



Procesiranje signala

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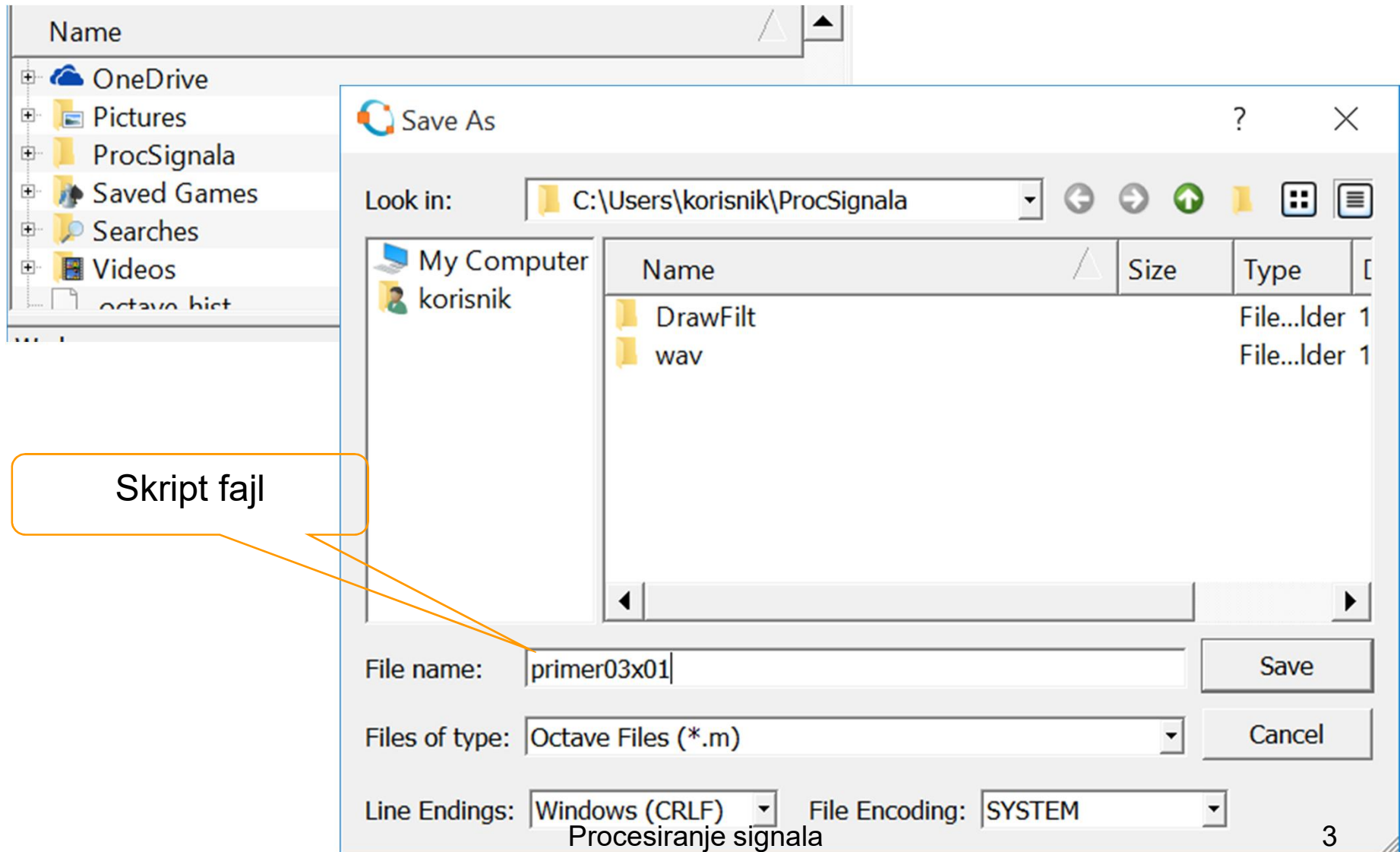
"This project has been funded with support from the European Commission. This publication [communication] reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein"

Sadržaj predmeta, Teorijska nastava

1. Uvodno predavanje. Upoznavanje sa planom i programom, ciljevima, ishodom i metodama.
2. Šta je procesiranje signala, istorijski pregled obrade signala, primeri primene.
3. Vizuelizacija signala (Python, Excel).
4. Kompleksni ekponencijalni diskretni signali. Primer sinteze muzičkog signala.
5. Furijeova analiza: Diskretna Furijeova transformacija (DFT) i serija (DFS). Brza Furijeova transformacija (Fast Fourier transform, FFT) i primena za spektralne analizatore i osciloskope.
6. Linearani filtri: konvolucija, idealni i realni filtri, dizajn filtra. Primena konvolucije u GPS sistemima.

...

Octave radni folder



Toolbox signal

```
>> pkg load signal
```

```
>> doc audioread
```

```
>> [y,fs]=audioread ("wav/dsp01.wav") ;
```

```
>> sound(y,fs)
```

```
-- : [Y, FS] = audioread (FILENAME)
-- : [Y, FS] = audioread (FILENAME, SAMPLES)
-- : [Y, FS] = audioread (FILENAME, DATATYPE)
-- : [Y, FS] = audioread (FILENAME, SAMPLES, DATATYPE)

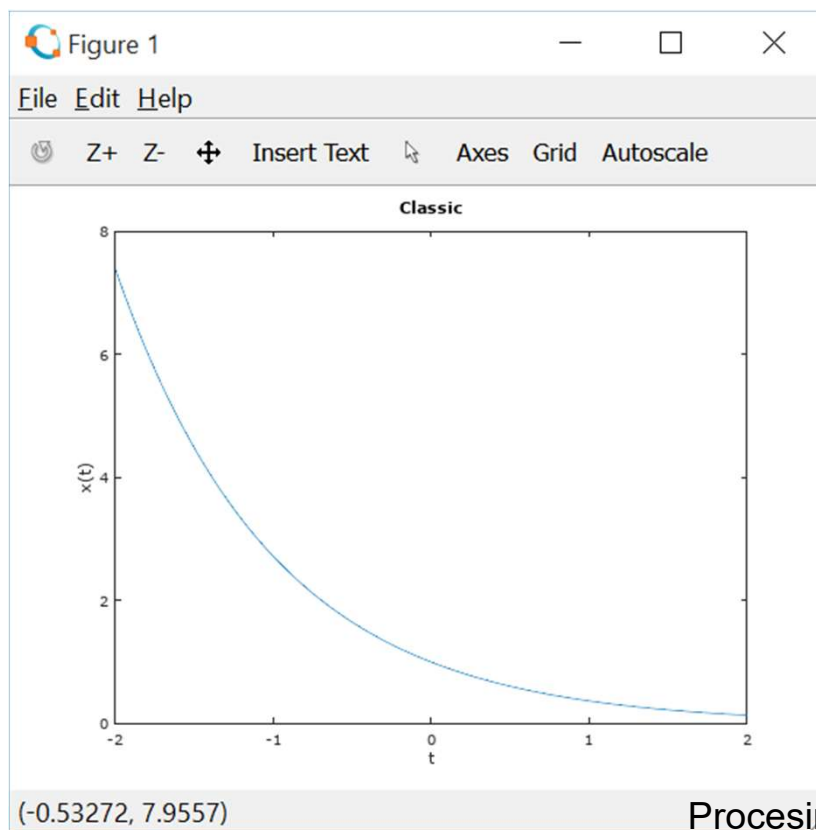
Read the audio file FILENAME and return the audio data
Y and sampling rate FS.
```

Workspace			
Filter ☐			
Name	Class	Dimension	Value
fs	double	1x1	44100
t	double	1x401	-2:0.01:2
x	double	1x401	[7.3891, 7...
y	double	165390x1	[0; 0; 0; 0;...

The audio data is stored as matrix with rows corresponding to audio frames and columns corresponding to channels.

The optional two-element vector argument SAMPLES specifies starting and ending frames.

Octave primer editovanja i izvršavanja



Procesiranje signala

Editor

File Edit View Debug Run Help

primer03x01.m

```
1 t=-2:0.01:2;  
2 x=exp(-t);  
3 plot(t, x)  
4 xlabel('t');  
5 ylabel('x(t)');  
6 title('Classic');  
7
```

line: 7 col: 1 encoding: SYSTEM eol: CRLF

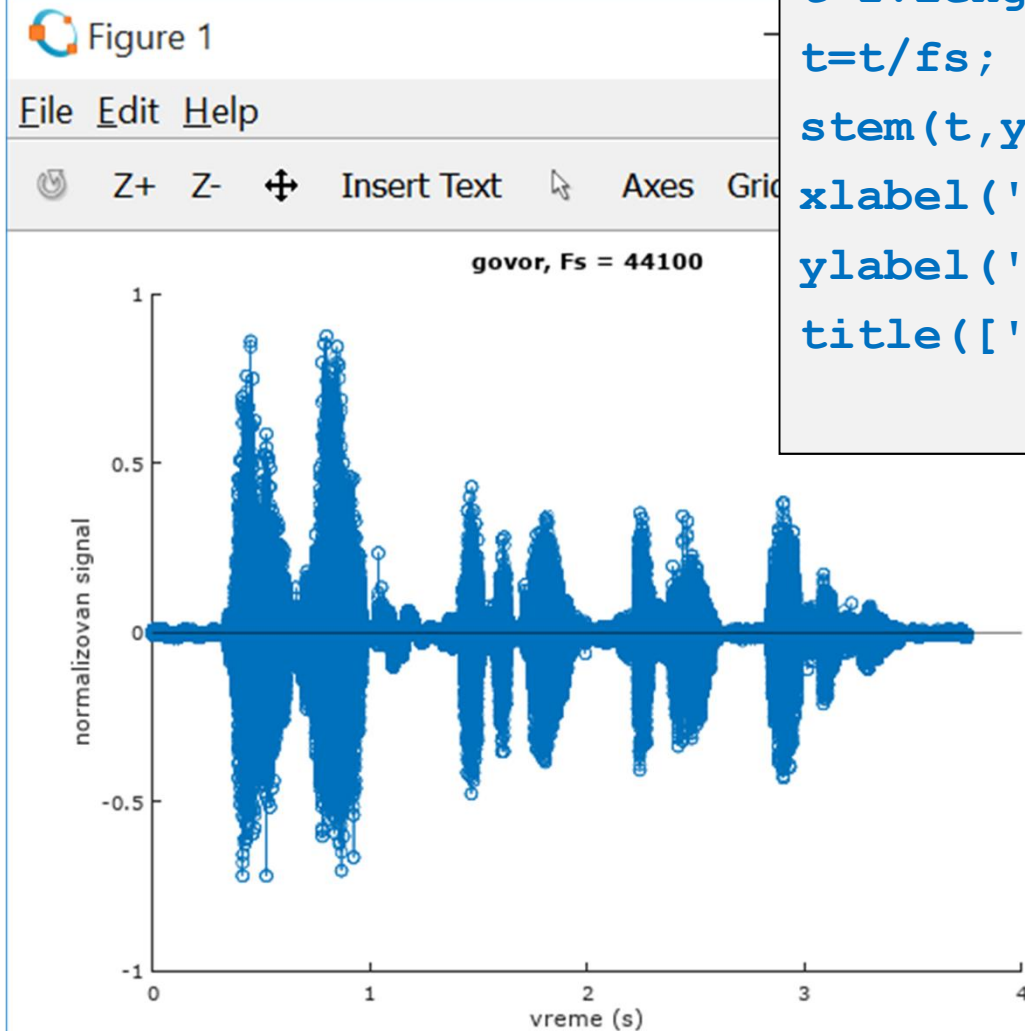
Command Window Editor Documentation

Izvrši

Editor

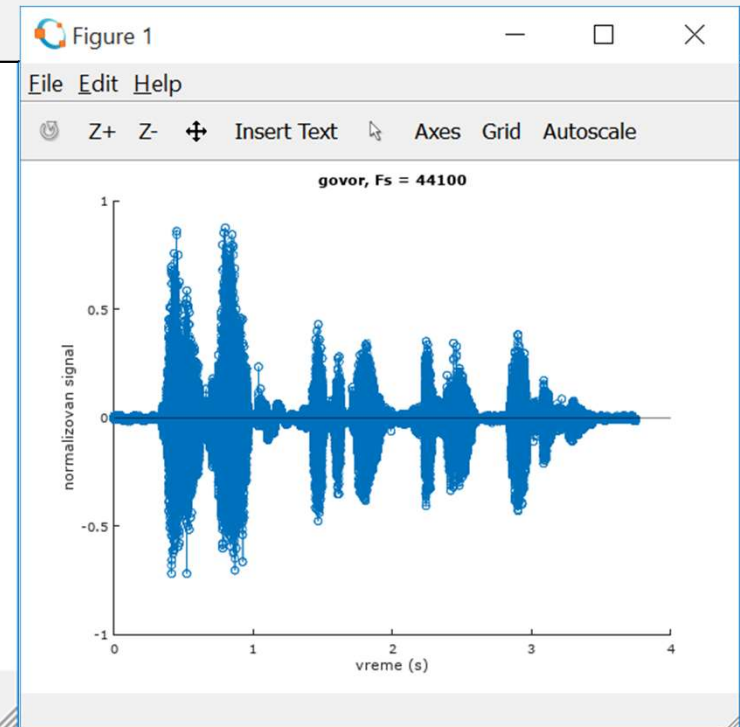
```
clear all, close all, clc
[y,fs]=audioread ("wav/dsp01.wav");
sound(y,fs)
t=1:length(y);
t=t/fs;
stem(t,y)
xlabel('vreme (s)');
ylabel('normalizovan signal');
title(['govor, Fs = ' num2str(fs)]);
```

primer03x02.m



(0.96959, 0.99483)

Procesiranje signala



```
clear all, close all, clc
```

```
...
```

```
fontsize = 24;
```

```
fontname = 'Times New Roman';
```

```
set(findobj('Type','axes'),'FontName',fontname,...
```

```
'FontSize',round(fontsize*0.85));
```

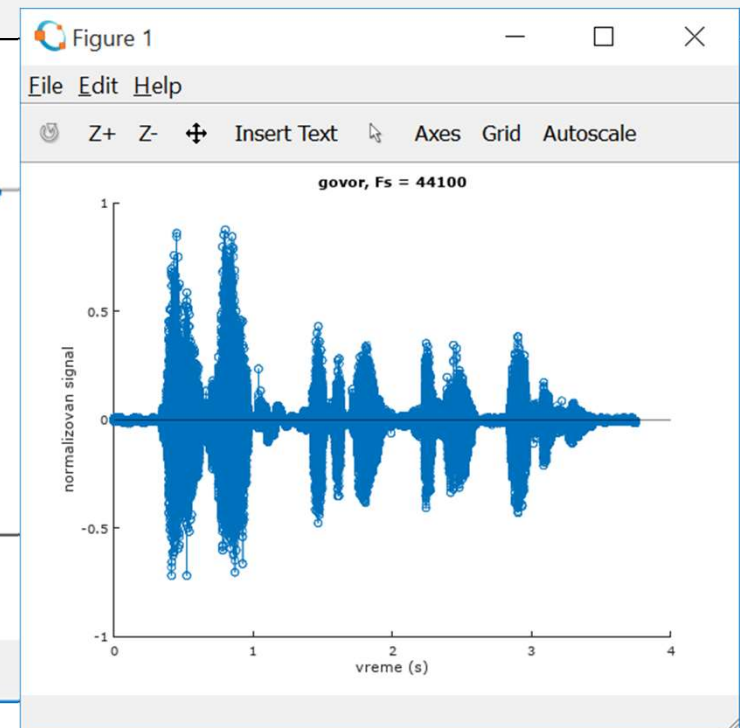
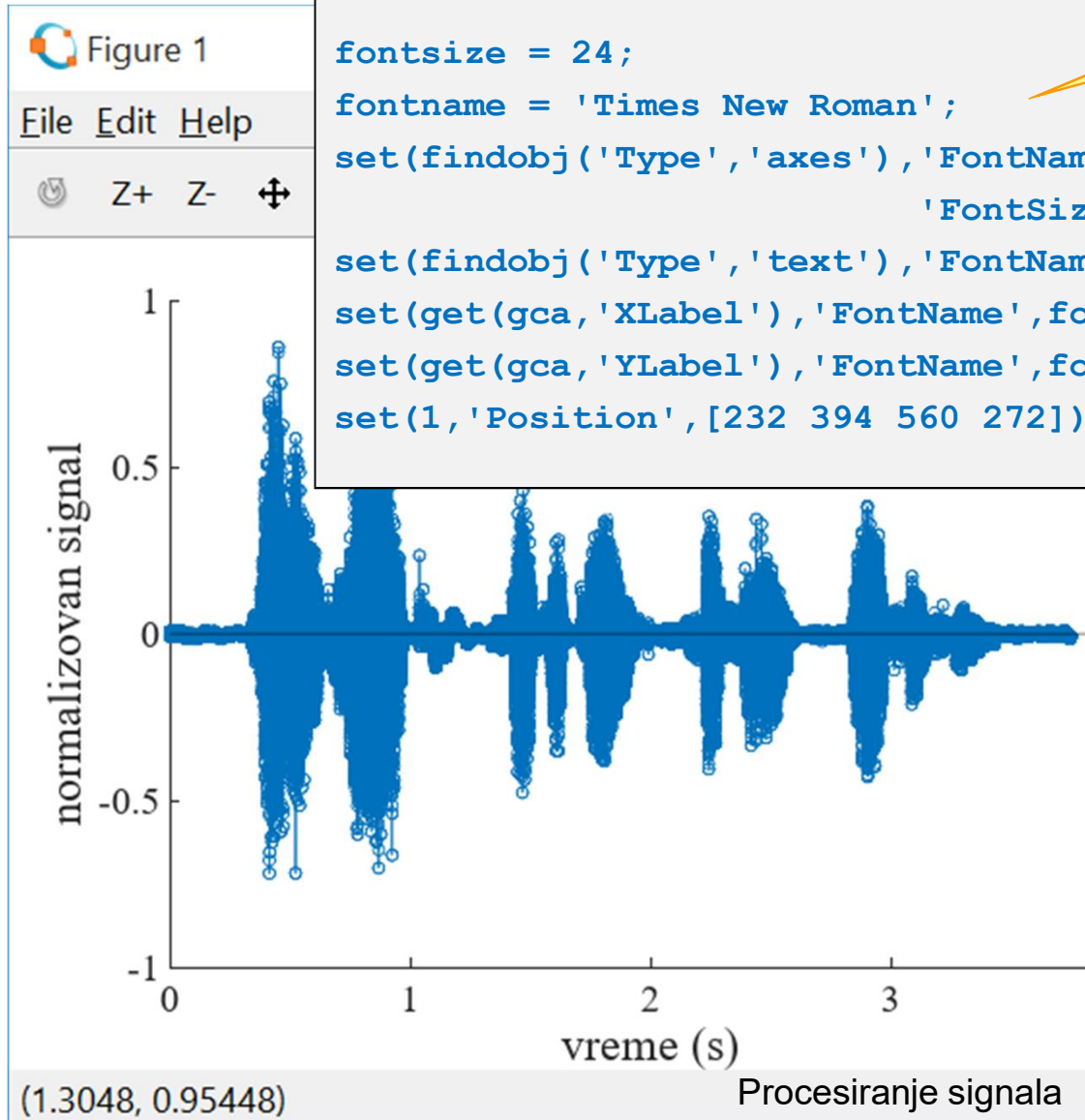
```
set(findobj('Type','text'),'FontName',fontname,'FontSize',fontsize);
```

```
set(get(gca,'XLabel'),'FontName',fontname,'FontSize',fontsize);
```

```
set(get(gca,'YLabel'),'FontName',fontname,'FontSize',fontsize);
```

```
set(1,'Position',[232 394 560 272]);
```

primer03x03.m



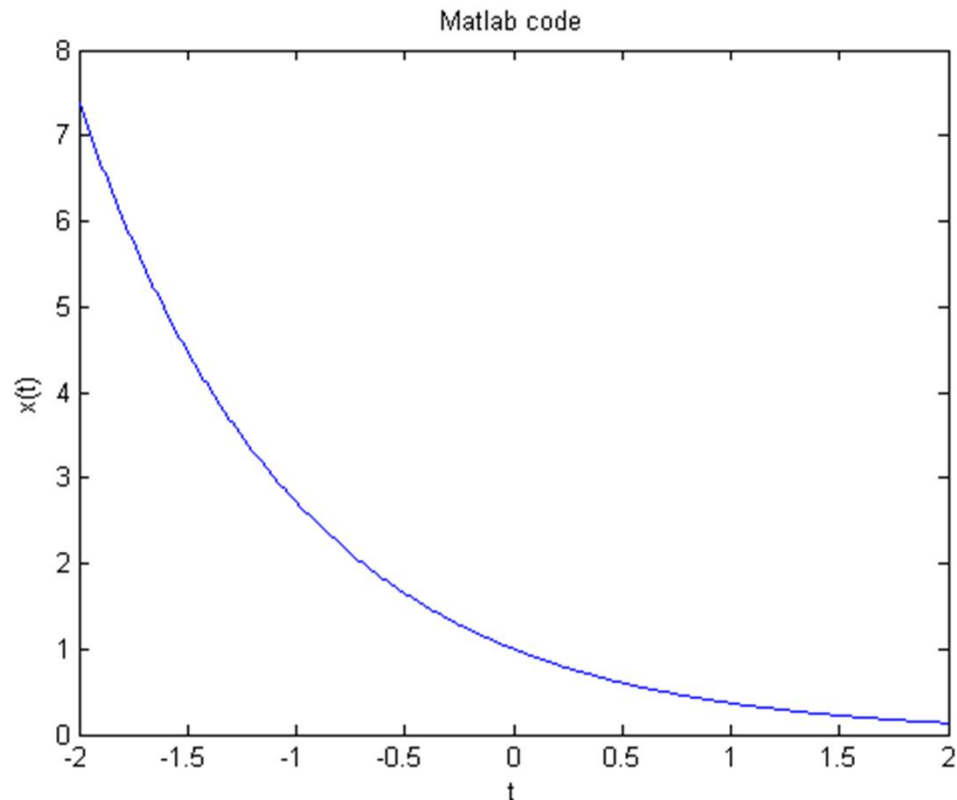
```
# Created by Octave 4.2.1, Wed Mar 21 14:01:36 2018 GMT
<unknown@unknown>
# name: a
# type: scalar struct
# ndims: 2
  1 1
# length: 130
...
# name: xscale
# type: sq_string
# elements: 1
# length: 6
linear
# name: xtick
# type: matrix
# rows: 1
# columns: 5
  0 1 2 3 4
```

```
a=get(gca)
save tmp.txt a
```

MATLAB skript - primer

Definiše se vreme

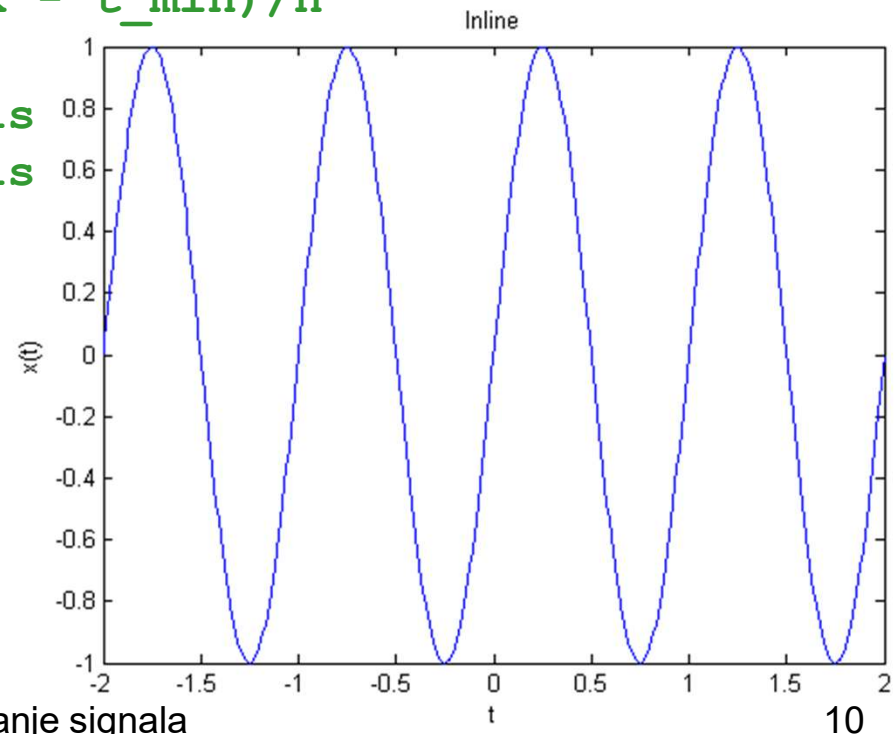
```
t=-2:0.01:2;  
x=exp(-t);  
plot(t, x)  
xlabel('t');  
ylabel('x(t)');  
title('Classic');
```



Inline function object

```
signal='sin(2*pi*t)'; % string with the function
time='t';             % string with variable
x=inline(signal, time);
t_min = -2; t_max = 2;
plotRange = [t_min t_max]; % the x-axis limits
N = 100; % step size is (t_max - t_min)/n
fplot(x, plotRange, n, 'b-')
xlabel('t'); % label x-axis
ylabel('x(t)'); % label y-axis
title('Inline '); % add title
```

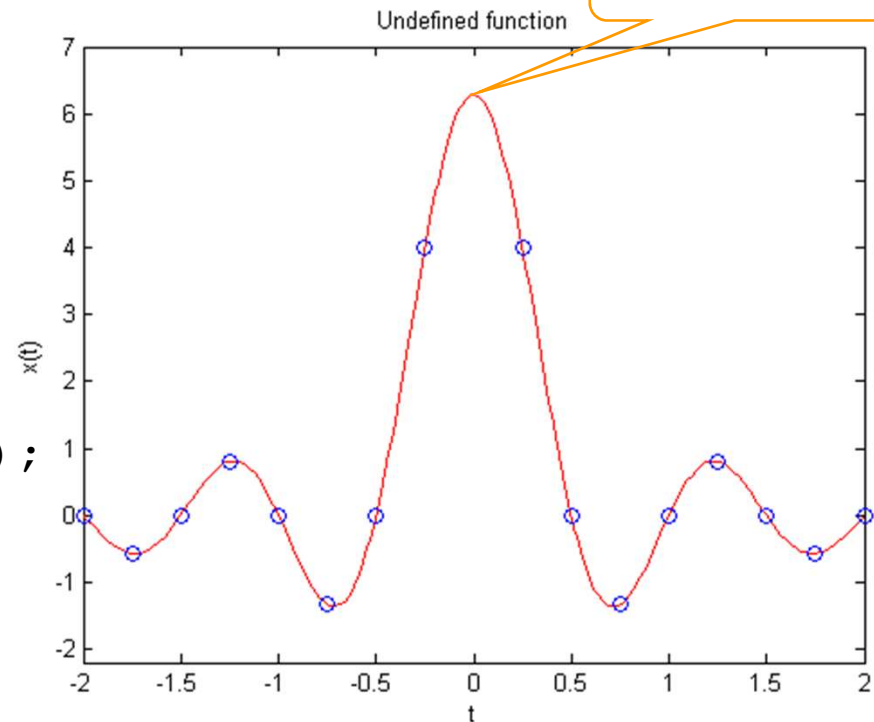
Define a function
before specifying the time



Procesiranje signala

Signal that is not defined at a point

```
signal='(1./t).*sin(2*pi*t)';  
time = 't';  
x = inline(signal, time);  
t1 = -2; t2 = 2;  
x1 = -2.2; x2 = 7;  
plotRange = [t1 t2 x1 x2];  
fplot(x,plotRange,100,'r-')  
xlabel('t'); ylabel('x(t)');  
title('Signal with singularity');  
hold on  
    t = -2:0.25:2;  
    x = (1./t).*sin(2*pi*t);  
    plot(t,x,'o')  
hold off
```



$$x(t) = \frac{1}{t} \sin(2\pi t)$$

Not defined
at t=0

Limit of the signal exists

```
syms t x
```

Define two symbolic variables

```
x = eval(signal)
```

```
t = eval(time)
```

```
t0 = 0;
```

```
x1 = subs(x,t,t0)
```

```
x2 = limit(x,t,t0)
```

Find limit of the symbolic expression

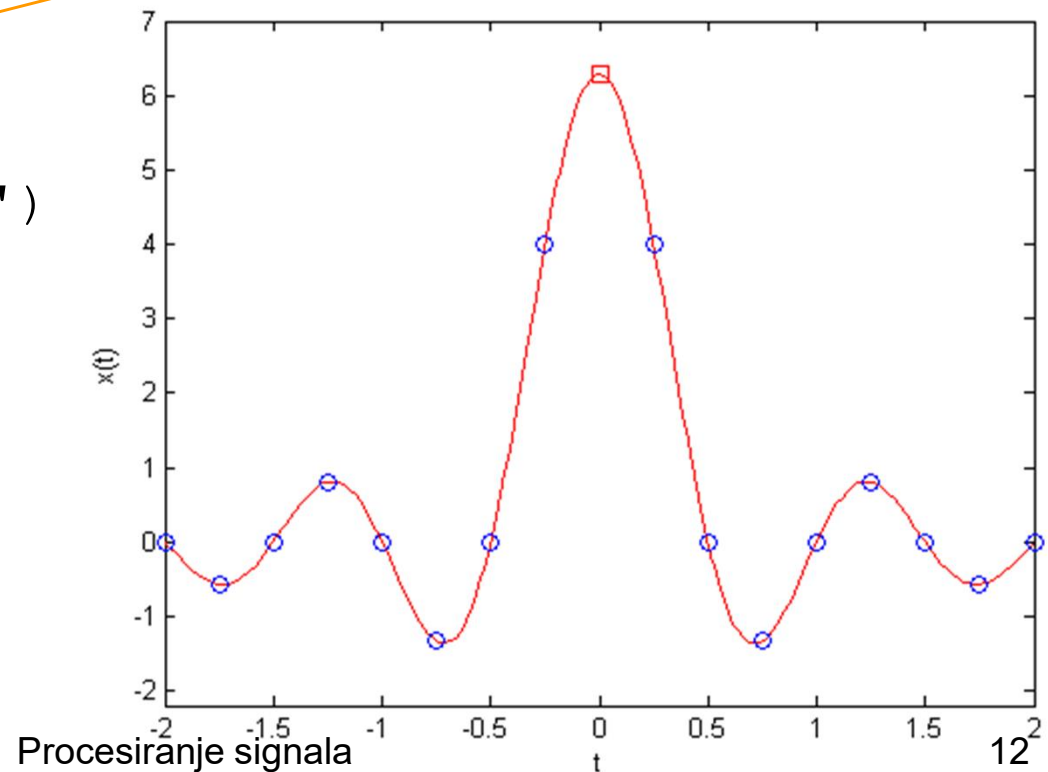
```
hold on
```

```
plot(t0,double(x2),'sr')
```

```
hold off
```

$$x_1 = x(0) = NaN$$

$$x_2 = \lim_{t \rightarrow 0} x(t) = 2\pi$$



Simplify an expression

```
syms t x
```

Define two symbolic variables

```
x = eval(signal)
```

```
t = eval(time)
```

```
t0 = 0;
```

```
x1 = subs(x,t,t0)
```

```
x_simp = simplify(x)
```

Simplify a symbolic expression

```
x2 = subs(x_simp,t,t0)
```

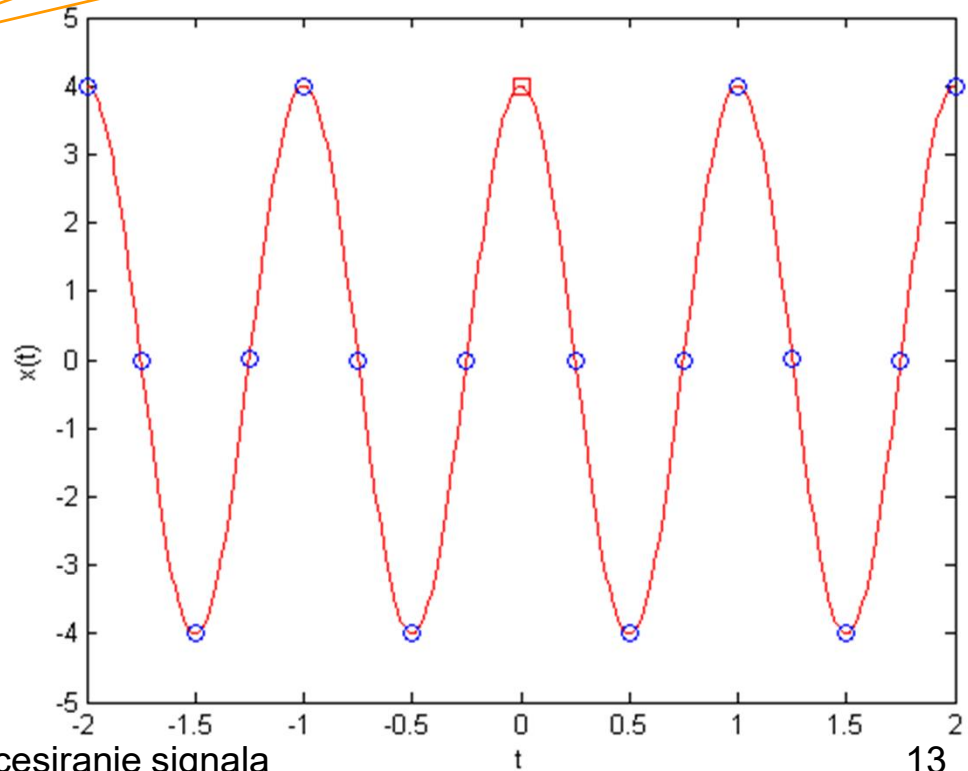
```
hold on
```

```
plot(t0,double(x2),'sr')
```

```
hold off
```

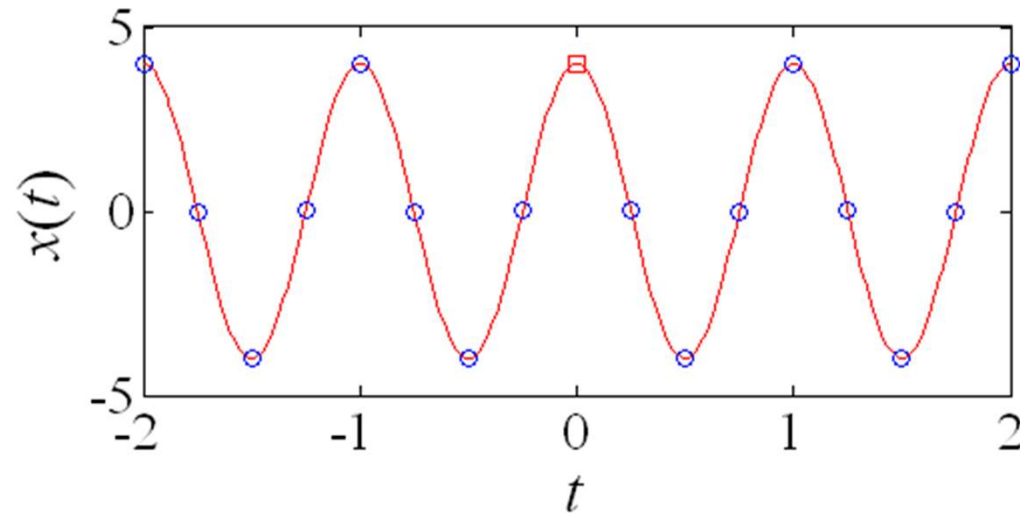
$$x(t) = \left(\frac{(t+1)^2}{t} - \frac{(t-1)^2}{t} \right) \cos(2\pi t)$$

$$x_{\text{simp}}(t) = 4 \cos(2\pi t)$$



Procesiranje signala

Refine figure typesetting



```
fontsize = 24;  
fontname = 'Times New Roman';  
set(findobj('Type','axes'),'FontName',fontname,...  
     'FontSize',round(fontsize*0.85));  
set(findobj('Type','text'),'FontName',fontname,'FontSize',fontsize);  
set(get(gca,'XLabel'),'FontName',fontname,'FontSize',fontsize);  
set(get(gca,'YLabel'),'FontName',fontname,'FontSize',fontsize);  
set(1,'Position',[232 394 560 272]);
```

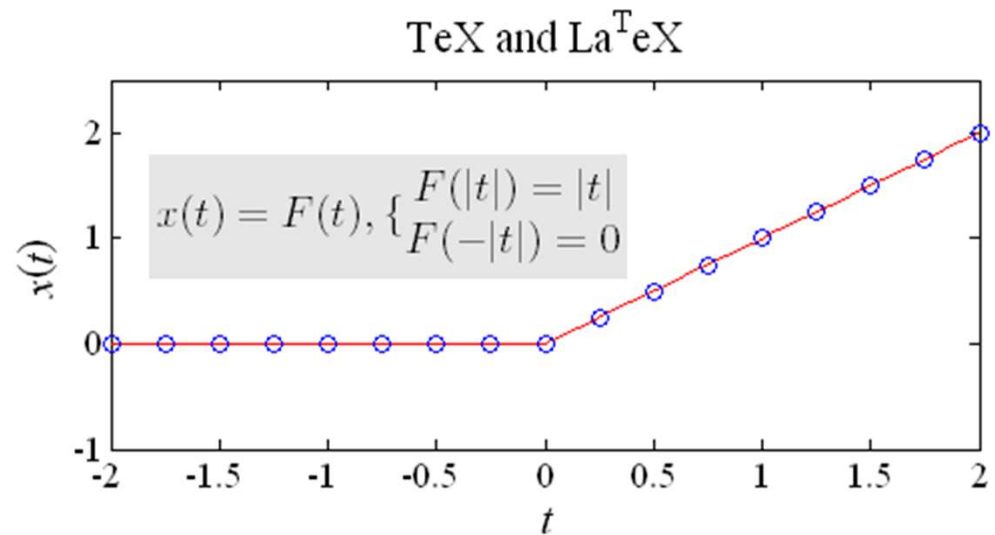
Procesiranje signala

LaTeX mathematical expressions

```

signal='t.*(t>0)';
time='t';
x=inline(signal,time);
t1=-2; t2=2;
x1=-1; x2=2.5;
plotRange=[t1 t2 x1 x2];
fplot(x,plotRange,100,'r-')
xlabel('\it t');
ylabel('{\it x}({\it t})');
tn = -2:0.25:2;
xn = feval(x, tn);
hold on, plot(tn,xn,'o'), hold off
texstr='$x(t)=F(t), \{\begin{array}{cl} F(|t|)=|t| \\ F(-|t|)=0 \end{array}\}$'
text('string',texstr,'interpreter','latex',...
    'FontSize',24,'pos',[-1.8 1.2],'BackgroundColor',[.9 .9 .9])

```

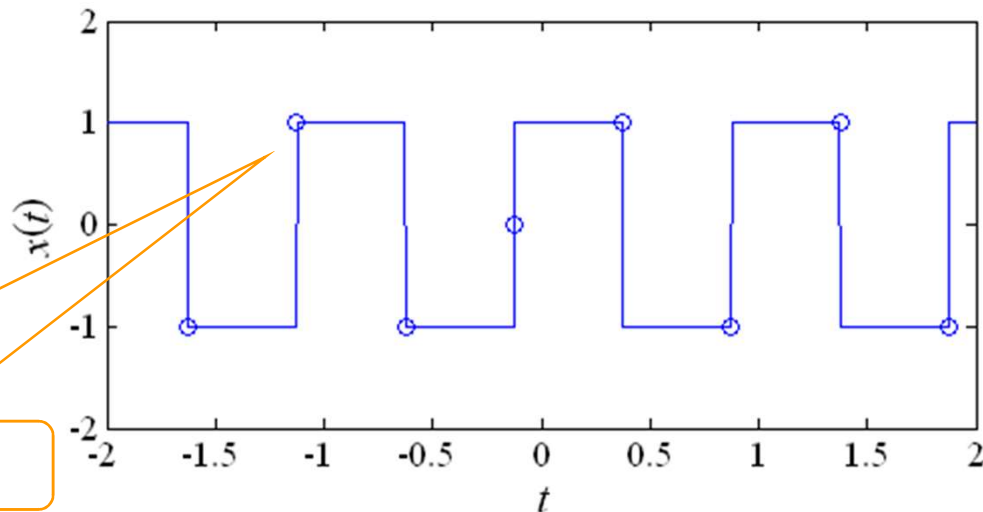


Anonymous function

$$x(t) = F(\sin(2\pi t + \pi/4))$$

$$F(0) = 0, F(|a|) = 1, F(-|a|) = -1$$

It should be 0, not 1

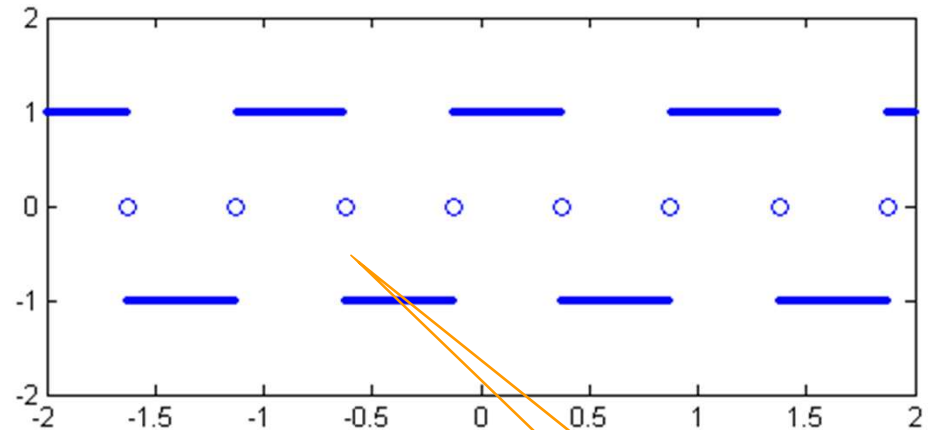


```
x = @(t) ((sin(2*pi*t+pi/4))>0) - ((sin(2*pi*t+pi/4))<0);
fplot(x, [-2 2 -2 2], 1000, 'b-')
xlabel('\it t'); ylabel('{\it x} ({\it t})');
t1 = [-13 -9 -5 -1 3 7 11 15]/8;
x1 = feval(x, t1);
hold on, plot(t1, x1, 'bo'), hold off
```

Correct plot of the signal

$$x(t) = F(\sin(2\pi t + \pi/4))$$

$$F(0) = 0, F(|a|) = 1, F(-|a|) = -1$$



```
x= @(t) ((sin(2*pi*t+pi/4))>0) - ((sin(2*pi*t+pi/4))<0);
f = @(t) (t>0) - (t<0);
fplot(x, [-2 2 -2 2], 1000, 'b.')
t1 = [-13 -9 -5 -1 3 7 11 15]/8;
sym xsin
xsin = 'sin(2*pi*t+pi/4)'
for ind = 1:length(t1)
    xs(ind)=eval(simplify(subs(xsin,sym(t1(ind) , 'r'))));
end
x2 = feval(f, xs);
hold on, plot(t1,x2,'ob'), hold off
```

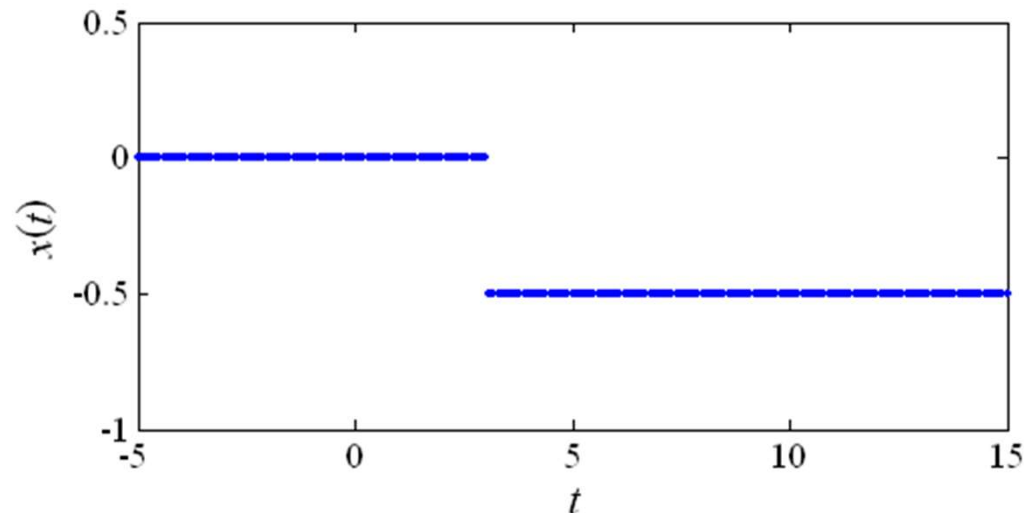
Correct
values

Unit step signal

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t \leq 0 \end{cases}$$

$$x(t) = -\frac{1}{2} u(t-3)$$

```
signal = 't>0';  
time = 't';  
x = inline(signal,time);  
k = (-5:0.1:15)';  
plot(k, -(1/2)*x(k-3), 'b.')
```



Procesiranje signala

Sinusoidalni signal

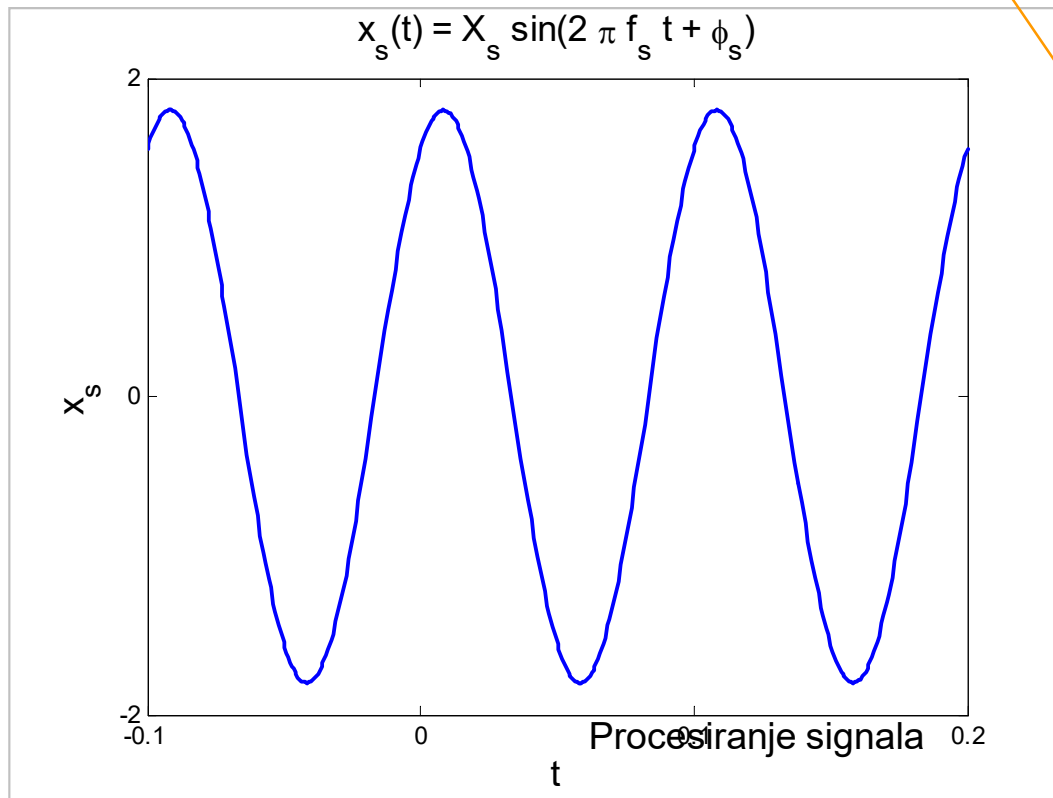
$$x_s(t) = X_s \sin(2\pi f_s t + \phi_s)$$

Amplituda

Faza u
radijanima

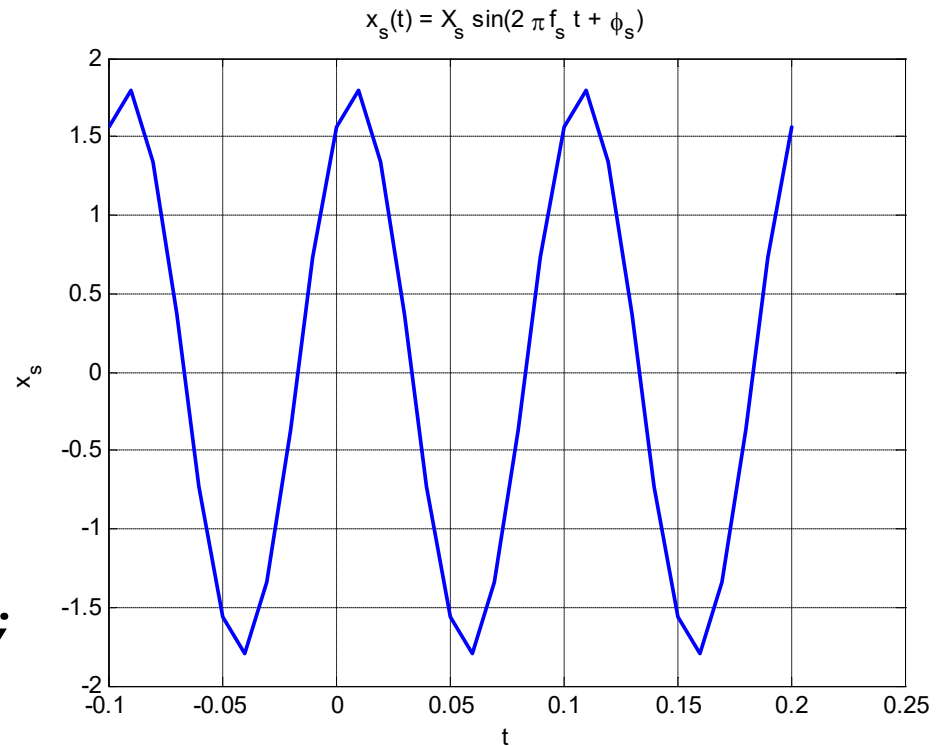
Vreme u
sekundama (s)

Frekvencija u
Hertz (Hz)



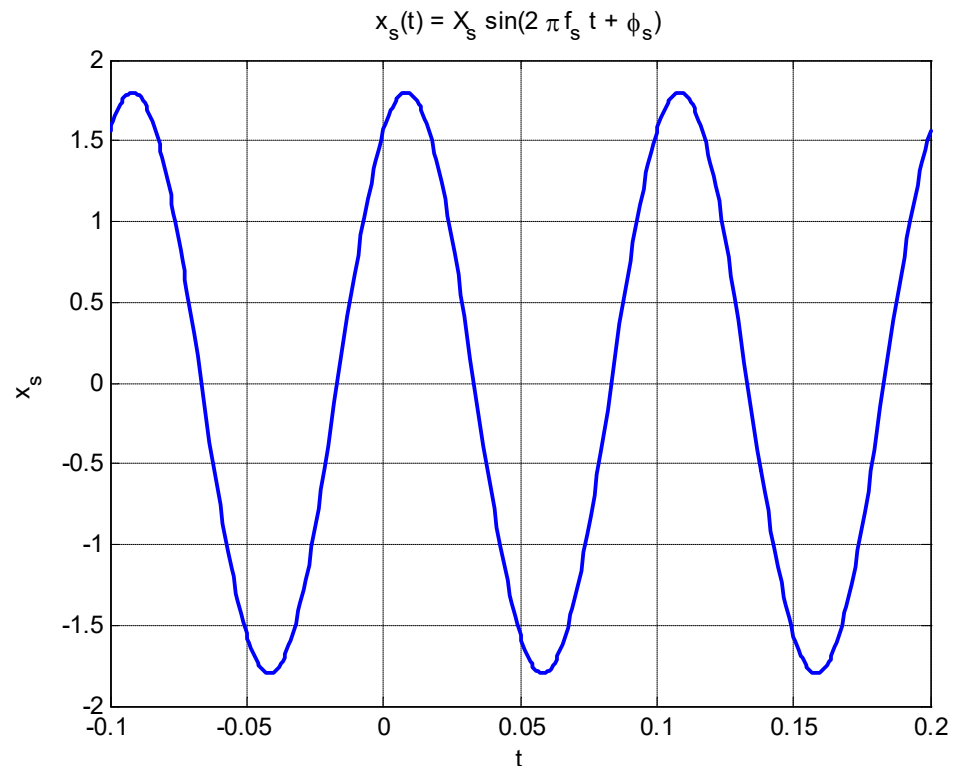
MATLAB kod za sin signal

```
Xs = 1.8;  
fs = 10;  
fi = pi/3;  
t1 = -0.1;  
tstep = 0.01;  
t2 = 0.2;  
t = t1:tstep:t2;  
x = Xs*sin(2*pi*fs*t+fi);  
plot(t, x)  
xlabel('t')  
ylabel('x_s')  
title('x_s(t) = X_s sin(2 \pi f_s t + \phi_s)')  
grid on
```



MATLAB kod za kontinualne

```
Xs = 1.8;  
fs = 10;  
fi = pi/3;  
  
t1 = -0.1;  
t2 = 0.2;  
t = [t1, t2];
```



```
x = inline('Xs*sin(2*pi*fs*t+fi)', 't', 'Xs', 'fs', 'fi');  
fplot(x,t,2e-3,1,'-',Xs,fs,fi)  
xlabel('t'); ylabel('x_s'); grid on  
title('x_s(t) = X_s sin(2 \pi f_s t + \phi_s)')
```

Eksponencijalni signal

$$x_e(t) = X_e e^{bt}$$

```
x = inline('Xe*exp(b*t)', 't', 'Xe', 'b');
```

```
Xe = 0.8;
```

```
b = -0.5;
```

```
t1 = 0;
```

```
t2 = 8;
```

```
t = [t1, t2];
```

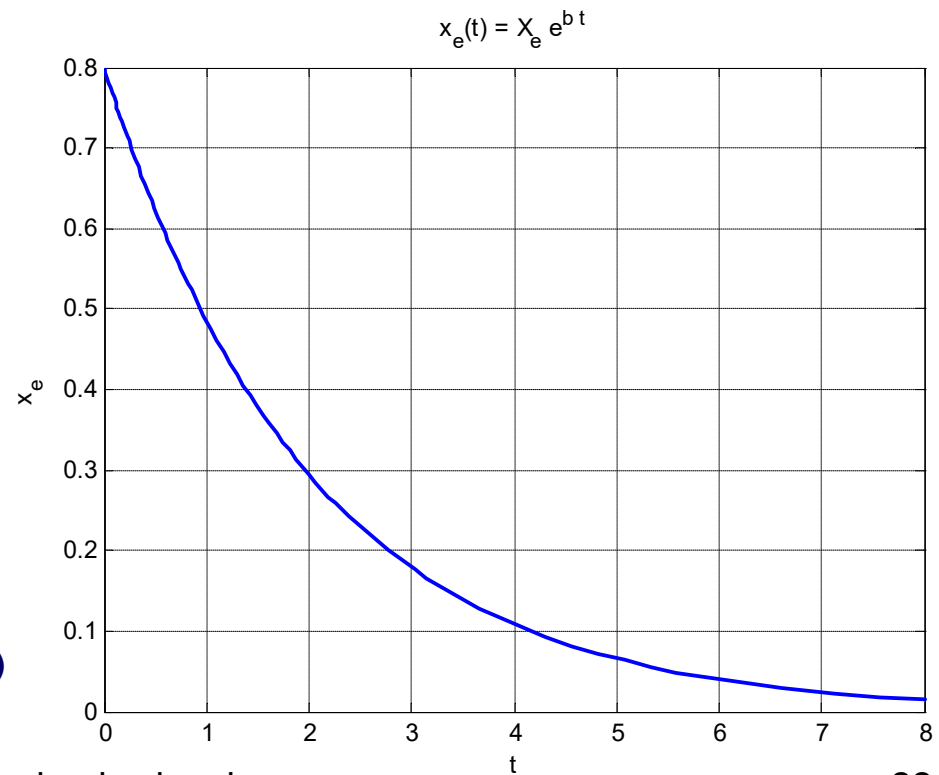
```
fplot(x,t,2e-3,1,'-',Xe,b)
```

```
xlabel('t')
```

```
ylabel('x_e')
```

```
title('x_e(t) = X_e e^{b t}')
```

```
grid on
```

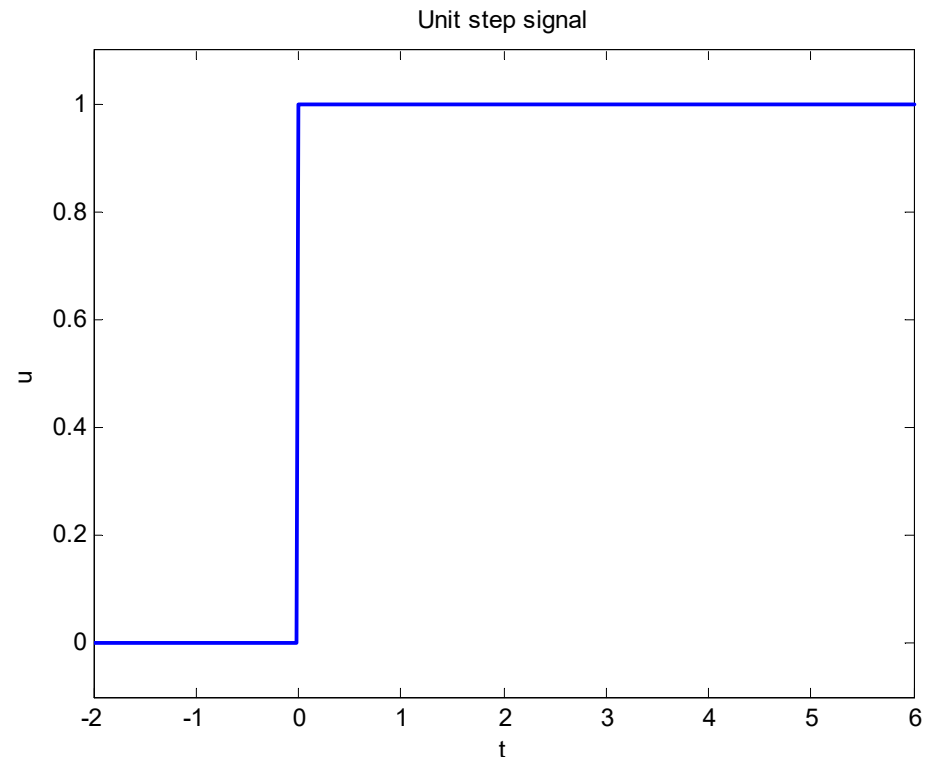


Unit step signal

Jedinična odskočna funkcija

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t \leq 0 \end{cases}$$

```
x = inline('t>0', 't');  
  
t1 = -2;  
t2 = 6;  
t = [t1, t2];  
  
fplot(x, t)  
  
xlabel('t')  
ylabel('u')  
title('Unit step signal')  
axis([t -0.1 1.1])
```



Pulse signal

Jedinični impulsni signal

$$p_{\varepsilon}(t) = \begin{cases} \frac{1}{\varepsilon}, & 0 < t \leq \varepsilon \\ 0, & \text{otherwise} \end{cases}$$

```
x = inline('(1/e)*((t>0)&(t<=e))','t','e');
```

```
e = 1/100;
```

```
t1 = -1;
```

```
t2 = 5;
```

```
t = [t1, t2];
```

```
fplot(x,t,1e-5,1000,'-',e)
```

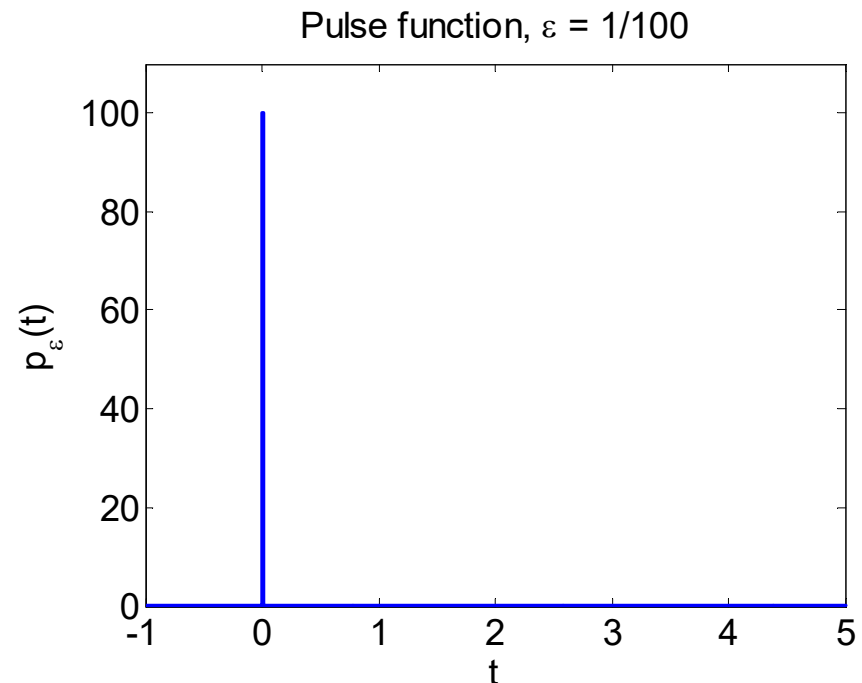
```
set(gca,'FontSize',16)
```

```
xlabel('t')
```

```
ylabel('p_\epsilon(t)')
```

```
axis([t -0.1 1.1/e])
```

```
title('Pulse function, \epsilon = 1/100')
```



Unit impulse signal (Dirac delta)

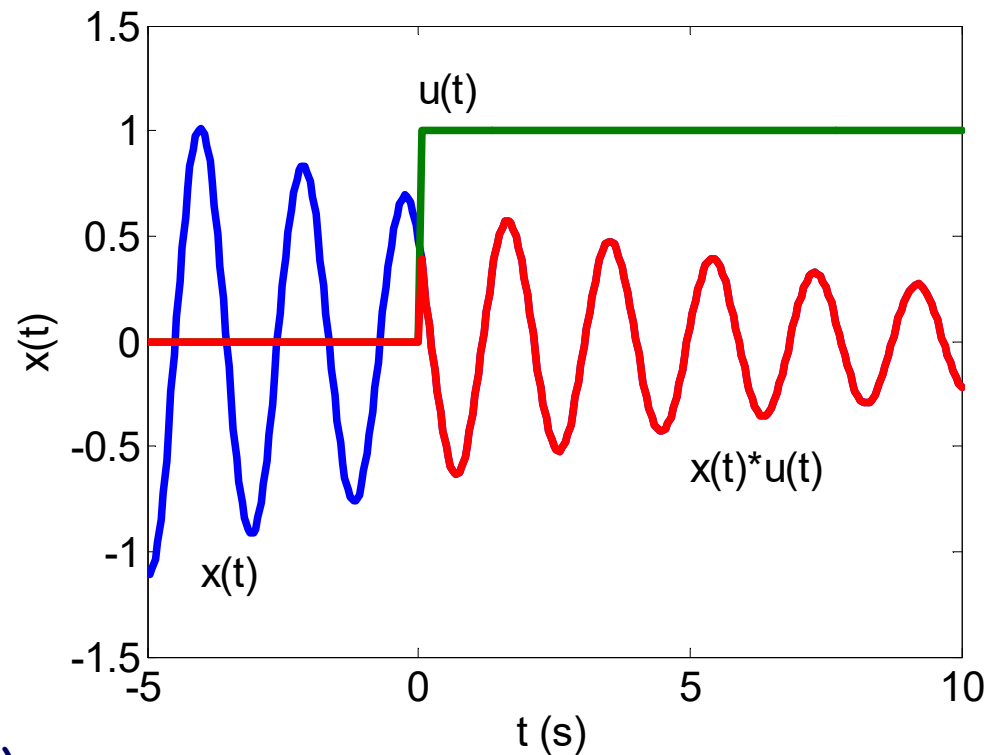
$$\delta(t) = \lim_{\varepsilon \rightarrow 0} p_{\varepsilon}(t)$$

$$\delta(t) = 0, \quad t \neq 0$$

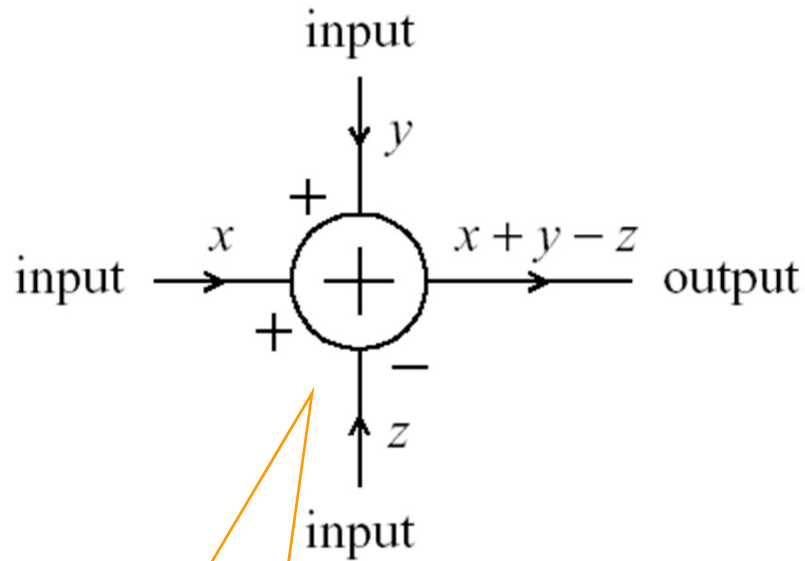
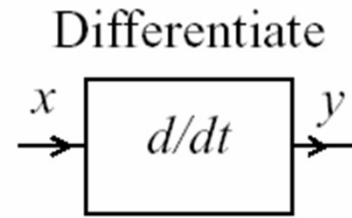
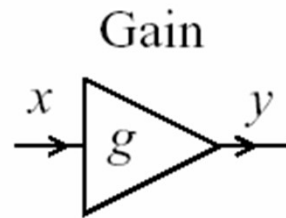
$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

Kauzalan signal u MATLAB-u

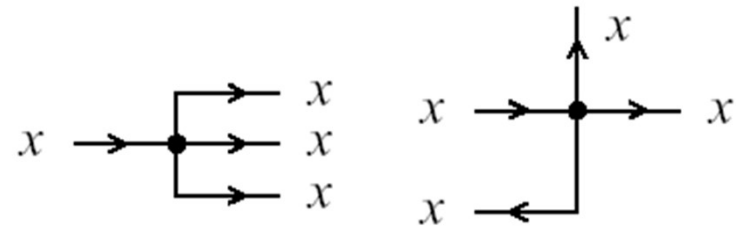
```
B = 0.02;  
a = 0.1;  
f = 0.53;  
phi = 3*pi/4;  
t = -5:0.05:10;  
x = B^a*exp(-a*t).*sin(...  
    2*pi*f*t+phi);  
xu = x.*(t>0);  
u = (t>0);  
plot(t,x,t,u,t,xu)  
ylabel('x(t)')  
xlabel('t (s)')  
text(0,1.2,'u(t)')  
text(-4,-1.1,'x(t)')  
text(5,-.6,'x(t)*u(t)')  
axis([t(1) t(end) -1.5 1.5])
```



Elementi blok dijagrama



Summing point

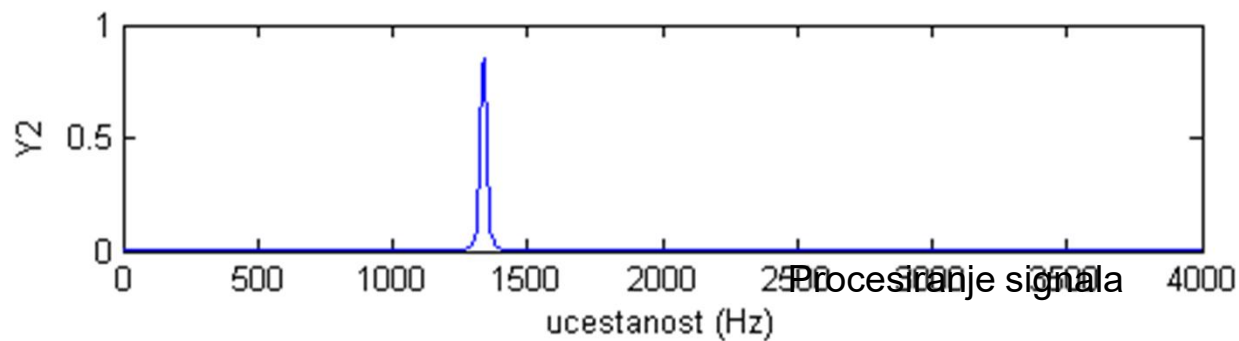
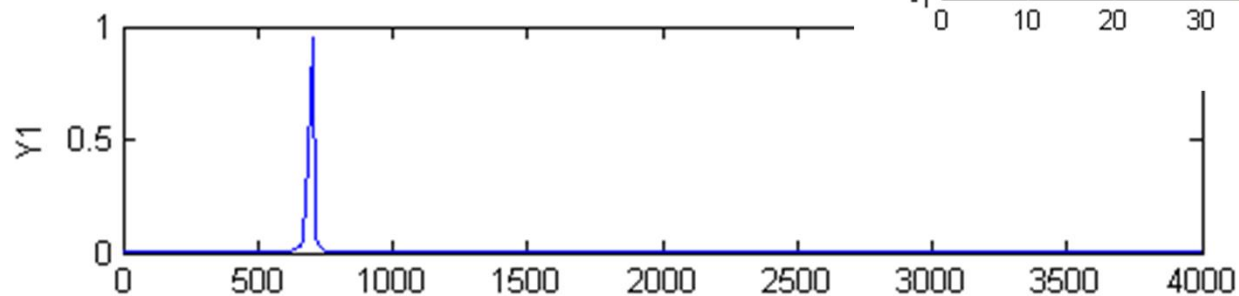
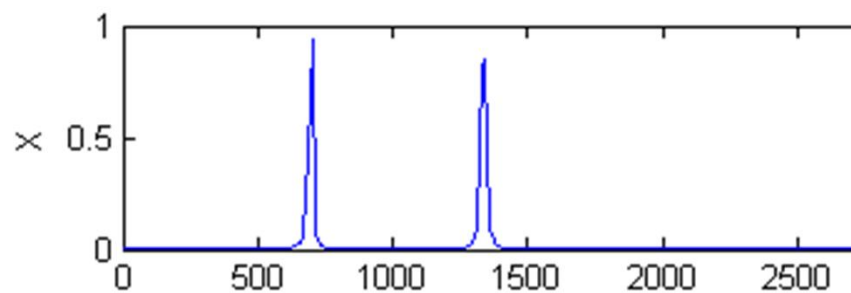
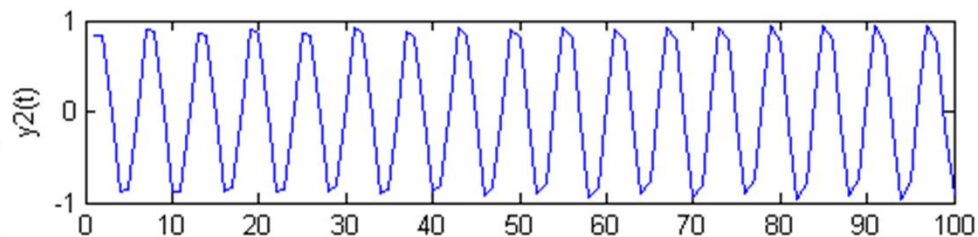
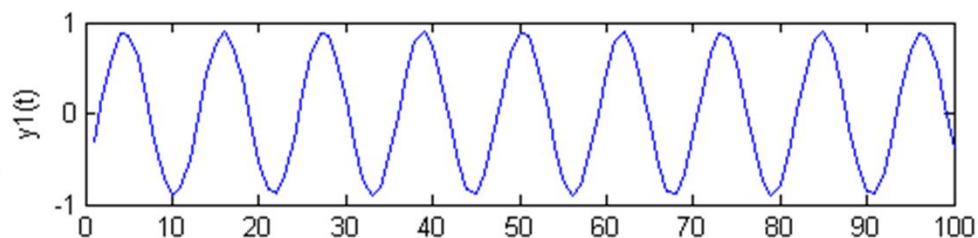
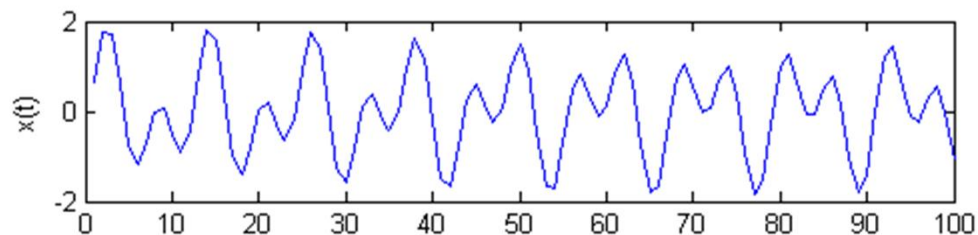


Takeoff point

Procesiranje signala

Telefon #2

697Hz + 1336 Hz



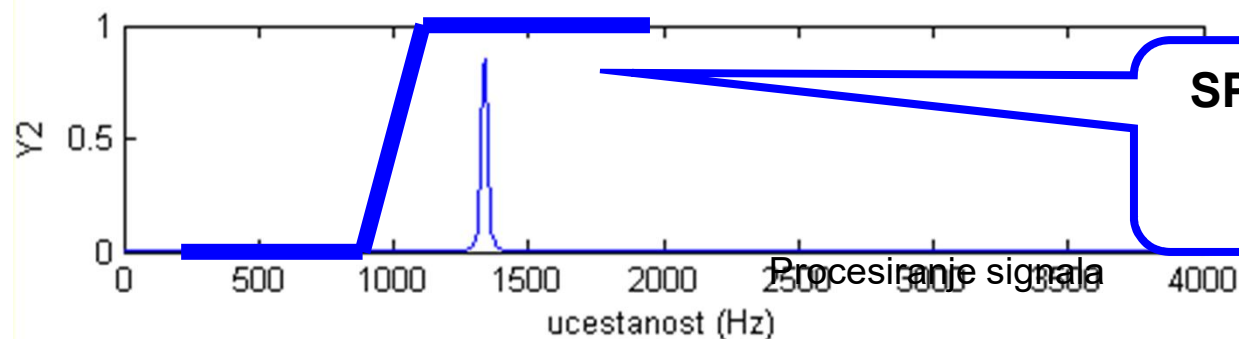
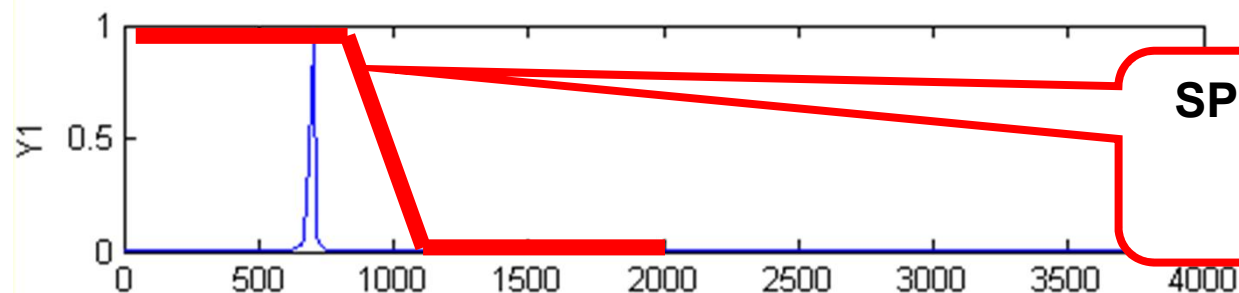
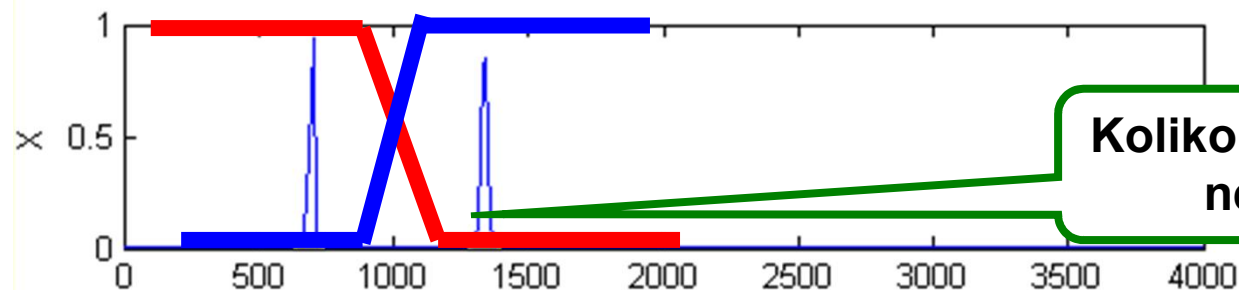
Procesiranje signala

Telefon #2

697Hz + 1336 Hz

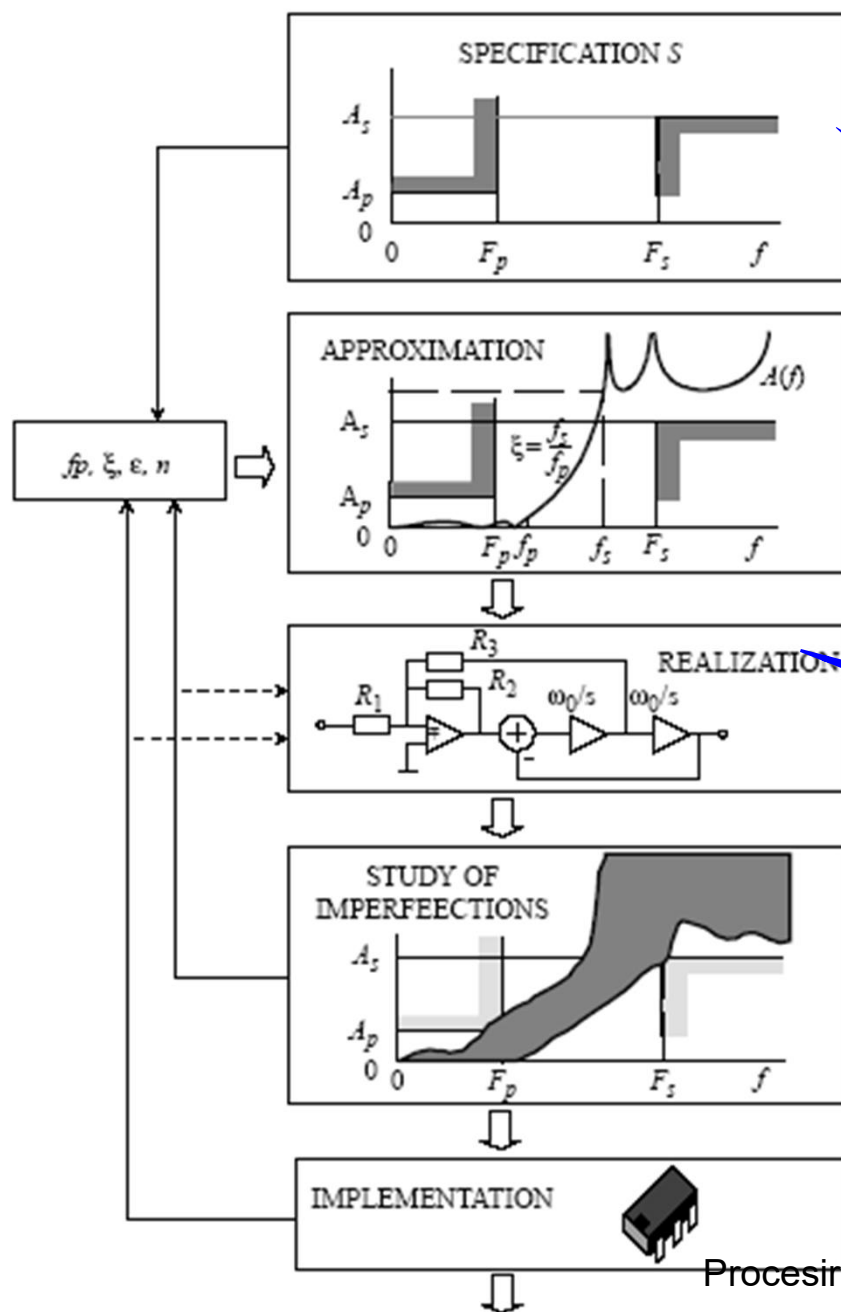
697, 770, 852, 941 Hz

1209, 1336, 1477, 1633 Hz



Procesiranje signala

Od specifikacija do realizacije



Specifikacije

Sinteza

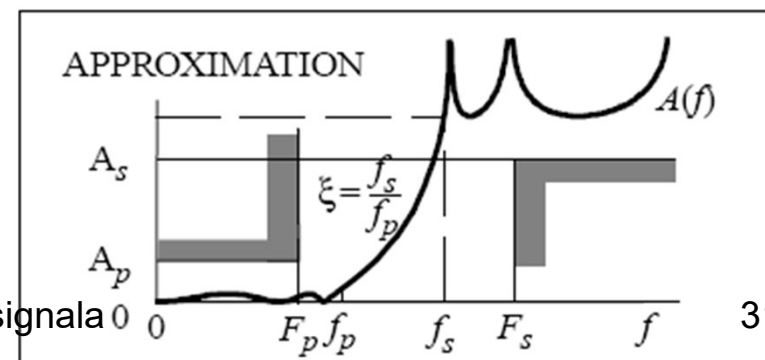
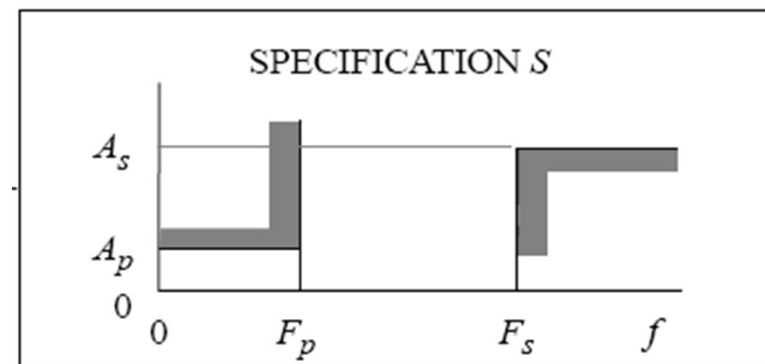
Procesiranje signala

Specifikacije

- Definišu se dozvoljene **tolerancije** u određenim frekvencijskim opsezima

$H(s)$ je racionalna funkcija kompleksne frekvencije
 $s = \delta + j\omega$, $s = j2\pi f$

$$a(f) = -20 \log(|H(j2\pi f)|)$$



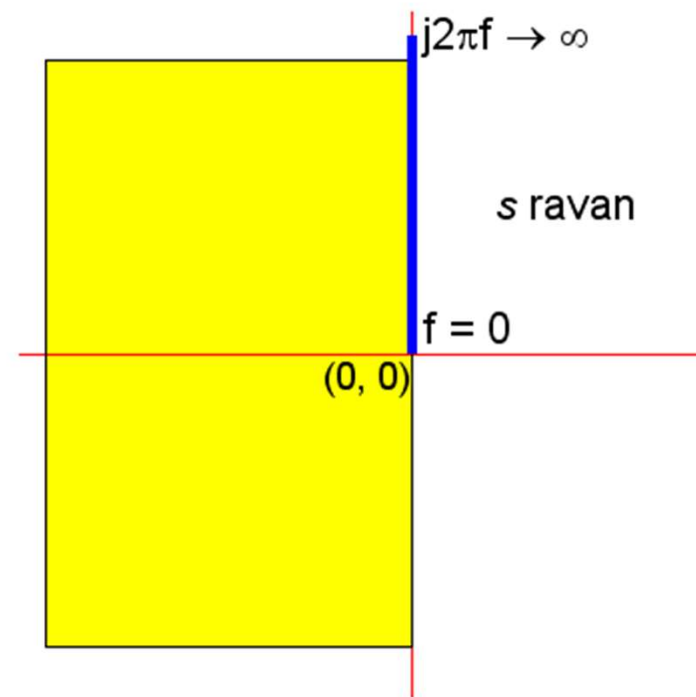
$$H(s) = \frac{\sum_{k=0}^M c_k s^k}{\sum_{k=0}^N d_k s^k} = \frac{C(s)}{D(s)}$$

Frekvencijski odziv

$$H(s) = \frac{\sum_{k=0}^M c_k s^k}{\sum_{k=0}^N d_k s^k} = \frac{C(s)}{D(s)}$$

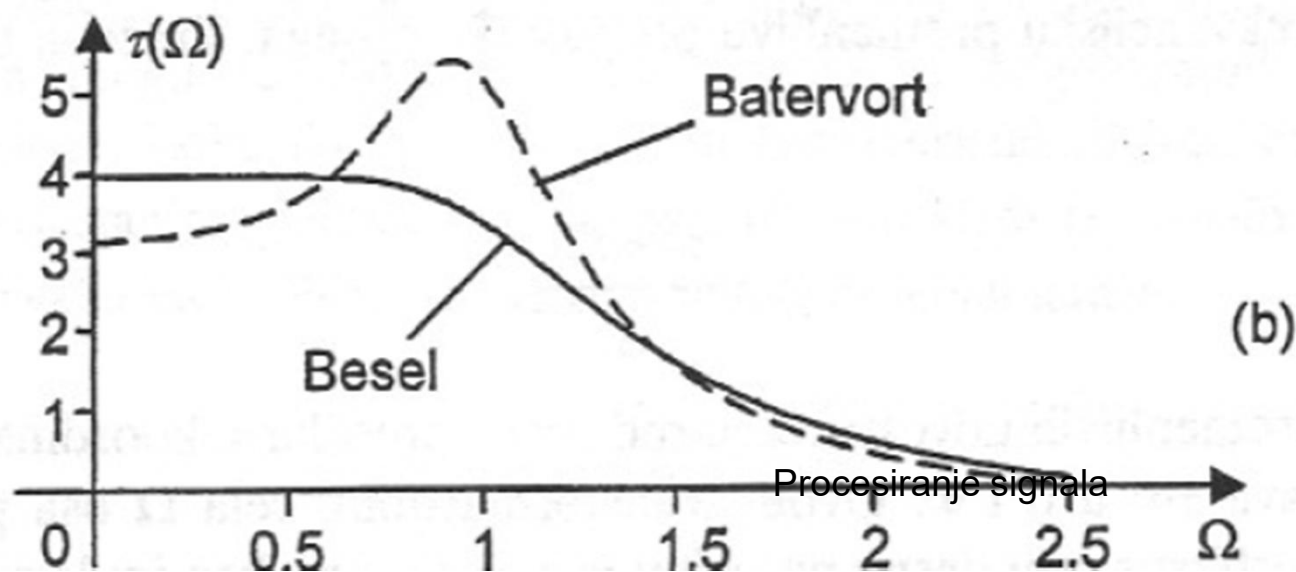
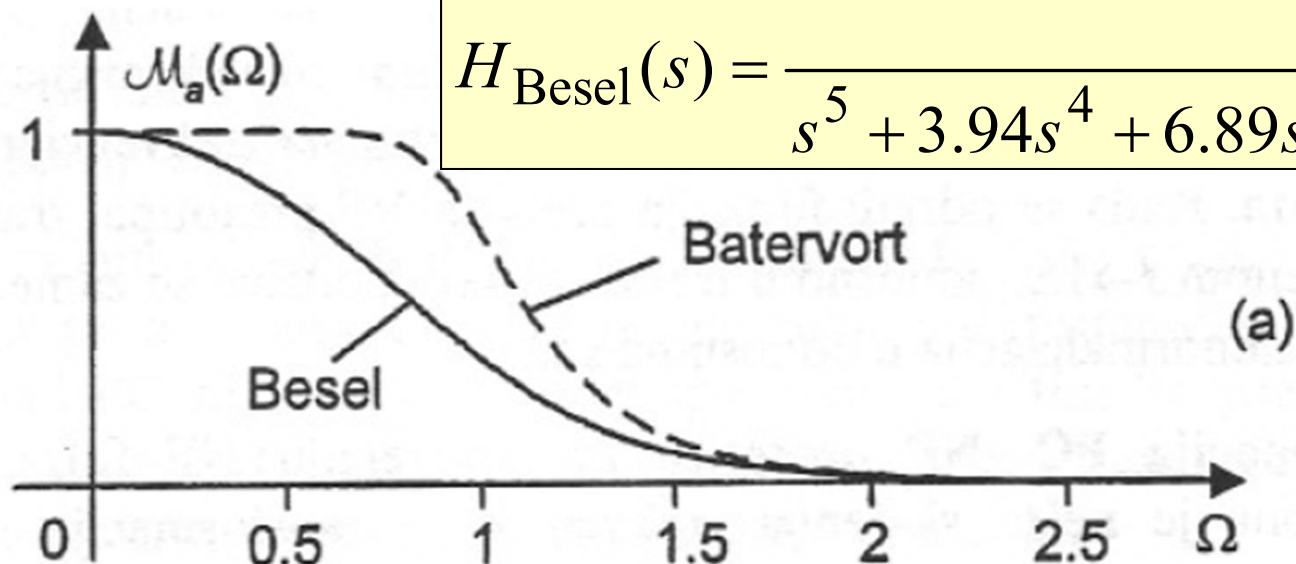
$$H(j2\pi f)$$

$$0 \leq f \leq \infty$$



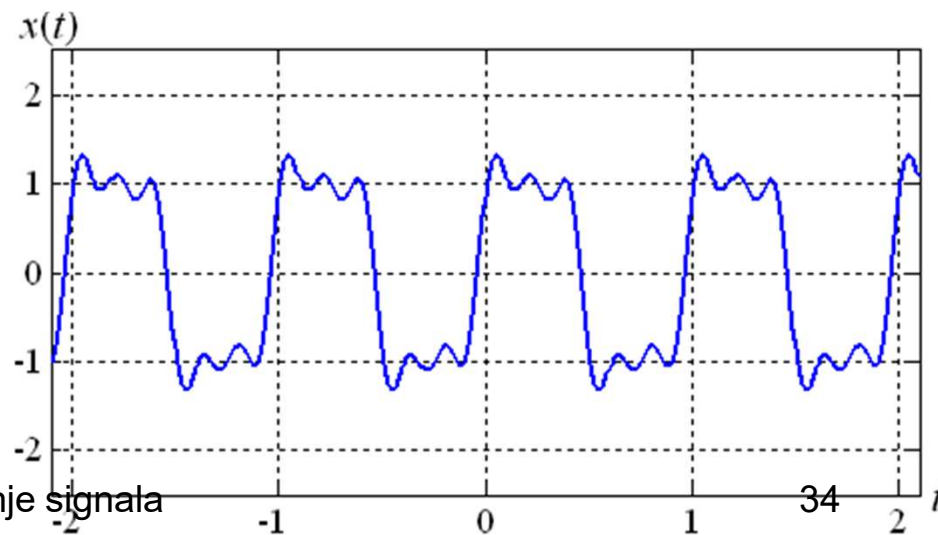
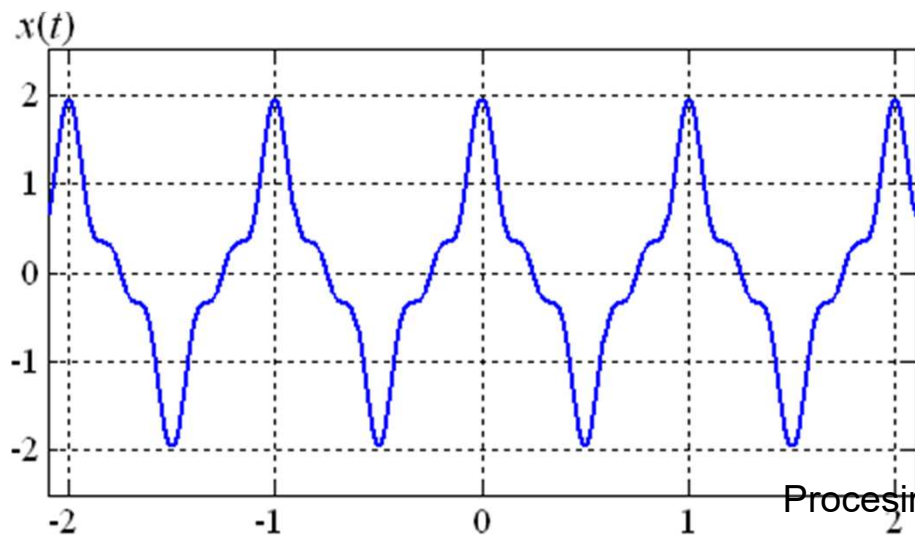
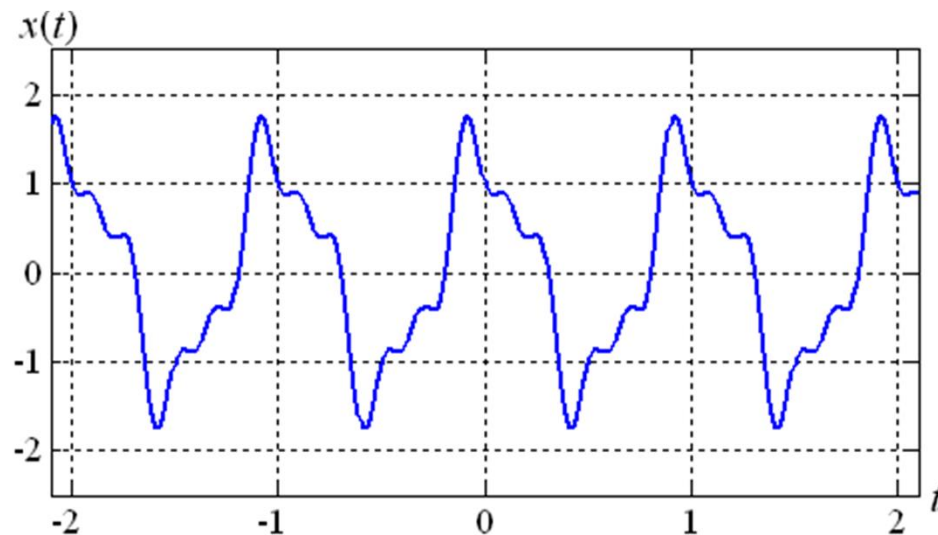
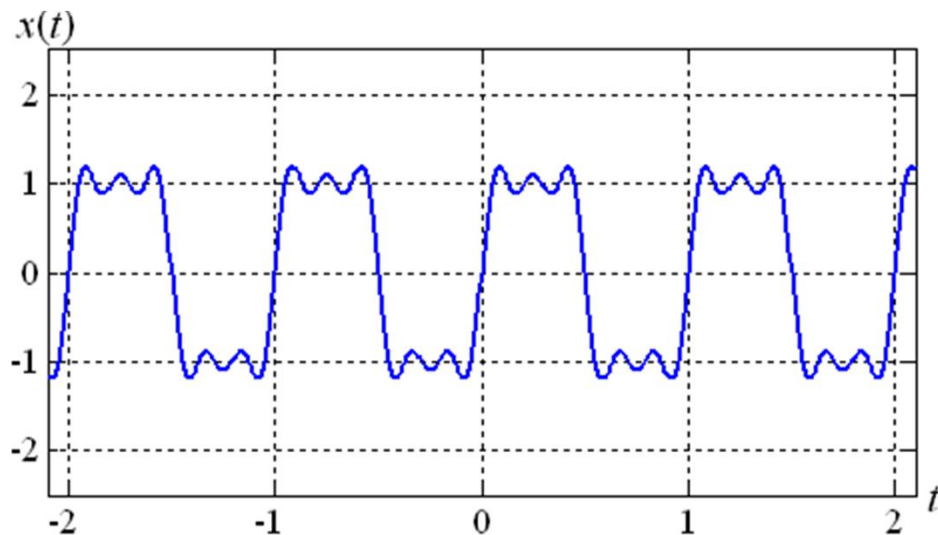
$$H_{\text{Batervort}}(s) = \frac{1}{s^5 + 3.24s^4 + 5.24s^3 + 5.24s^2 + 3.24s + 1}$$

$$H_{\text{Besel}}(s) = \frac{1}{s^5 + 3.94s^4 + 6.89s^3 + 6.78s^2 + 3.81s + 1}$$



Procesiranje signala

Uticaj fazne karakteristike



Procesiranje signala

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Ova prezentacija je nekomercijalna.

Slajdovi mogu da sadrže materijale preuzete sa Interneta, stručne i naučne građe, koji su zaštićeni Zakonom o autorskim i srodnim pravima.

Ova prezentacija se može koristiti samo privremeno tokom usmenog izlaganja nastavnika u cilju informisanja i upućivanja studenata na dalji stručni, istraživački i naučni rad i u druge svrhe se ne sme koristiti –

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(1) javno izvođenje ili predstavljanje objavljenih dela u obliku neposrednog poučavanja na nastavi;

- ZAKON O AUTORSKOM I SRODNIM PRAVIMA

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