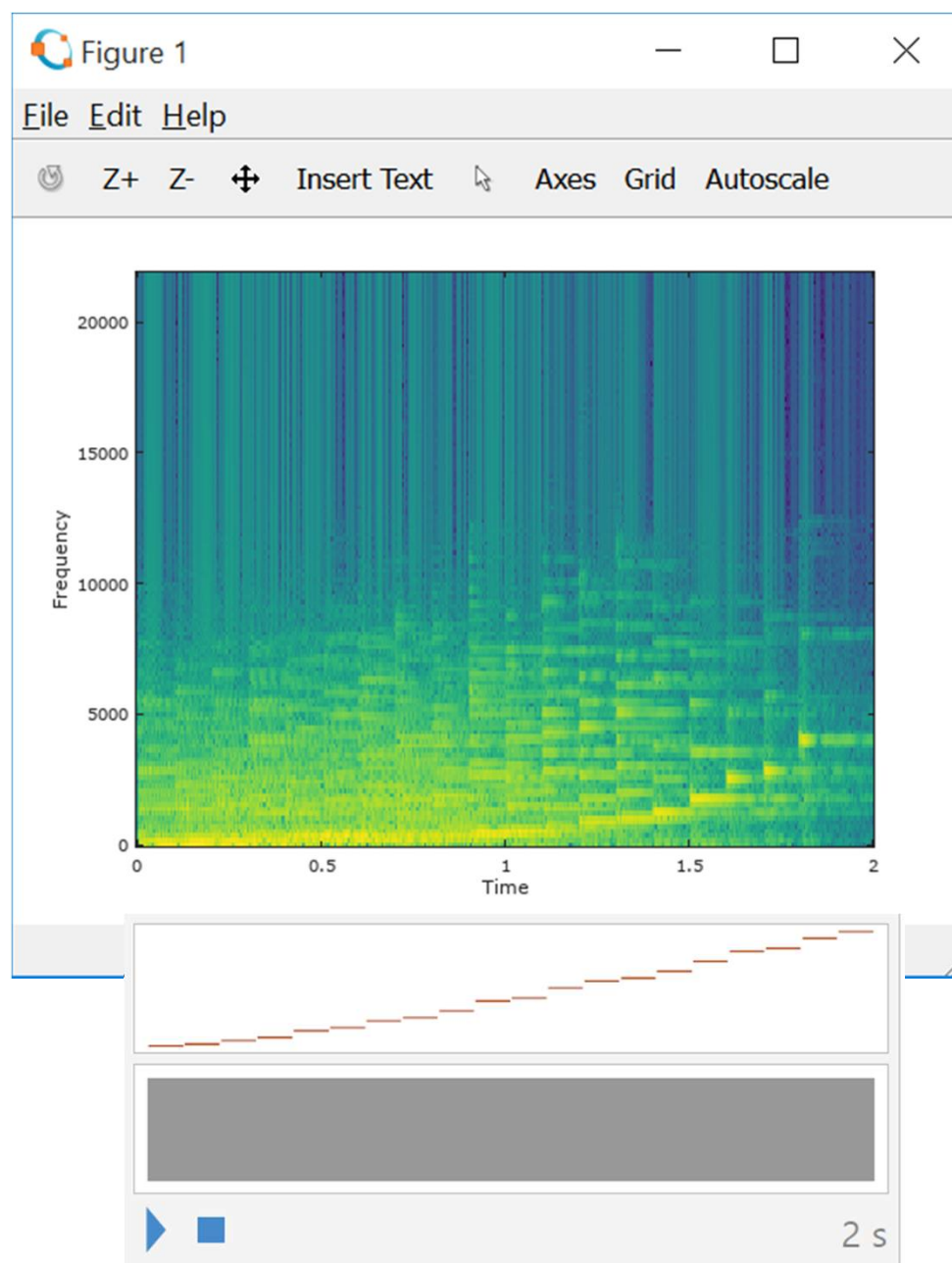
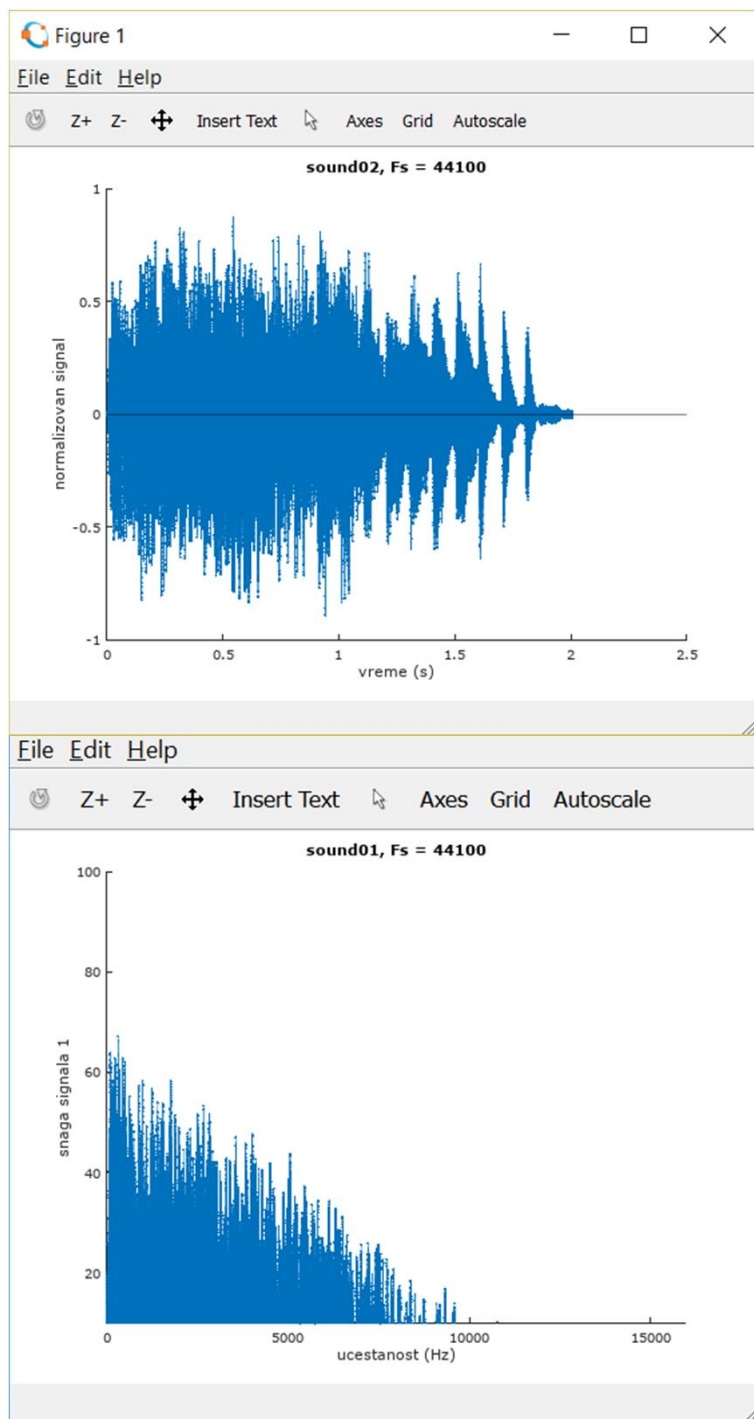
Procesiranje signala

pkg load signal

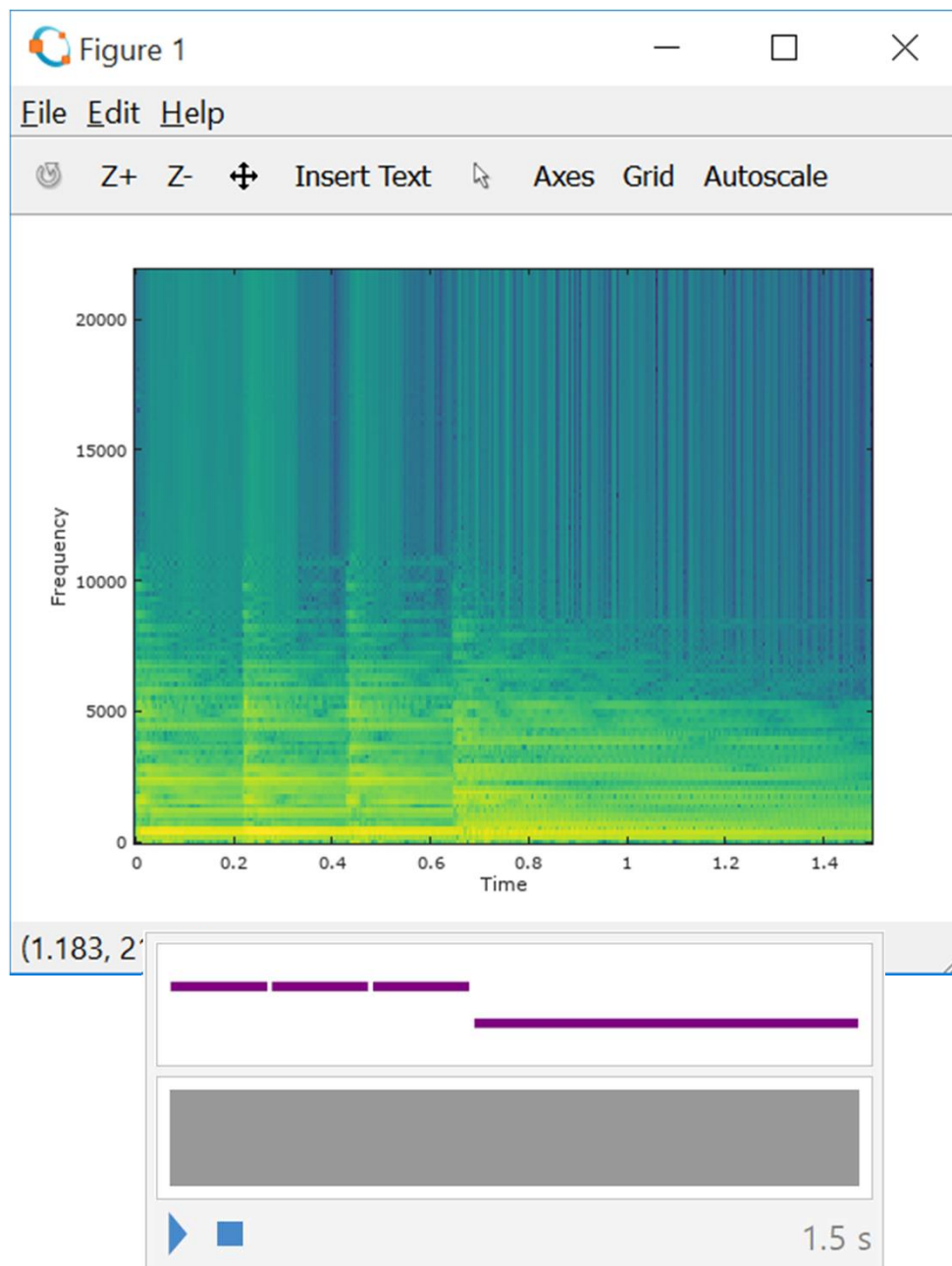
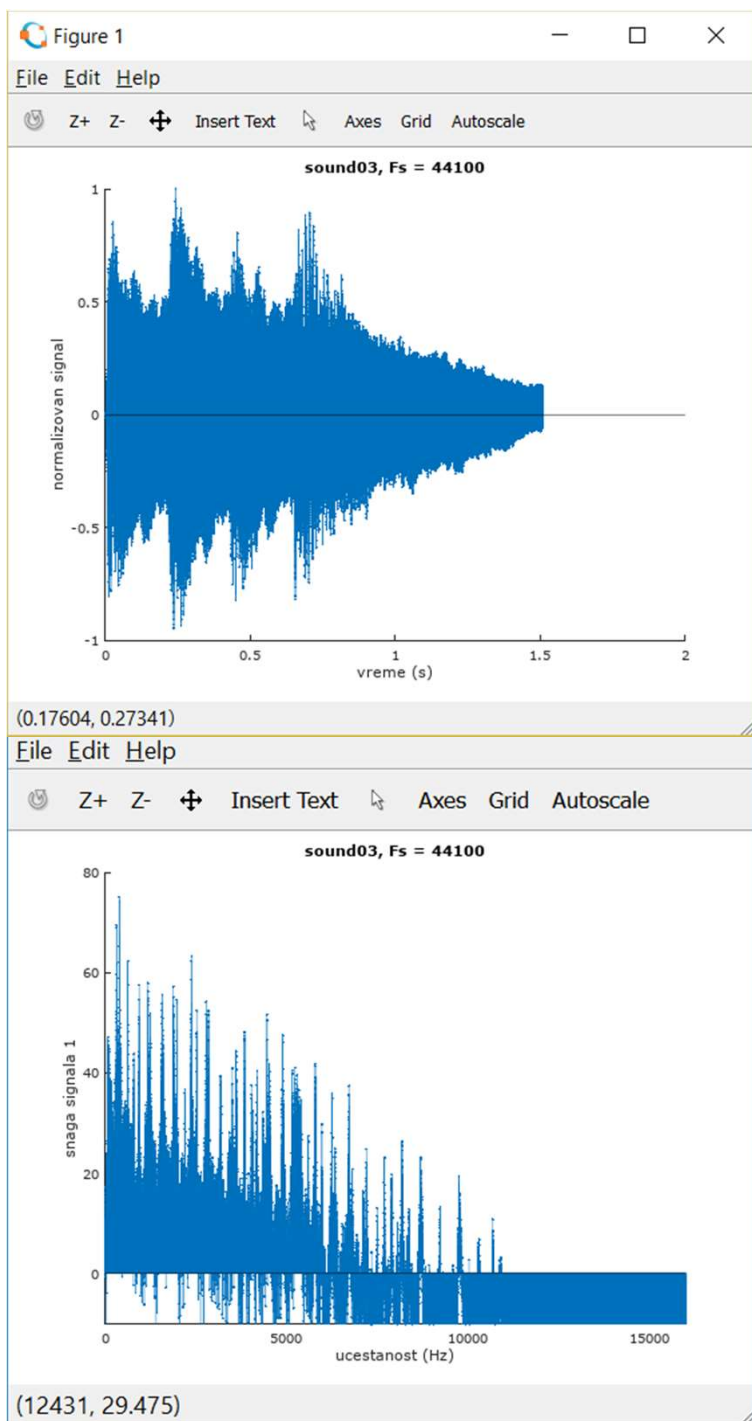
```
clear all, close all, clc
[y,fs]=audioread ("wav/sound01.wav");
y1=y(:,1);
y2=y(:,2);
nfft = 256;
overlap = 32;
specgram(y1,nfft,fs,hamming(nfft),overlap)
```

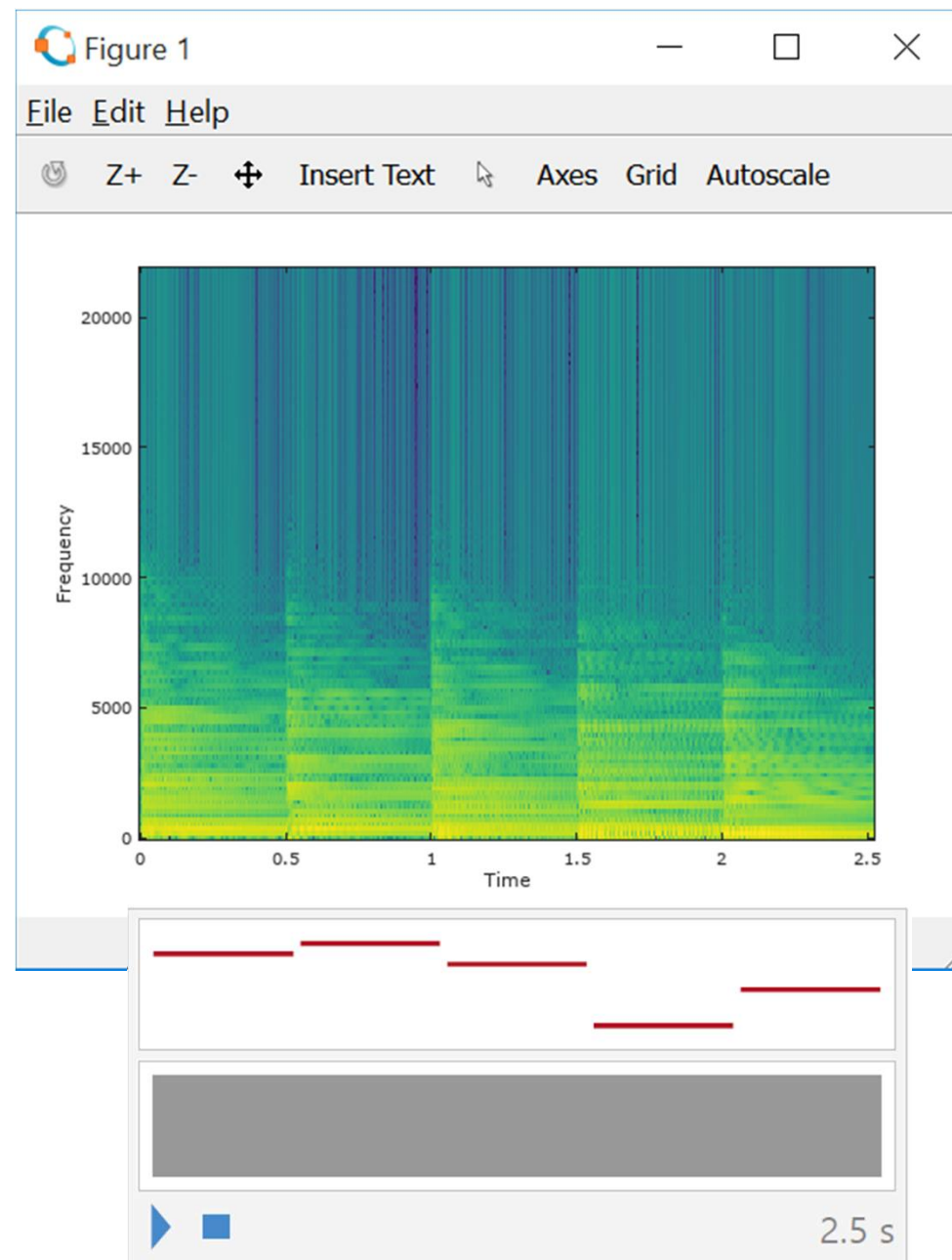
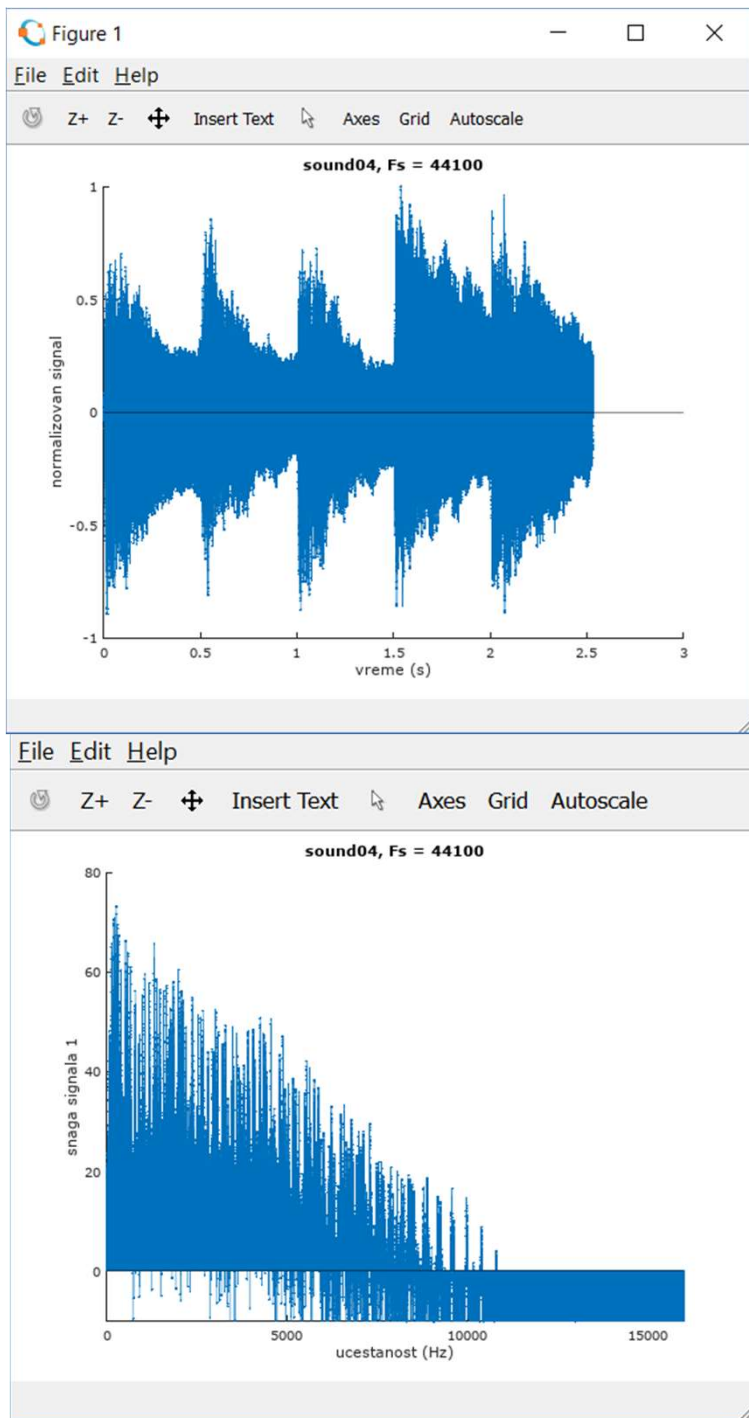
Procesiranje signala



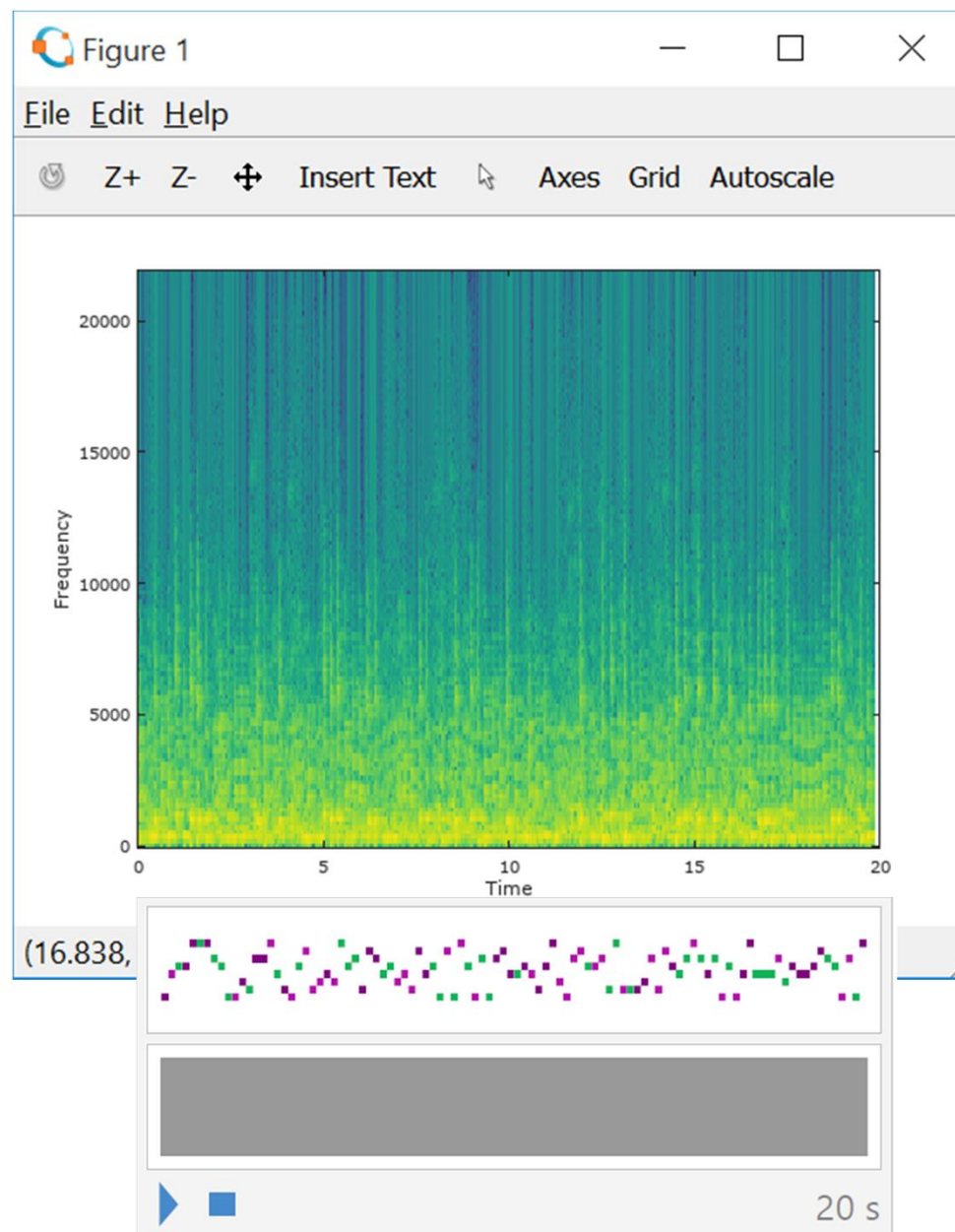
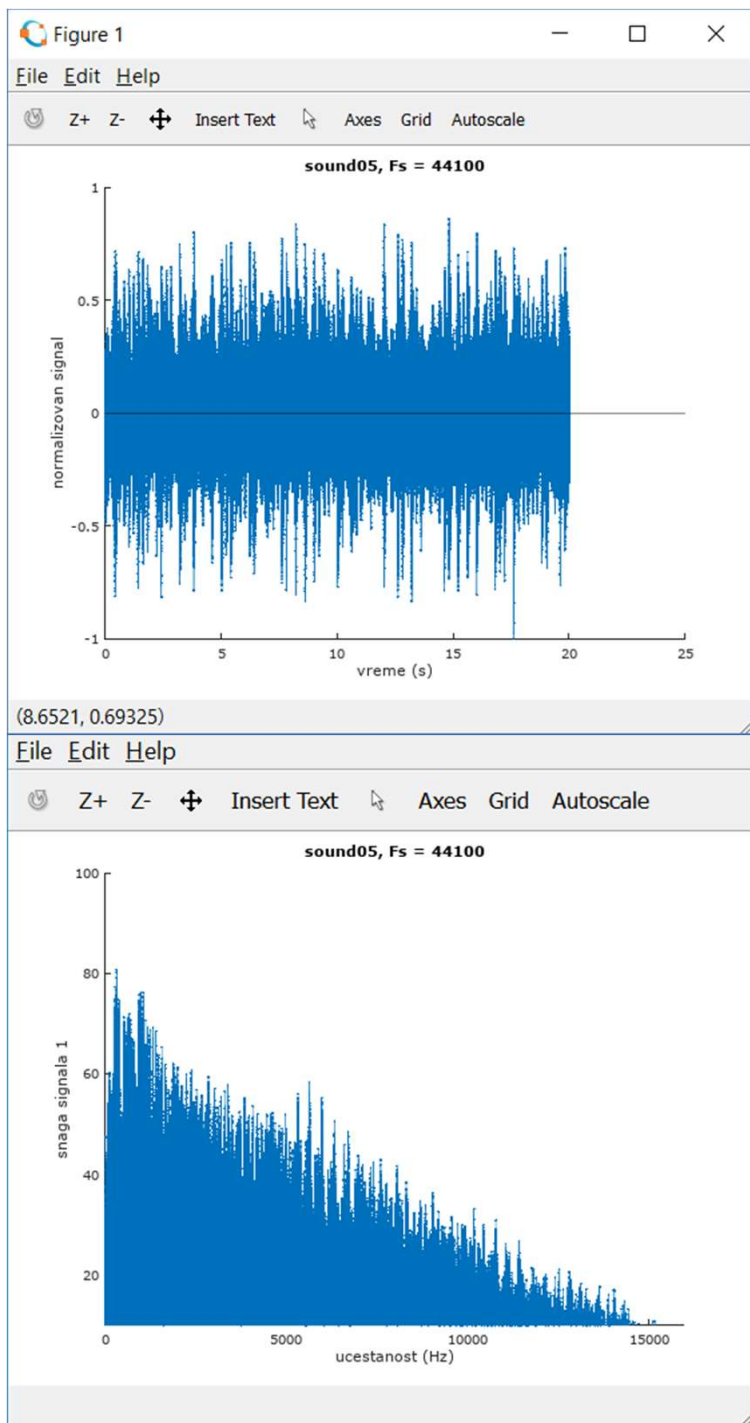
Procesiranje signala



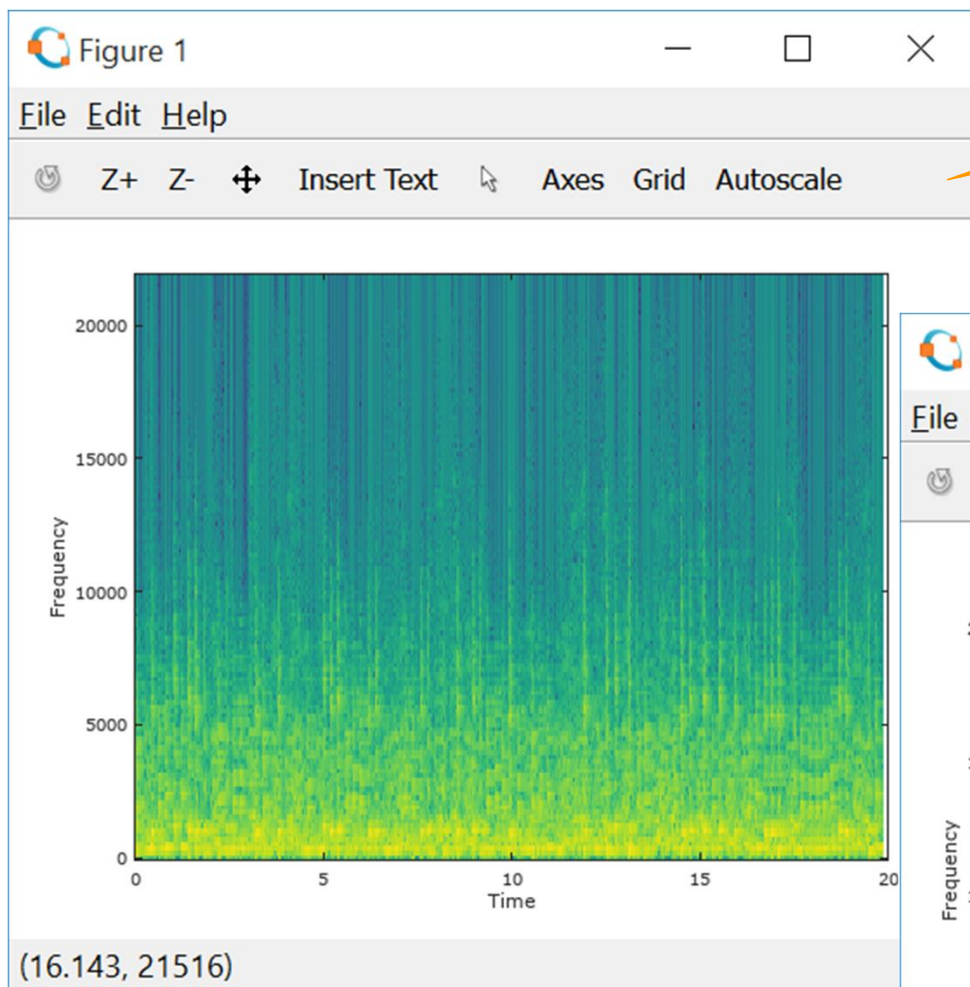
Procesiranje signala



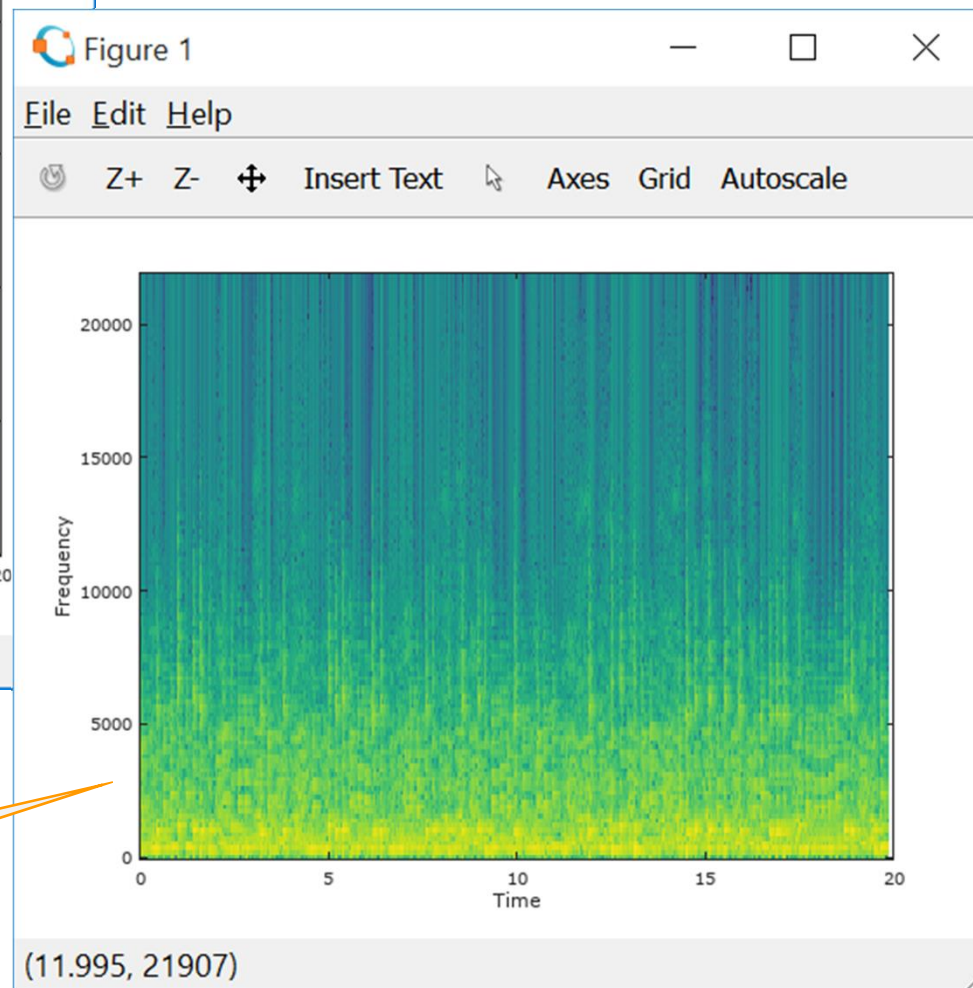
Procesiranje signala



Procesiranje signala



ovlap = 0;



ovlap = 128;

Procesiranje signala

30

Diskretna Furijeova transformacija

Sekvenca u vremenskom domenu $x[n]$ dužine N preslikava se u sekvenču $X[k]$, iste dužine gde se članovi računaju po formuli

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} kn}, \quad 0 \leq k \leq N-1$$

Matlab `fft` realizuje DFT

`X=fft(x)`

- Iz zadate sekvence u vremenskom domenu $x[n]$ dobija se nova sekvenca u $X[k]$

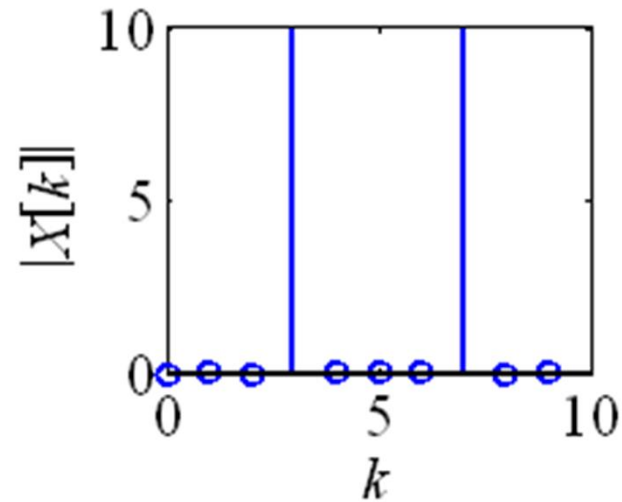
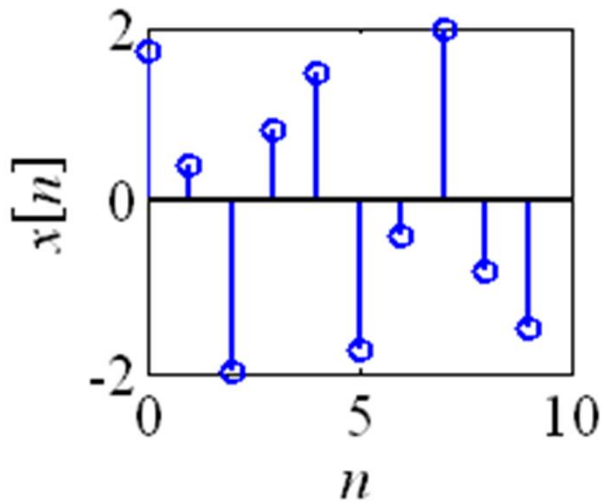
```
x = [1 1/2 1/4 1/8 1/16 1/32 1/64 1/128]
X = fft(x)
```

```
x =
1.0000 0.5000 0.2500 0.1250 0.0625 0.0313 0.0156 0.0078
```

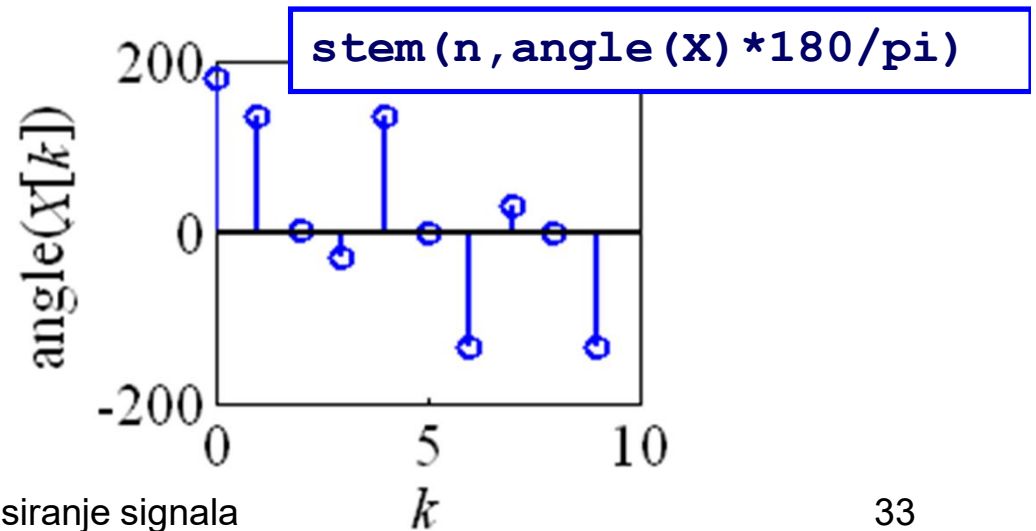
```
X =
Columns 1 through 4
1.9922 1.1861-0.6487i 0.7969-0.3984i 0.6889-0.1799i
Columns 5 through 8
0.6641 0.6889+0.1799i 0.7969+0.3984i 1.1861+0.6487i
```

*Kompleksni
brojevi*

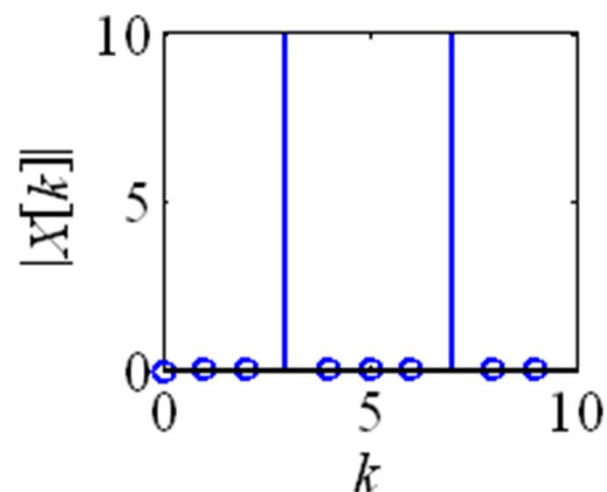
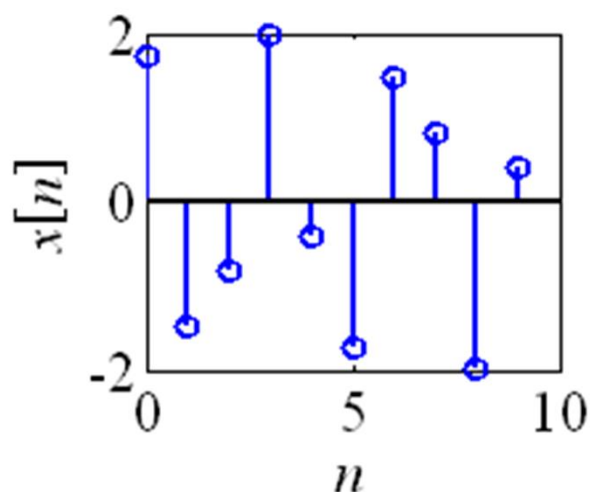
FFT



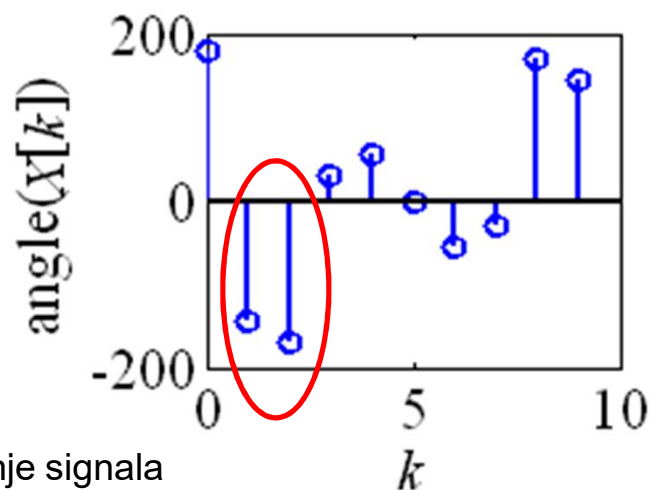
```
N = 10;
n = 0:N-1;
k = 3;
f = k/N;
fi = -pi/6;
x = 2*cos(2*pi*f*n+fi);
stem(n,x)
X = fft(x)
stem(n,abs(X))
```



FFT - promena faze

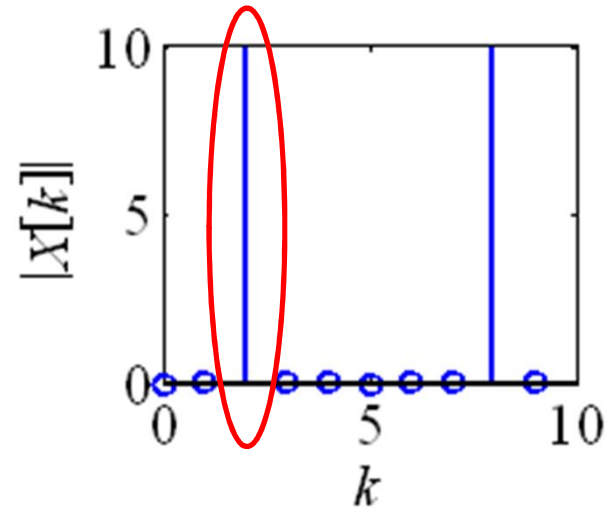
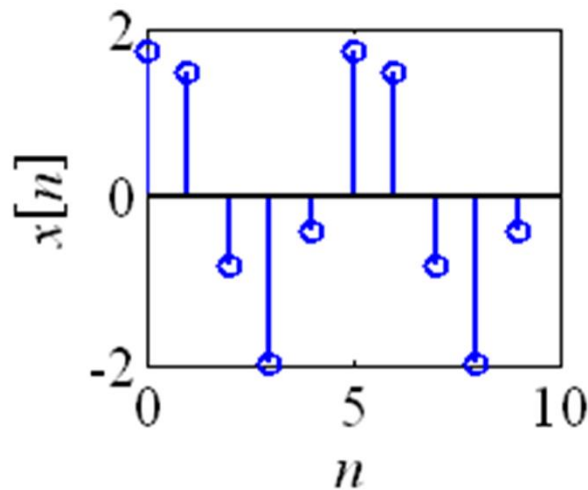


```
N = 10;  
n = 0:N-1;  
k = 3;  
f = k/N;  
fi = pi/6;  
x = 2*cos(2*pi*f*n+fi);  
stem(n,x)  
X = fft(x)  
stem(n,abs(X))
```

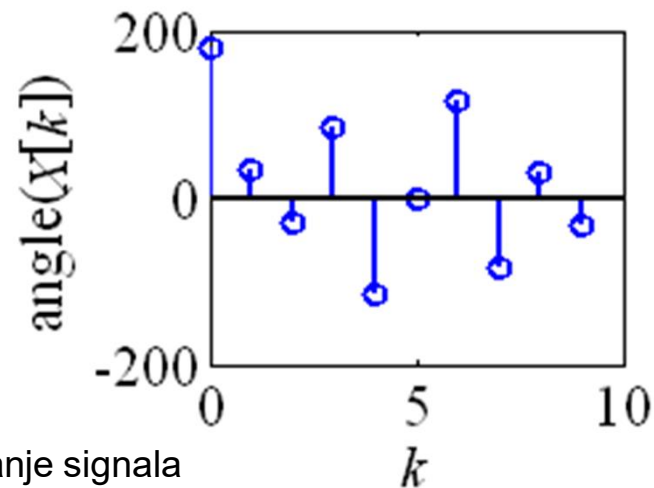


Procesiranje signala

FFT – promena k

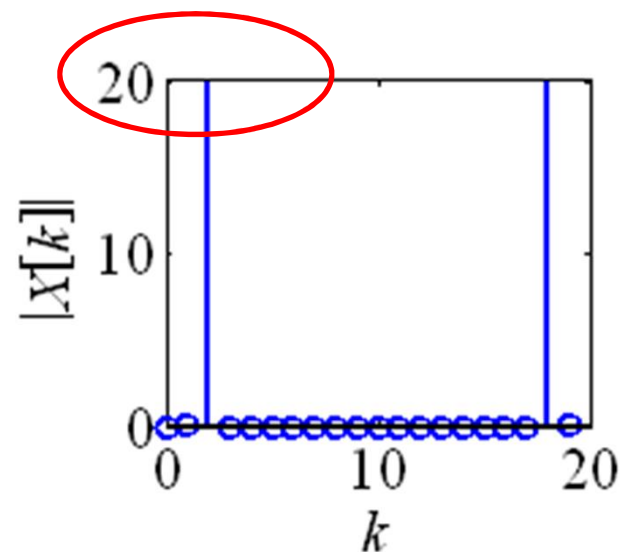
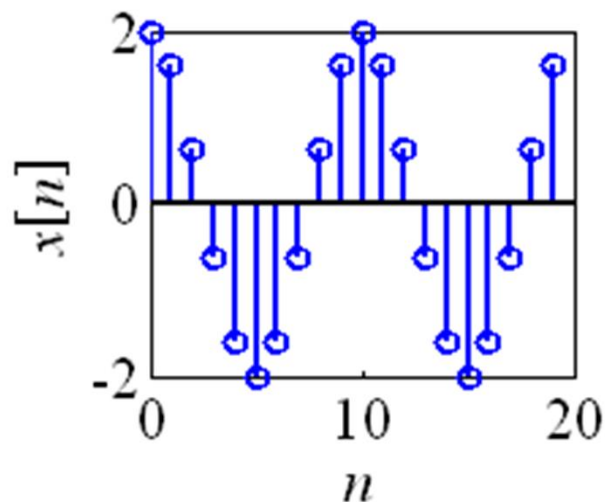


```
N = 10;  
n = 0:N-1;  
k = 2;  
f = k/N;  
fi = -pi/6;  
x = 2*cos(2*pi*f*n+fi);  
stem(n,x)  
X = fft(x)  
stem(n,abs(X))
```

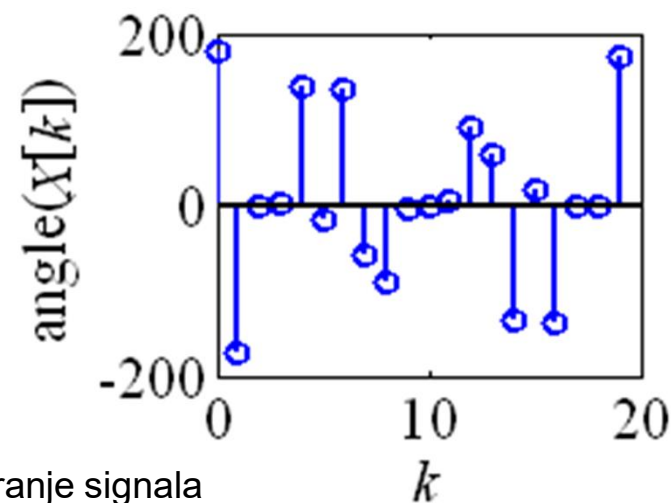


Procesiranje signala

FFT – promena N

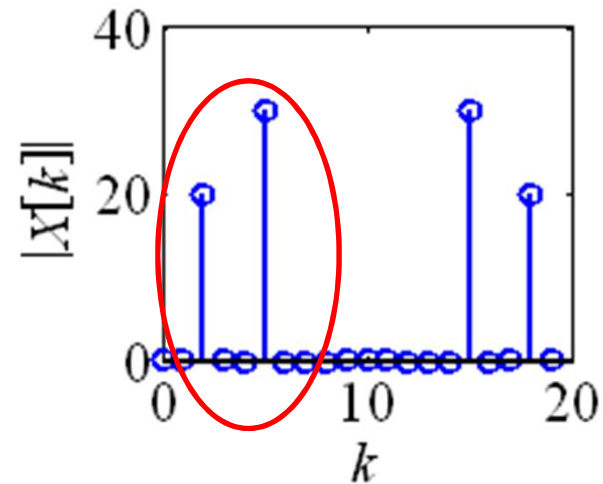
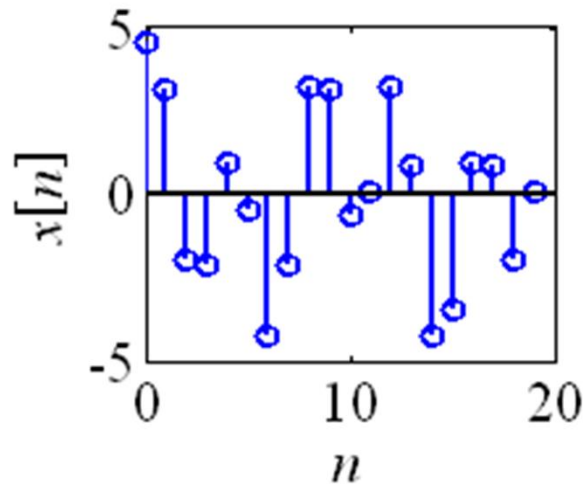


```
N = 20;  
n = 0:N-1;  
k = 3;  
f = k/N;  
fi = -pi/6;  
x = 2*cos(2*pi*f*n+fi);  
stem(n,x)  
X = fft(x)  
stem(n,abs(X))
```

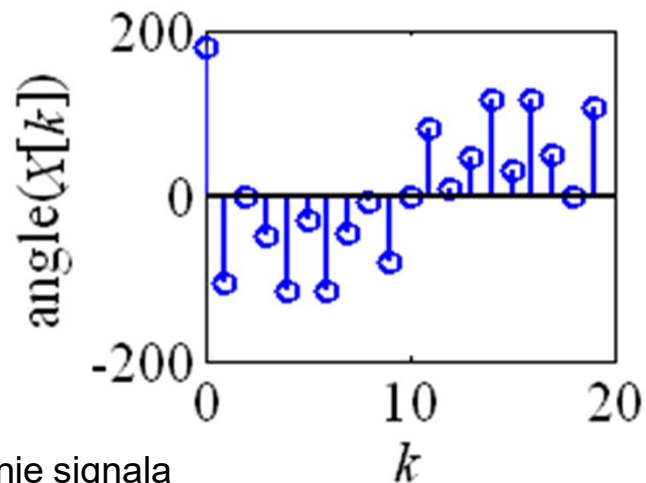


Procesiranje signala

FFT – dve sinusoide



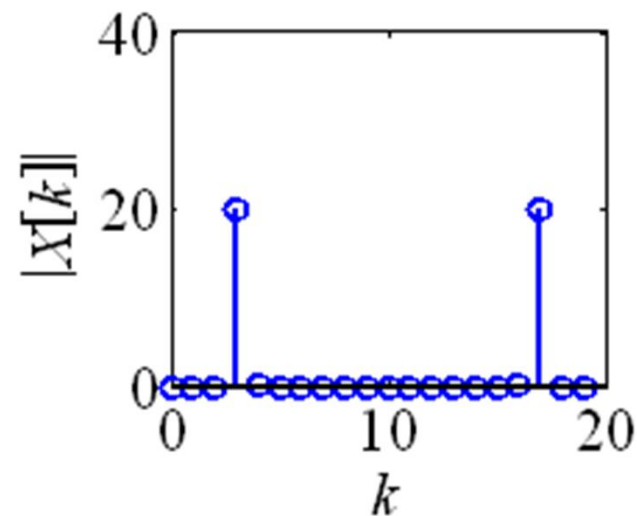
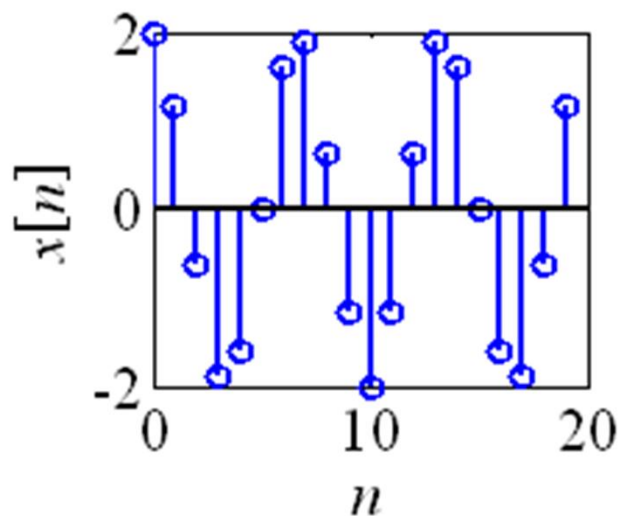
```
N=20;  
n=0:N-1;  
k1=2;    k2=5;  
f1=k1/N; f2=k2/N;  
fi2=-pi/6;  
x = 2*cos(2*pi*f1*n+0)+...  
    3*cos(2*pi*f2*n+fi2);  
X = fft(x)
```



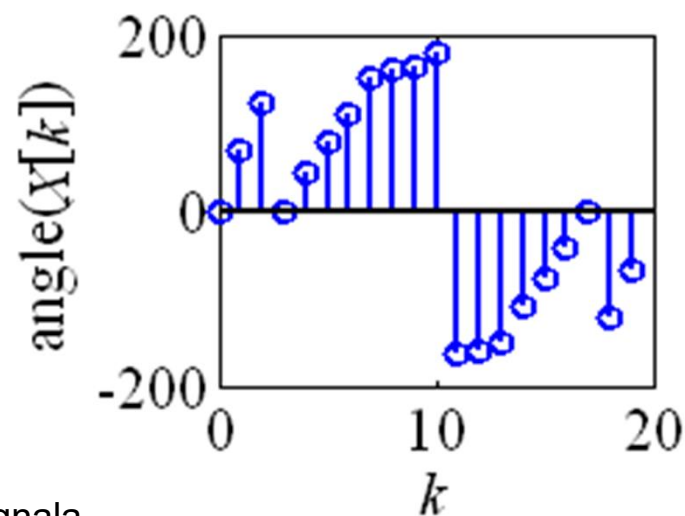
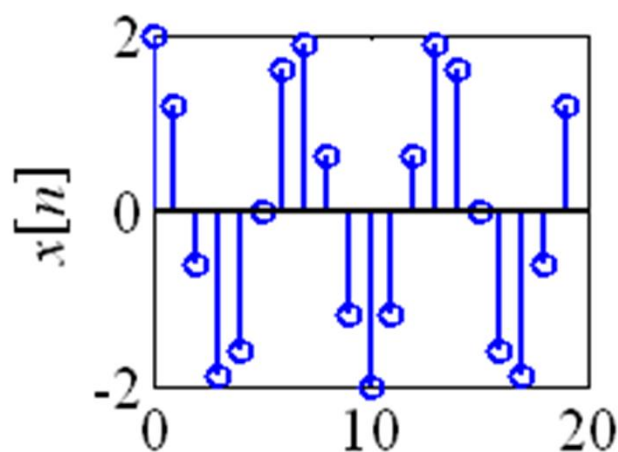
Procesiranje signala

FFT - aliasing

$k=3;$
 $f=k/N;$



$k=N+3;$
 $f=k/N;$

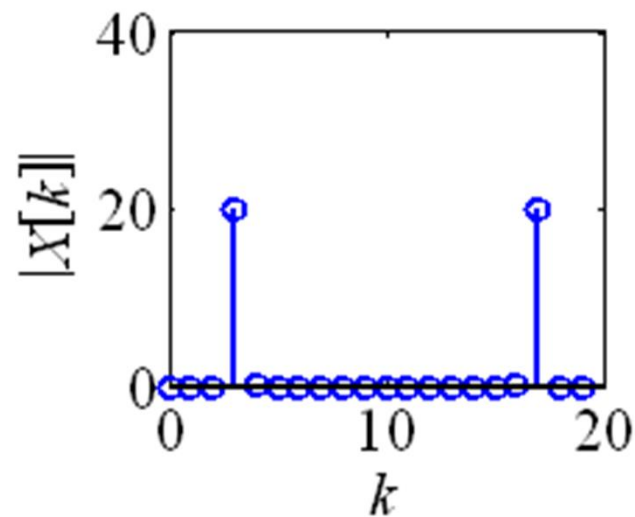
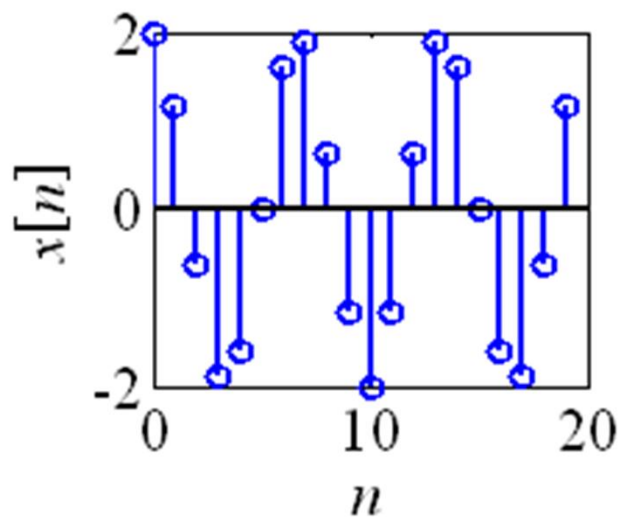


$x = 2 \cdot \cos(2 \cdot \pi \cdot f \cdot n);$

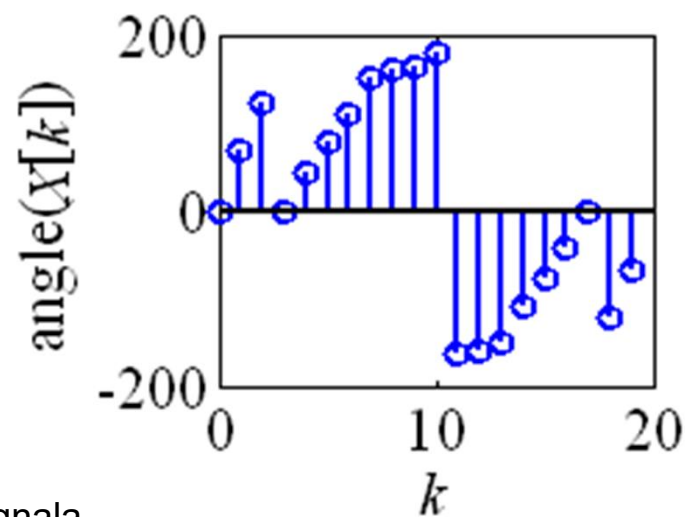
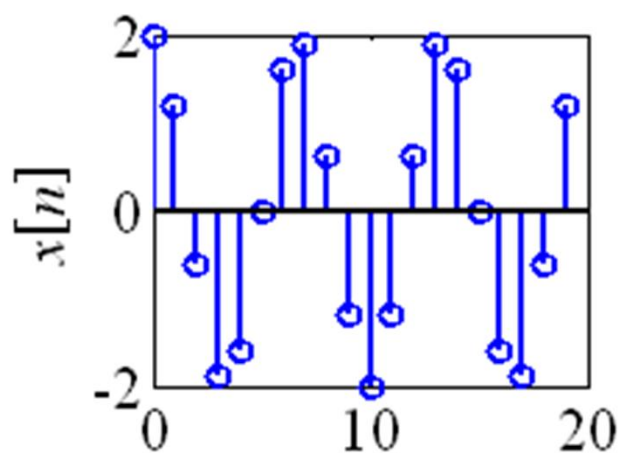
n Procesiranje signala

FFT - folding

$k=3;$
 $f=k/N;$



$k=N-3;$
 $f=k/N;$



$x = 2 \cdot \cos(2 \cdot \pi \cdot f \cdot n);$

Procesiranje signala

Jednoznačnost FFT

```
k=3;  
f=k/N;
```

$$k < N/2$$

```
x = 2*cos(2*pi*f*n) ;
```

$$f < 1/2$$

Jednoznačnost FFT

```
x = 2*cos(2*pi*f*n);
```

$$f < 1/2$$

$$f = 1-f$$

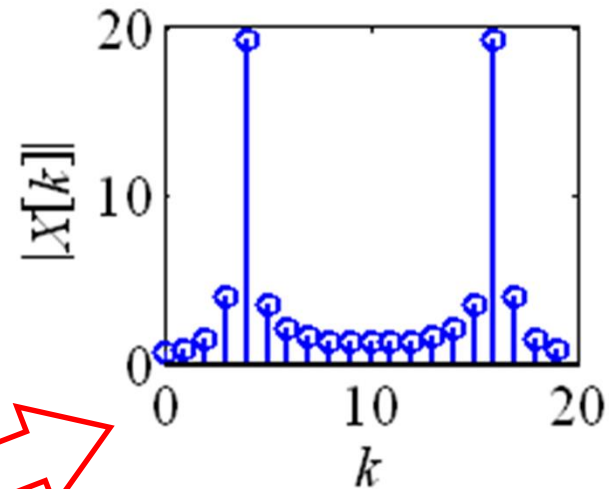
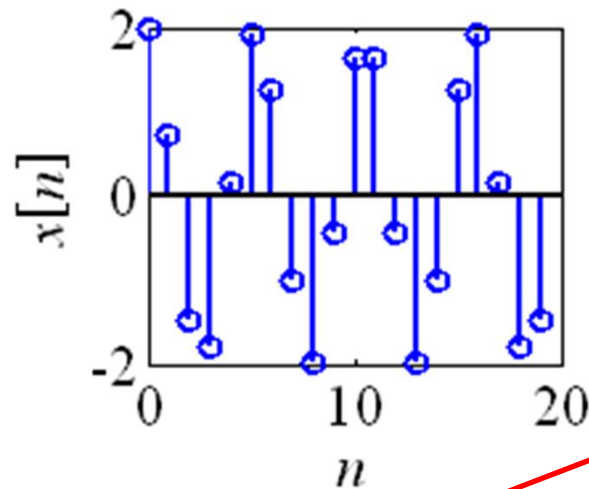
$$f = 1+f$$

$$f = 2-f$$

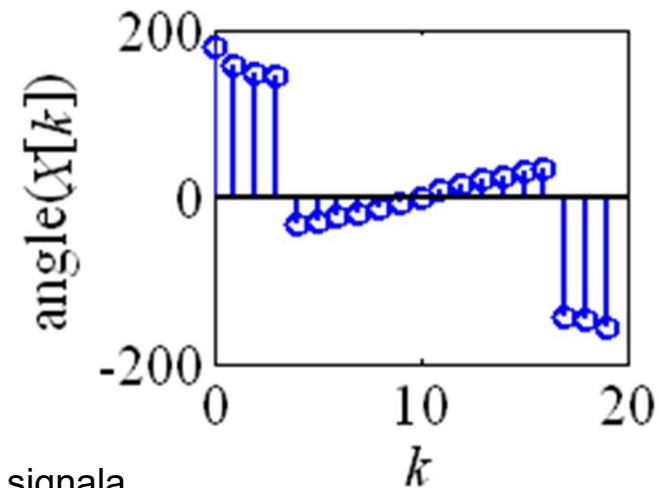
$$f = 2+f$$

FFT – curenje spektra

frequency leakage



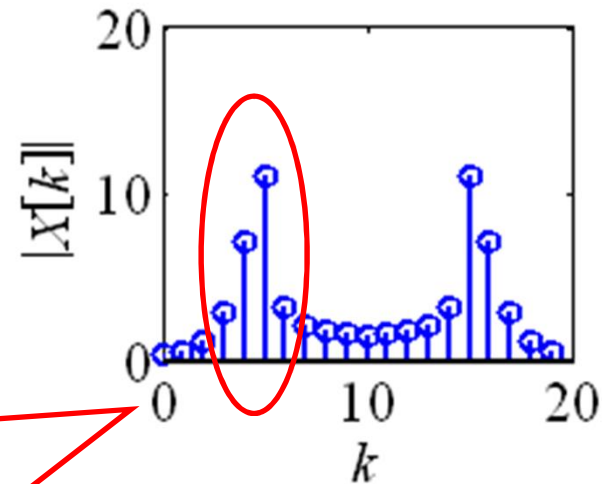
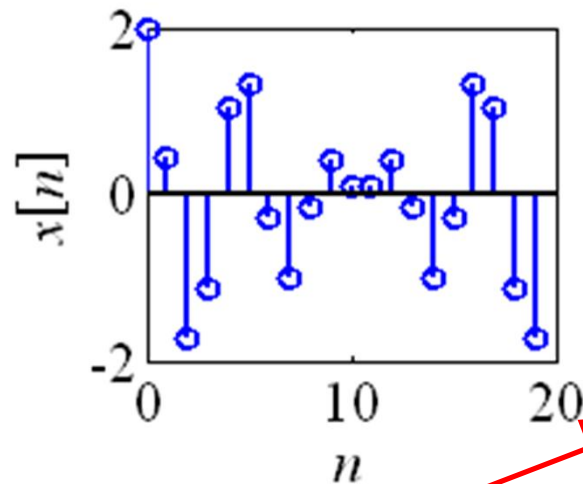
```
N = 20;  
M = 21;  
n = 0:N-1;  
k = 4;  
f = k/M;  
x = 2*cos(2*pi*f*n);  
X = fft(x)
```



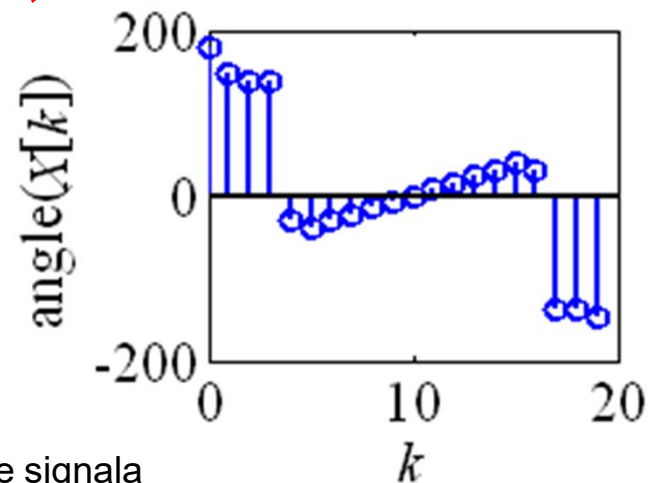
Procesiranje signala

FFT – frekvencijska rezolucija

*frequency
resolution*



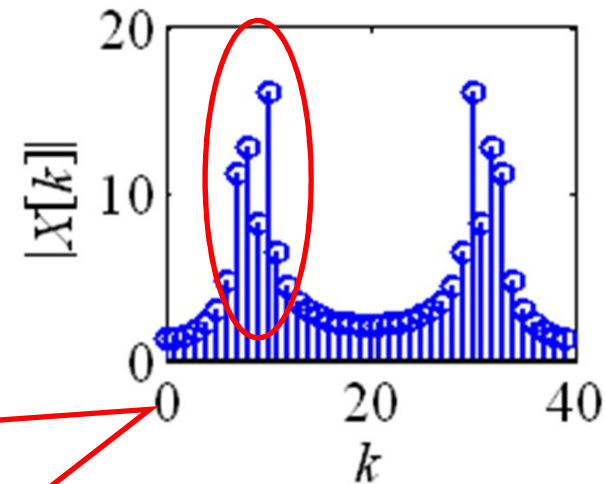
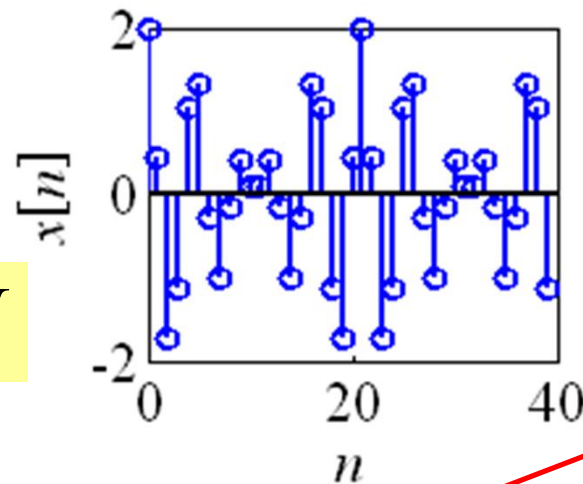
```
N = 20;  
M = 21;  
n = 0:N-1;  
k1=4; f1=k1/M;  
k2=5; f2=k2/M;  
x=cos(2*pi*f1*n)+...  
    cos(2*pi*f2*n);  
X = fft(x)
```



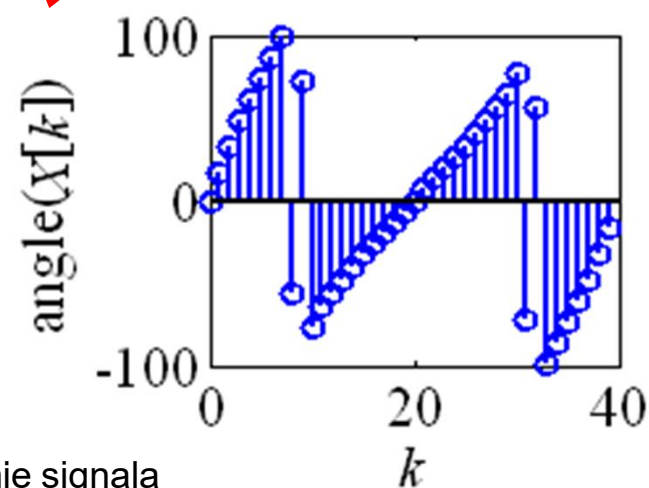
Procesiranje signala

FFT – frekvencijska rezolucija

$$\Delta f = 1/N$$

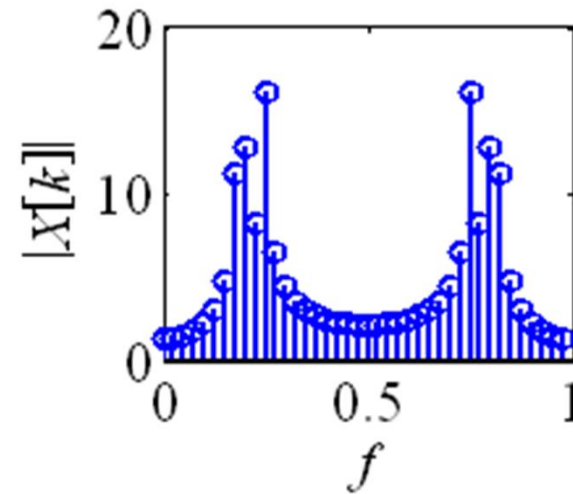
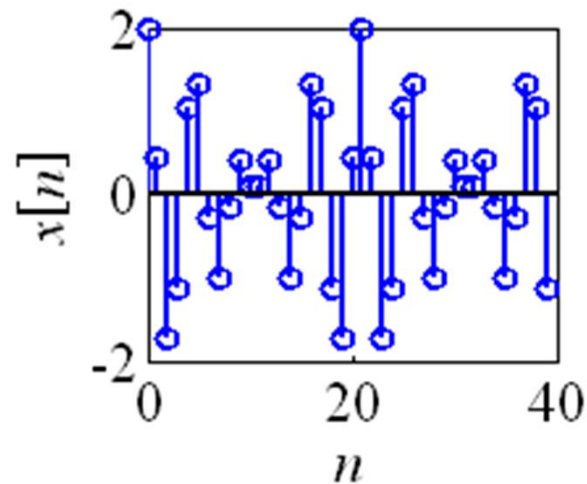


```
N = 40;  
M = 21;  
n = 0:N-1;  
k1=4; f1=k1/M;  
k2=5; f2=k2/M;  
x=cos(2*pi*f1*n)+...  
    cos(2*pi*f2*n);  
X = fft(x)
```

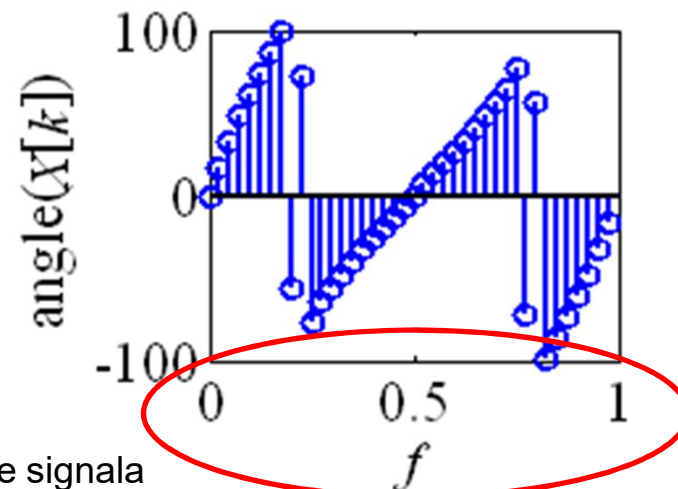


Procesiranje signala

FFT – digitalna frekvencija

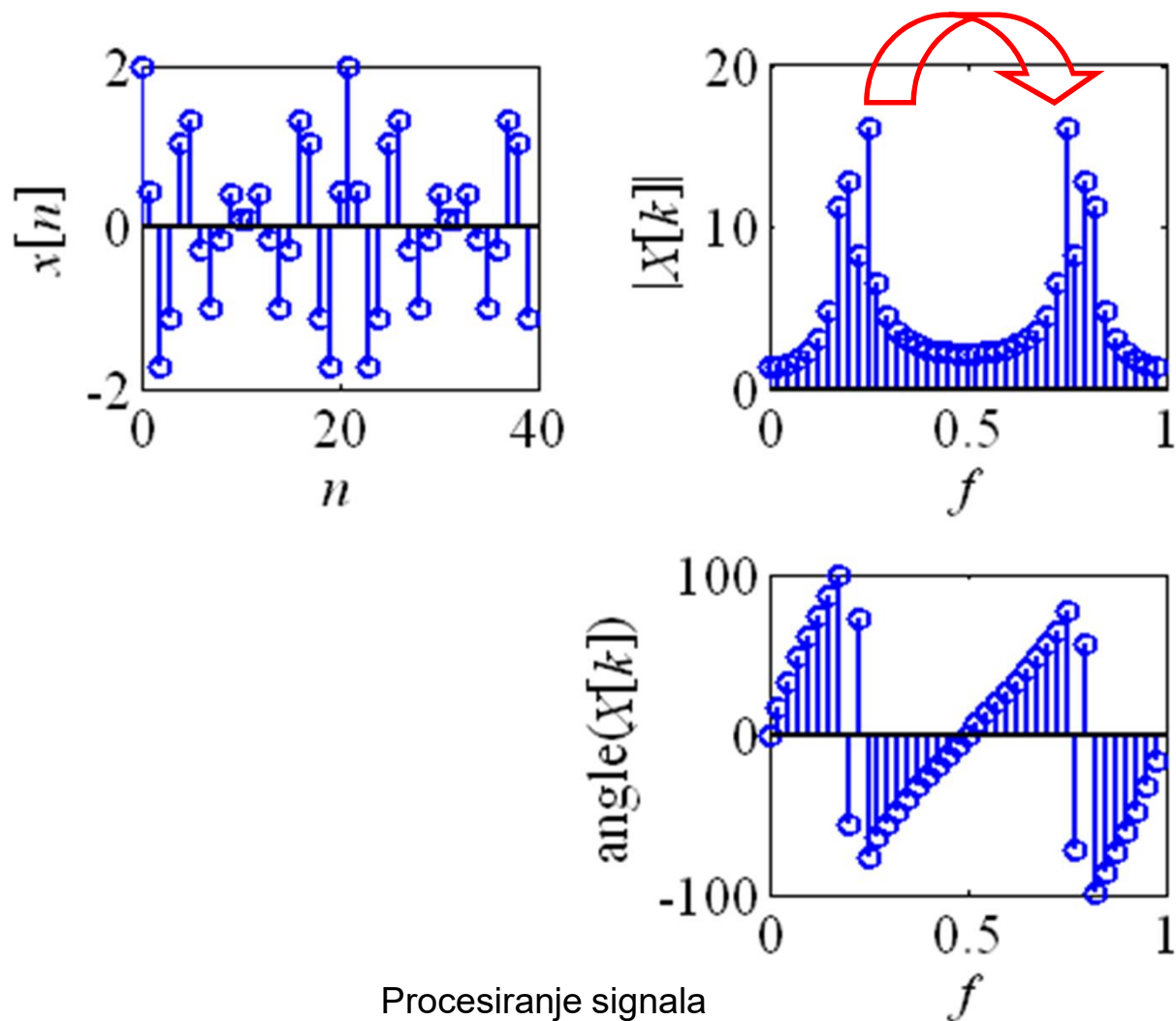


```
stem(n/N,abs(y))  
xlabel('f')
```

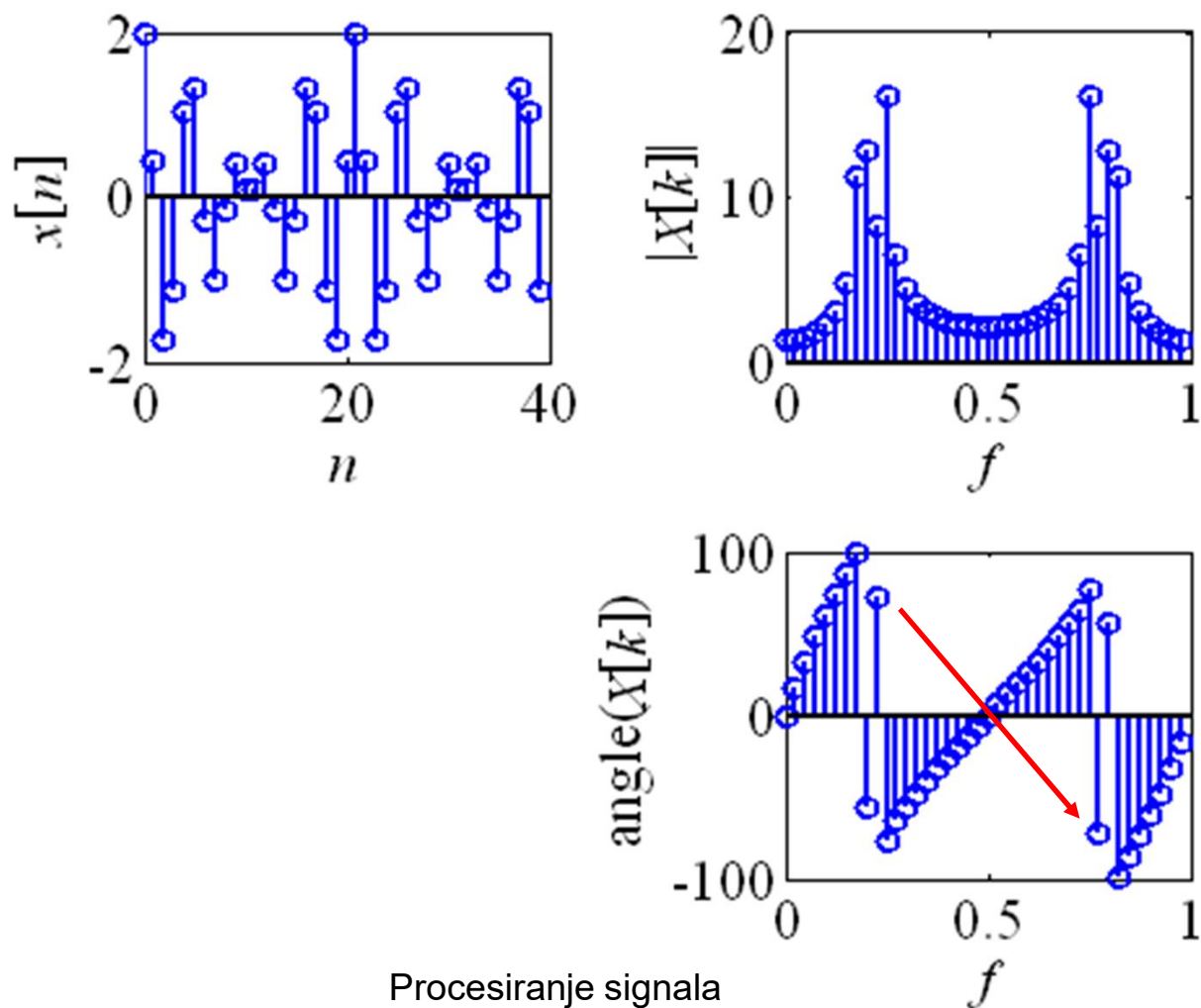


Procesiranje signala

FFT – simetrija

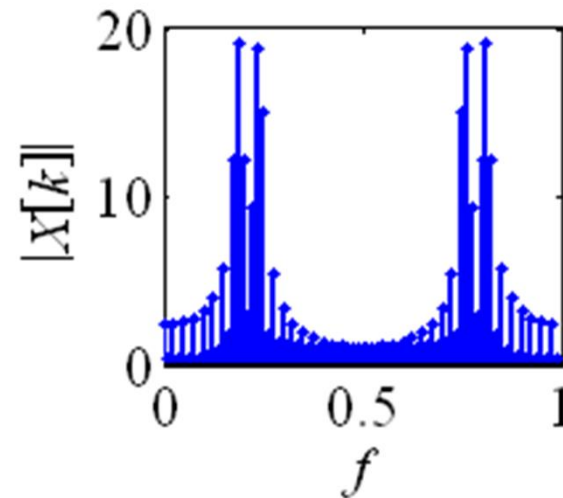
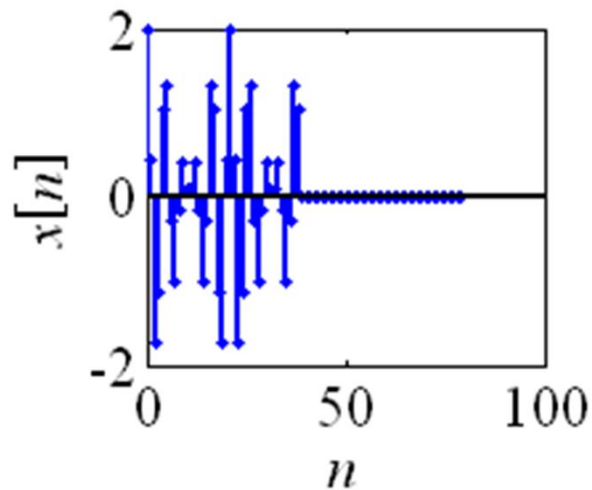


FFT – antisimetrija

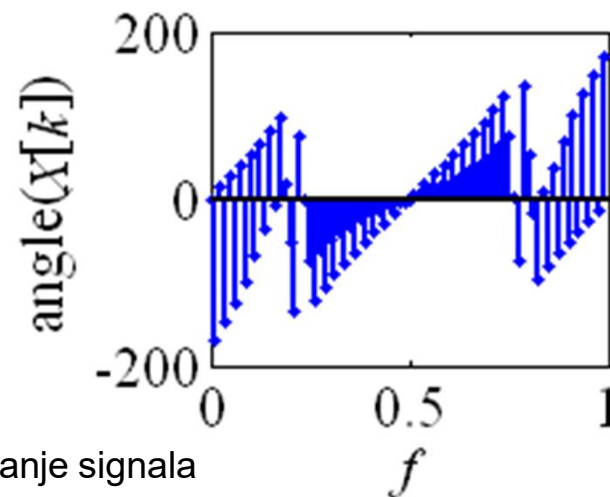


Procesiranje signala

FFT – dopuna nulama

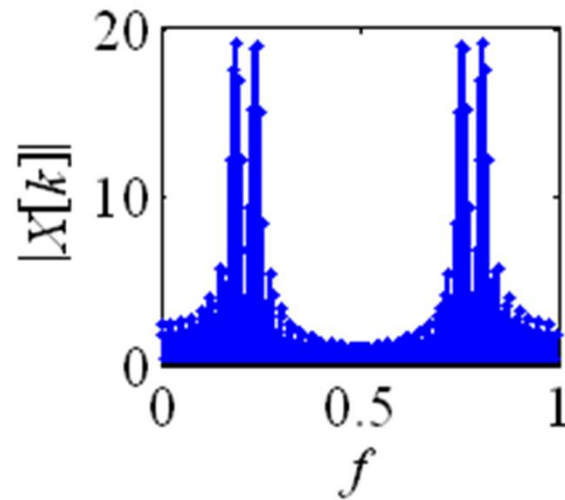
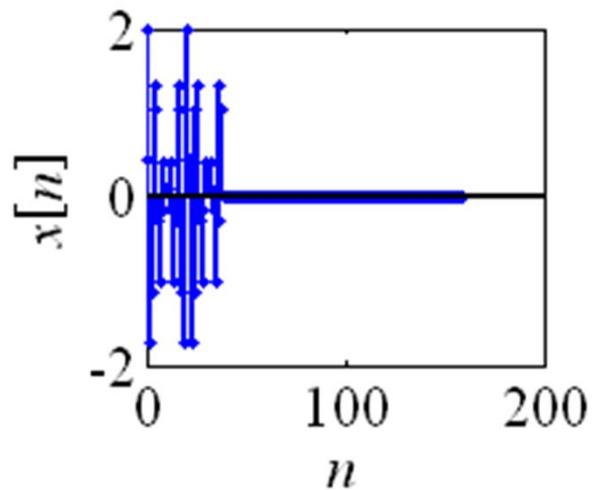


```
N = 80;  
.  
.  
.  
x = x.* (n < (N/2-1)) ;
```

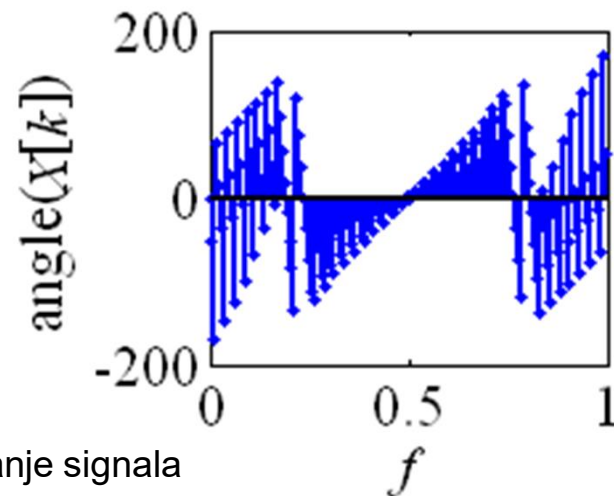


Procesiranje signala

FFT – dopuna nulama (2)

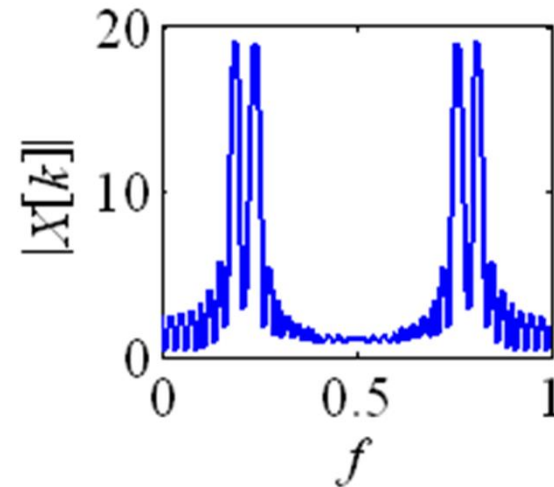
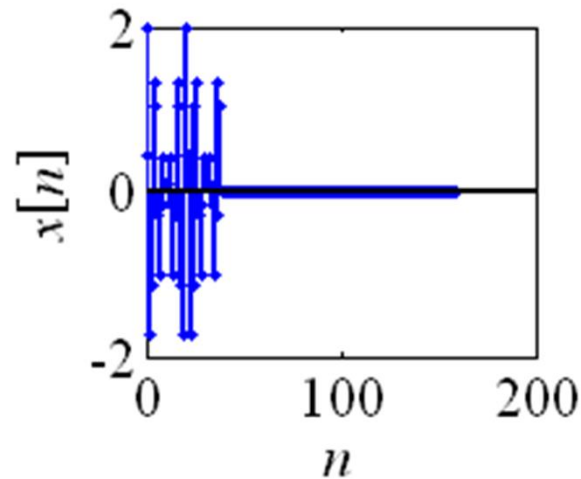


```
N = 160;  
.  
.  
.  
x = x.* (n<39) ;
```

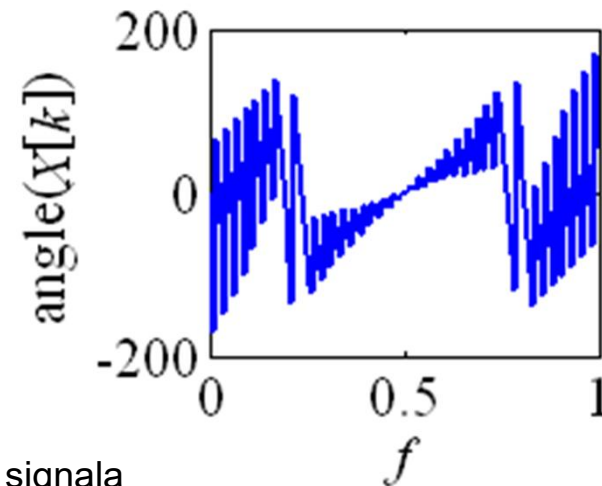


Procesiranje signala

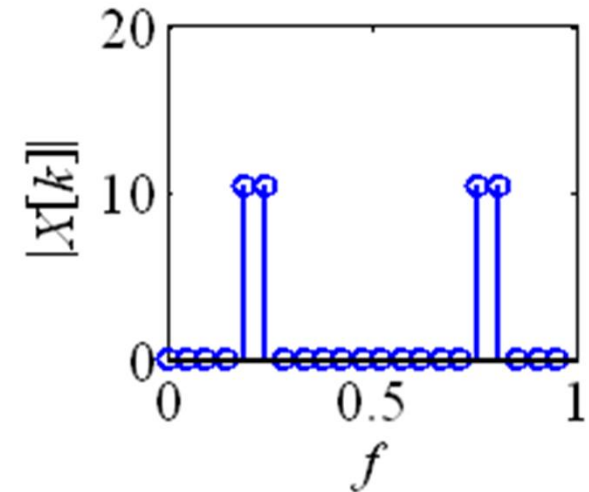
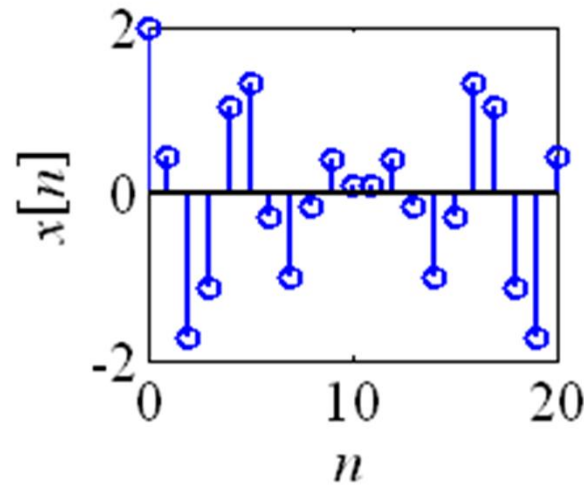
FFT – dopuna nulama (2)



```
plot(n/N,abs(y),'-')  
.  
.  
.  
plot(n/N,angle(y)*180/pi,'-')
```

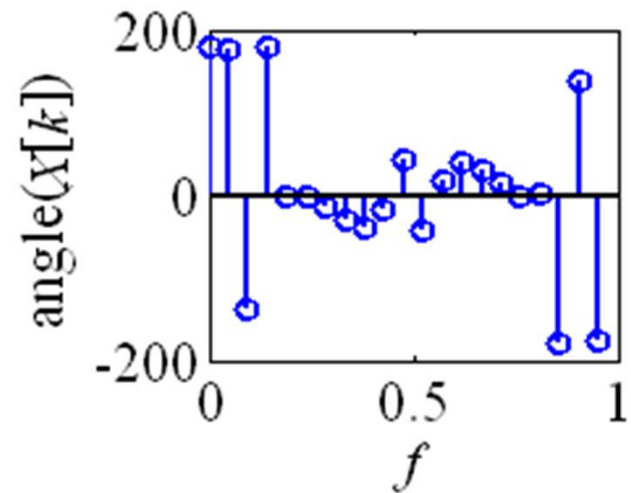


FFT – perfektna dopuna



```
N = 21;  
.  
.  
.  
k1 = 4;  
f1 = k1/M;  
k2 = 5;  
f2 = k2/M;  
x = cos(2*pi*f1*n) + cos(2*pi*f2*n);  
x = x.*(n<21);
```

Procesiranje signala



Parseval-ova teorema

Pojam snage

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

Profesor dr Miroslav Lutovac

mlutovac@viser.edu.rs

Ova prezentacija je nekomercijalna.

Slajdovi mogu da sadrže materijale preuzete sa Interneta, stručne i naučne građe, koji su zaštićeni Zakonom o autorskim i srodnim pravima.

Ova prezentacija se može koristiti samo privremeno tokom usmenog izlaganja nastavnika u cilju informisanja i upućivanja studenata na dalji stručni, istraživački i naučni rad i u druge svrhe se ne sme koristiti –

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(1) javno izvođenje ili predstavljanje objavljenih dela u obliku neposrednog poučavanja na nastavi;

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("Sl. glasnik RS", br. 104/2009 i 99/2011)