



Procesiranje signala

Profesor dr Miroslav Lutovac

"This project has been funded with support from the European Commission. This publication [communication] reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein"

Sadržaj predmeta, Teorijska nastava

1. Uvodno predavanje. Upoznavanje sa planom i programom, ciljevima, ishodom i metodama.
2. Šta je procesiranje signala, istorijski pregled obrade signala, primeri primene.
3. Vizuelizacija signala (Python, Excel).
4. Kompleksni ekponencijalni diskretni signali. Primer sinteze muzičkog signala.
5. Furijeova analiza: Diskretna Furijeova transformacija (DFT) i serija (DFS). Brza Furijeova transformacija (Fast Fourier transform, FFT) i primena za spektralne analizatore i osciloskope.
6. Linearani filtri: konvolucija, idealni i realni filtri, dizajn filtra. Primena konvolucije u GPS sistemima.

...

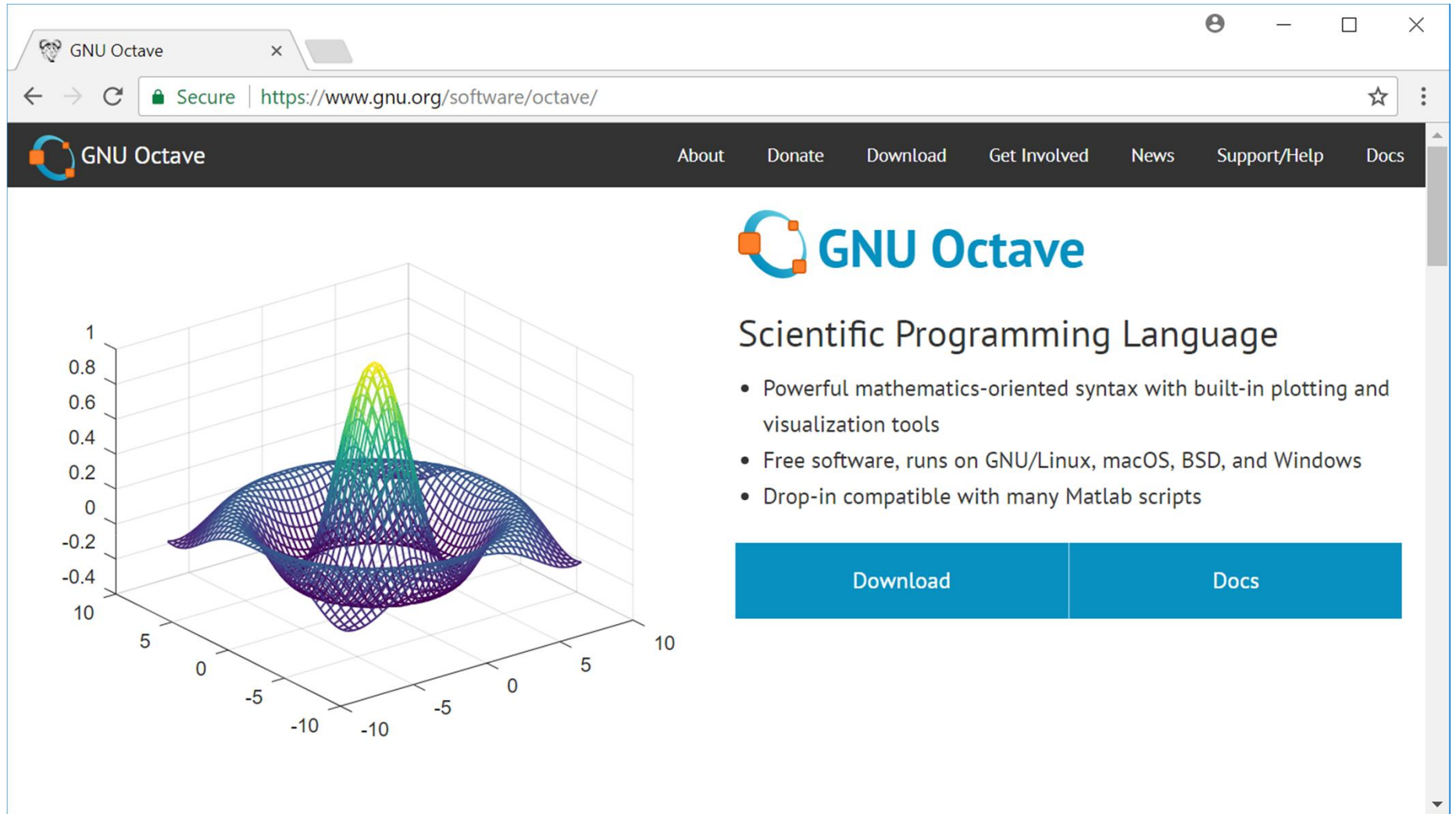
Sadržaj

- Spektralna analiza sinusoidalnih, nestacionarnih i slučajnih signala
- Procesiranje muzičkih signala
- Sinteza digitalnih muzičkih signala
- Kompresija signala
- Trans-multiplekseri
- Diskretni višetonski prenos digitalnih podataka
- Konvertori sa oversamplingom

Obrada muzičkih signala

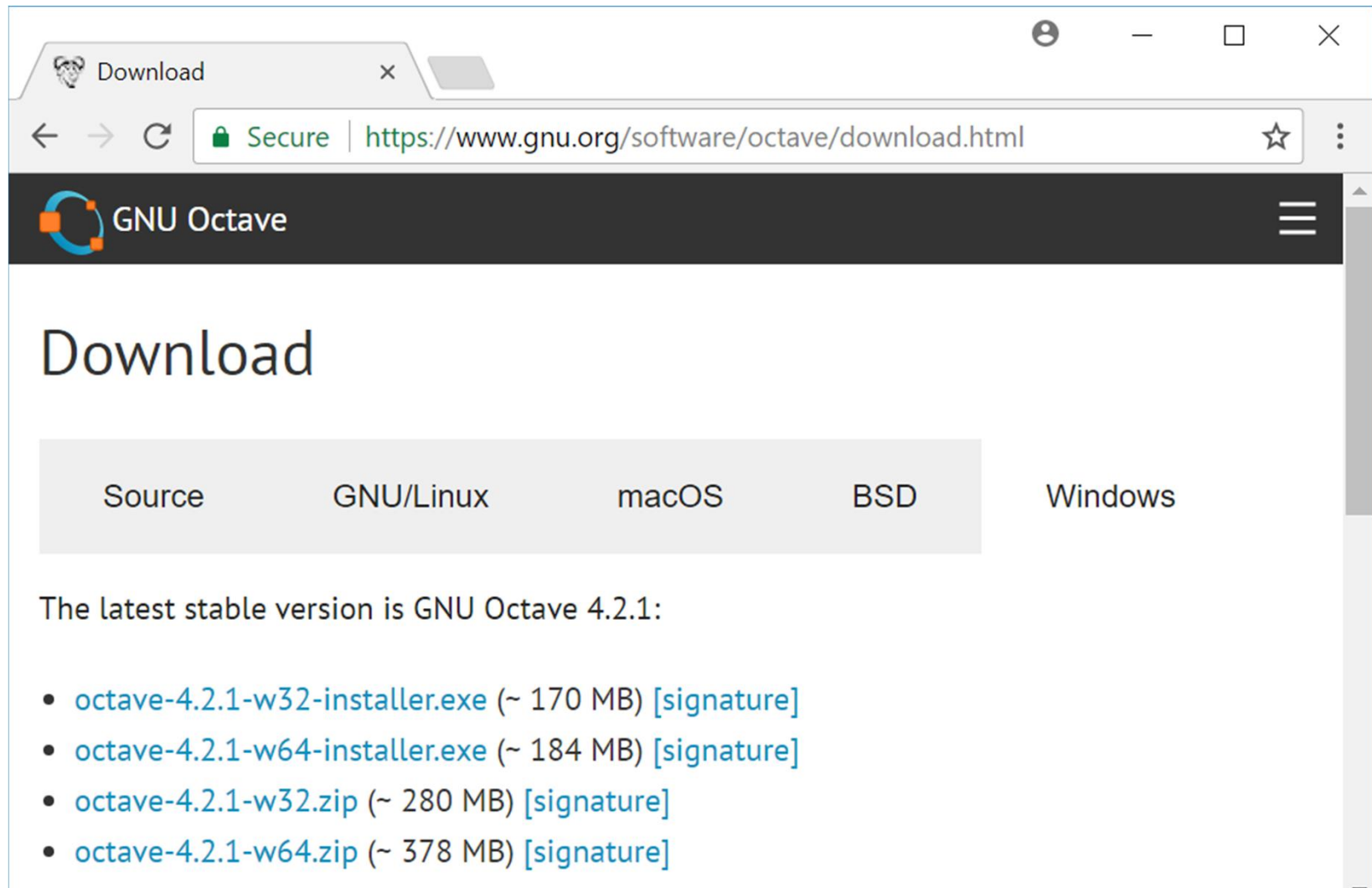
- Muzički signali se generišu u akustičkoj prostori za svaki instrument posebno i pamte se kao poseban zapis
- Dodaju se specijalni efekti
- Pravi se miks signala
- Efekti: eho, reverberacija

GNU Octave



<https://www.gnu.org/software/octave/>

GNU Octave

A screenshot of a web browser displaying the GNU Octave download page. The browser's address bar shows the URL https://www.gnu.org/software/octave/download.html. The page has a dark header with the GNU Octave logo and name. Below the header, the word "Download" is prominently displayed. A horizontal navigation bar contains five tabs: "Source", "GNU/Linux", "macOS", "BSD", and "Windows". The "Source" tab is currently selected. The main content area states that the latest stable version is GNU Octave 4.2.1 and lists four download options with their respective sizes and links to signatures.

Download

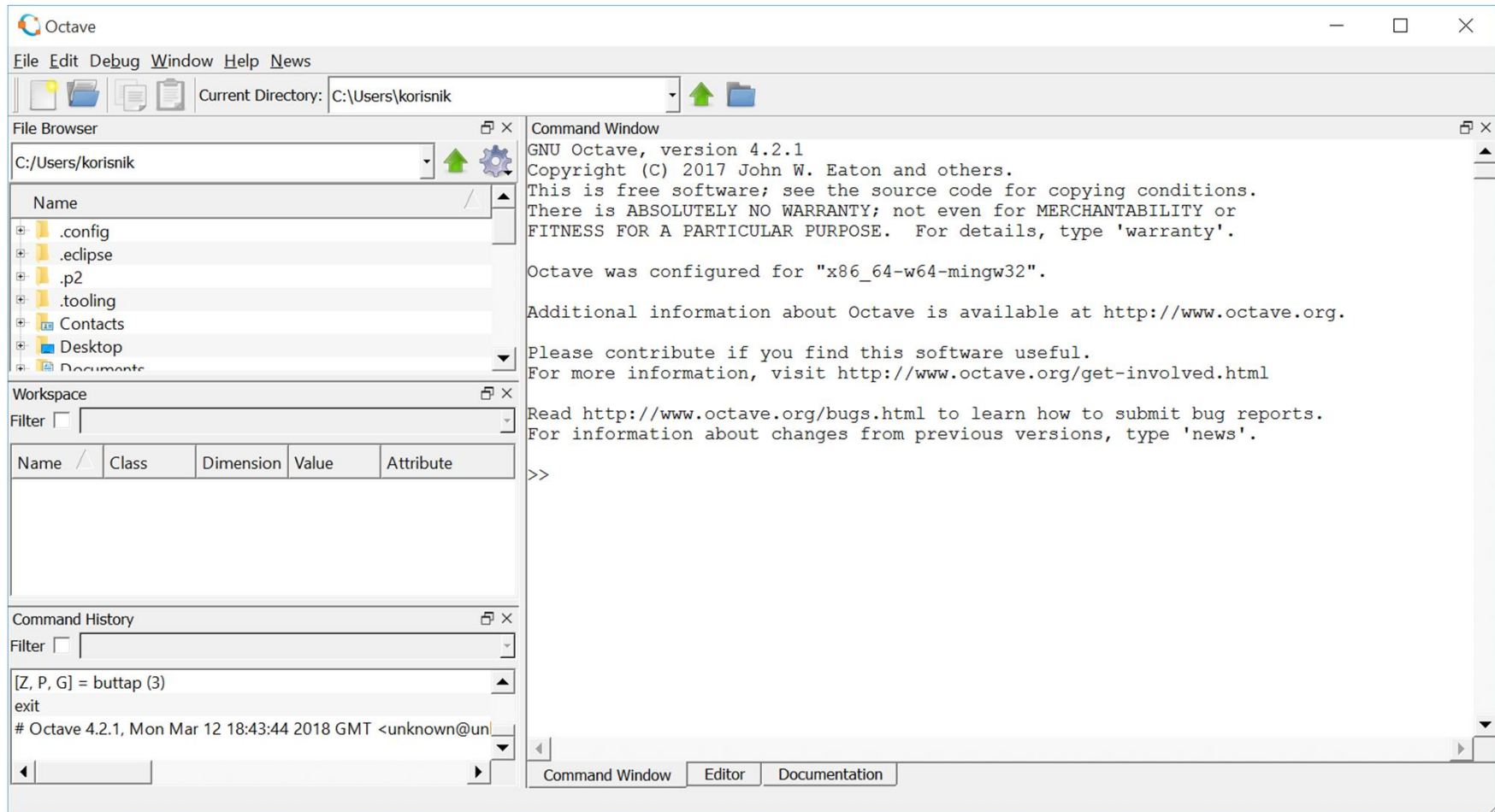
Source GNU/Linux macOS BSD Windows

The latest stable version is GNU Octave 4.2.1:

- [octave-4.2.1-w32-installer.exe](#) (~ 170 MB) [\[signature\]](#)
- [octave-4.2.1-w64-installer.exe](#) (~ 184 MB) [\[signature\]](#)
- [octave-4.2.1-w32.zip](#) (~ 280 MB) [\[signature\]](#)
- [octave-4.2.1-w64.zip](#) (~ 378 MB) [\[signature\]](#)

<https://www.gnu.org/software/octave/download.html>

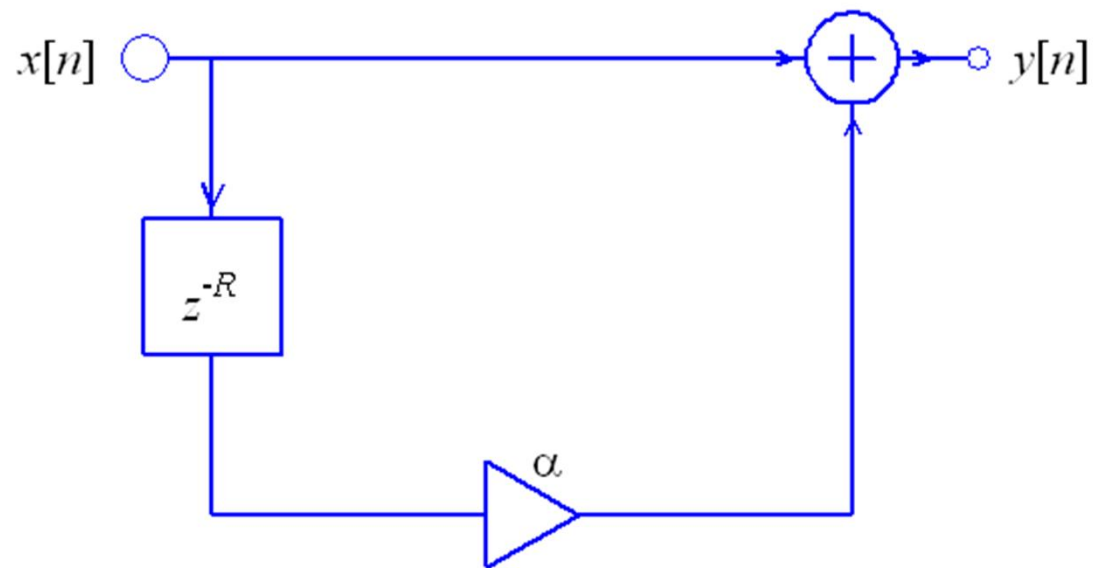
GNU Octave 4.2.1



Filtar za eho

- Eho se generiše kolima za kašnjenje
- Zakašnjeni signal je oslabljen
- Filtar se naziva **comb** filter

Program_echo (1)

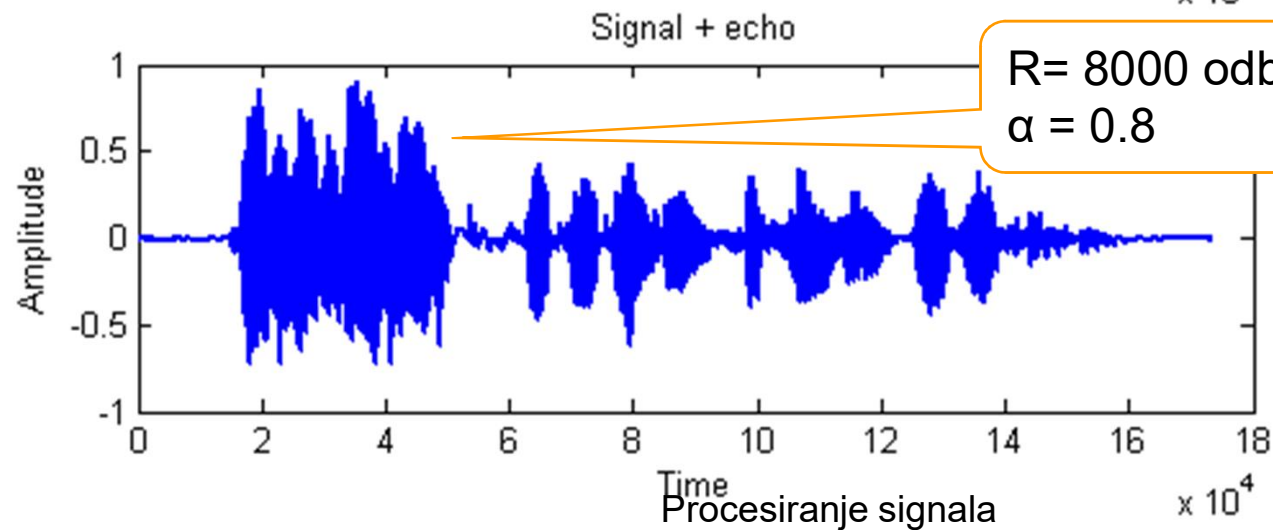
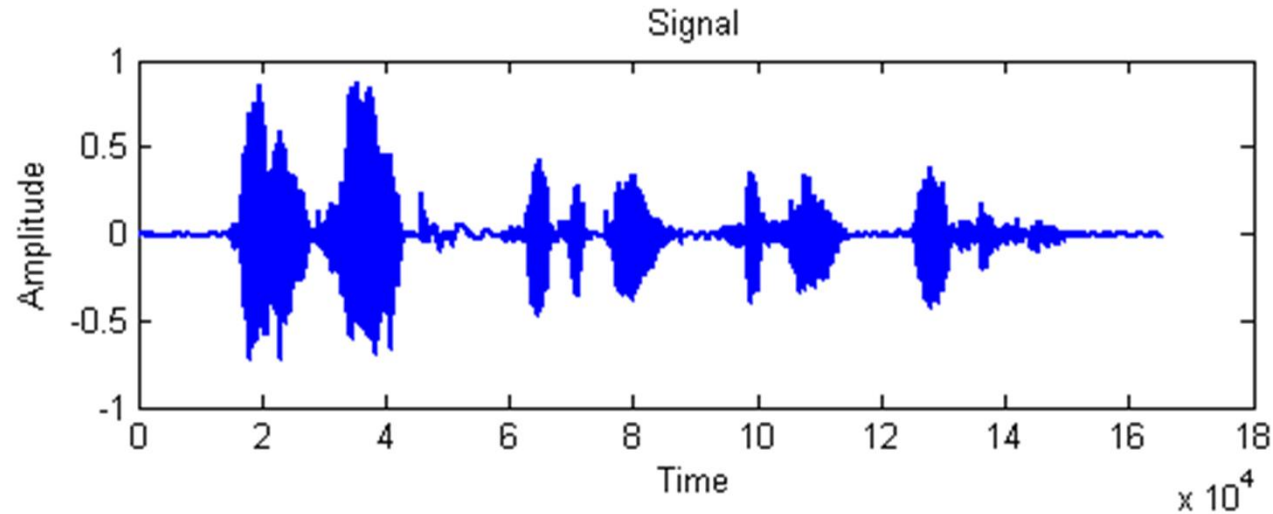


Program_echo (2)

```
[x,fs,nbits] = wavread('dsp01.wav');  
wavplay(x,fs);  
nDelay = 8000;  
aGain = 0.8;  
y = singleecho(x,nDelay,aGain);  
wavplay(y,fs);  
subplot(2,1,1)  
plot(1:length(x),x)  
xlabel('Time'); ylabel('Amplitude')  
title('Signal')  
subplot(2,1,2)  
plot(1:length(y),y)  
xlabel('Time'); ylabel('Amplitude')  
title('Signal + echo')
```

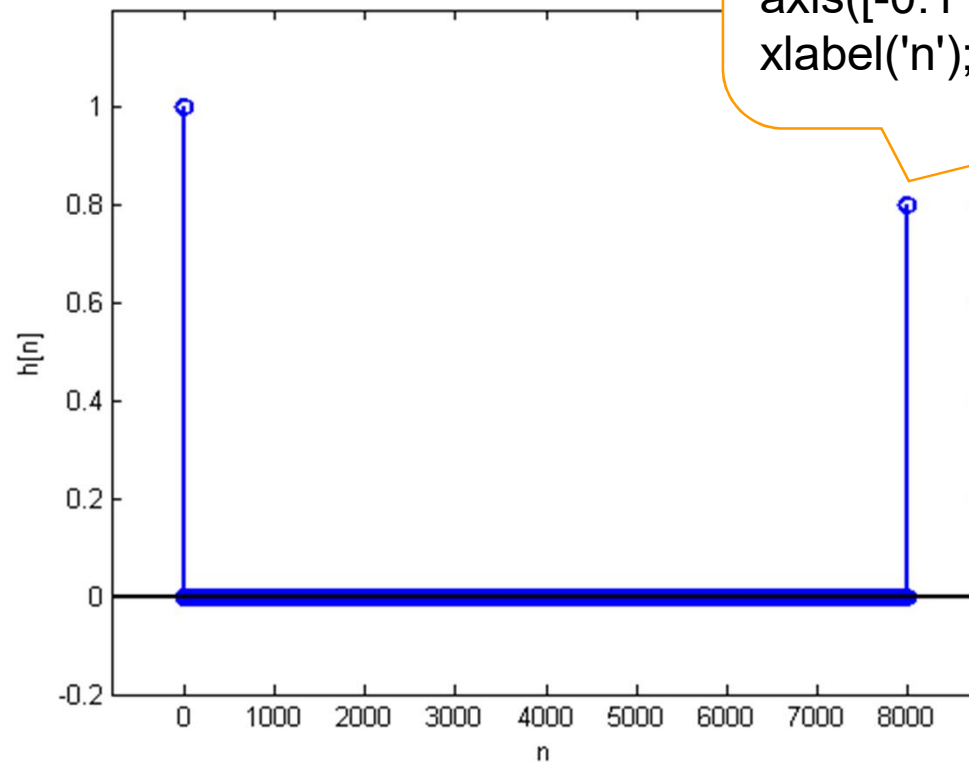
```
function y = singleecho(x, R, a);  
y =x;  
y(end+R) = 0;  
y(R+1:end) = y(R+1:end) + a*x;
```

Program_echo (3)



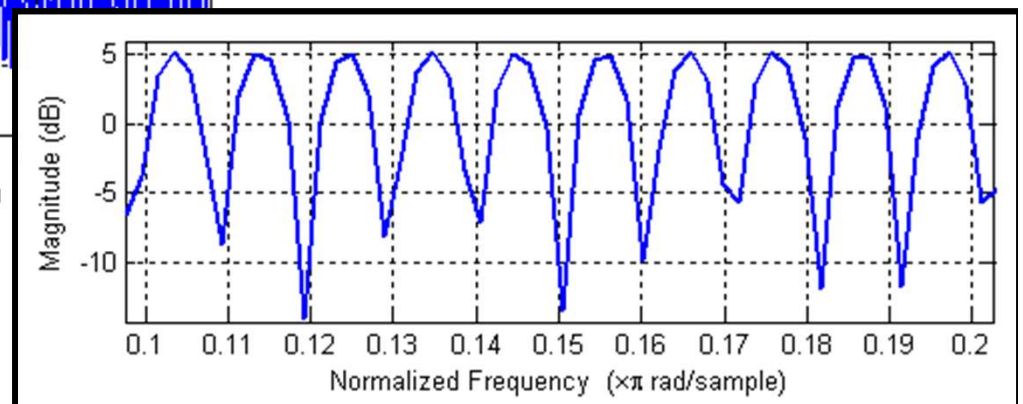
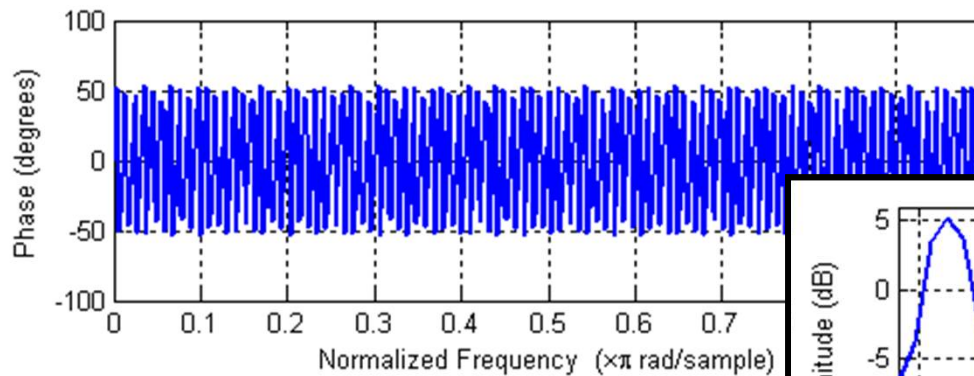
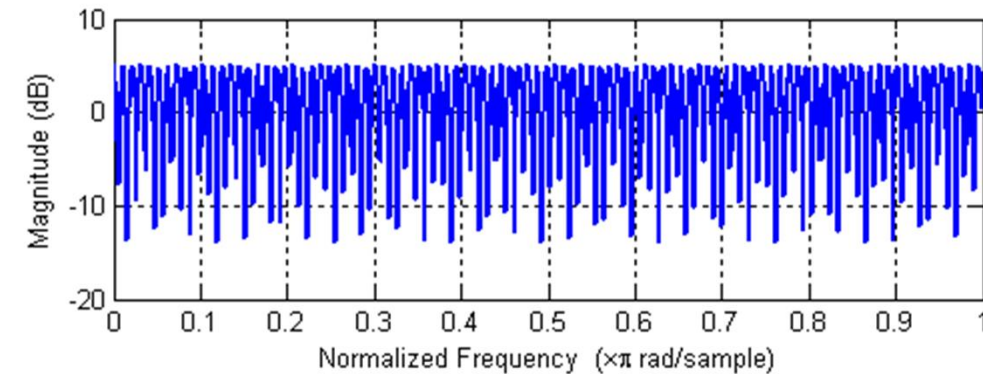
Program_echo (4)

```
h = 1;  
h(nDelay)=aGain;  
stem(h);  
axis([-0.1*nDelay 1.1*nDelay -0.2 1.2]);  
xlabel('n'); ylabel('h[n]')
```

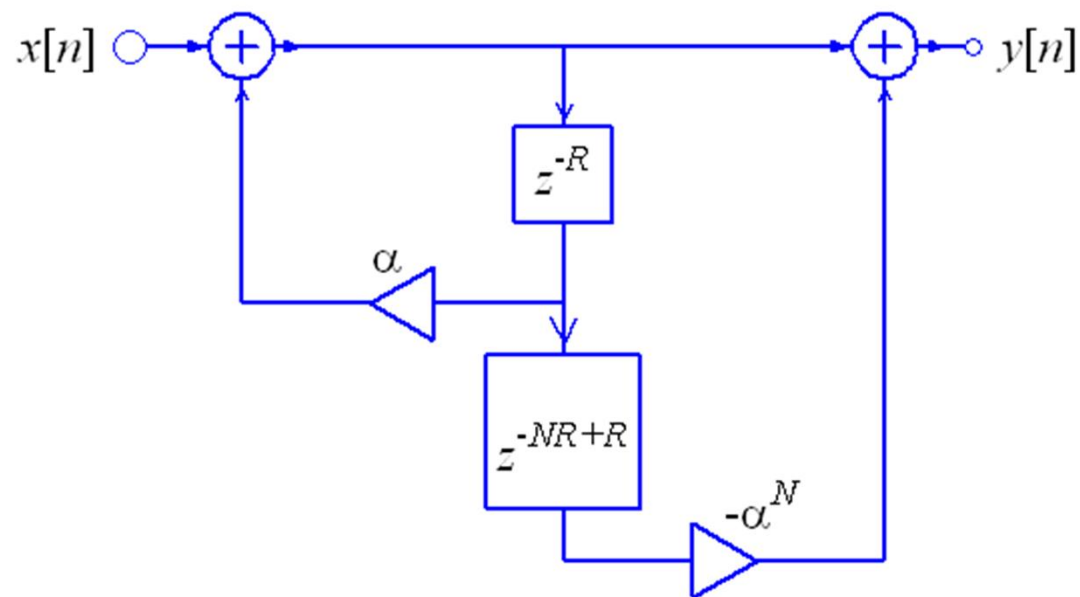


Program_echo (5)

figure
freqz(h);



Program_m_echo (1)

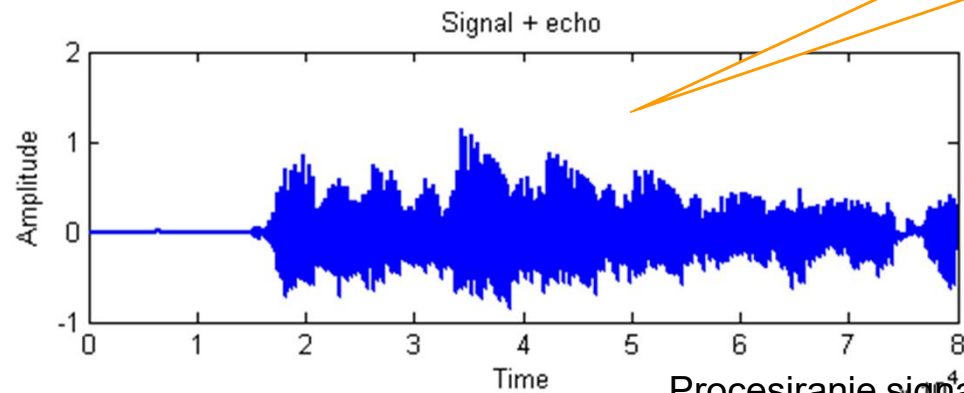
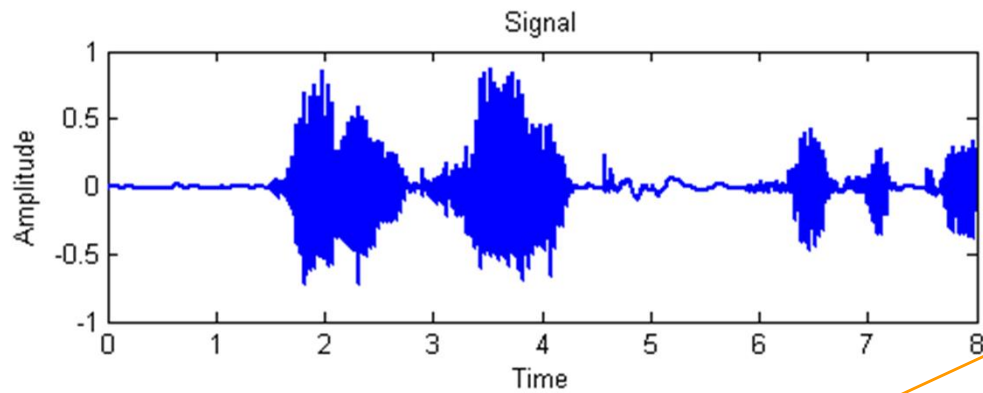


Program_m_echo (2)

```
[x,fs,nbits] = wavread('dsp01.wav');  
x(80000:end)=[];  
wavplay(x,fs);  
nDelay = 8000;  
nEcho = 3;  
aGain = 0.8;  
y = multiecho(x,nDelay,aGain,nEcho);  
wavplay(y,fs);  
subplot(2,1,1)  
plot(1:length(x),x)  
subplot(2,1,2)  
plot(1:length(y),y)
```

```
function y = multiecho(x,R,a,N);  
num = 1;  
if N > 0  
    num(N*R+1) = -a^N;  
end  
den = 1;  
den(R+1) = -a;  
y=filter(num,den,x);
```

Program_m_echo (3)

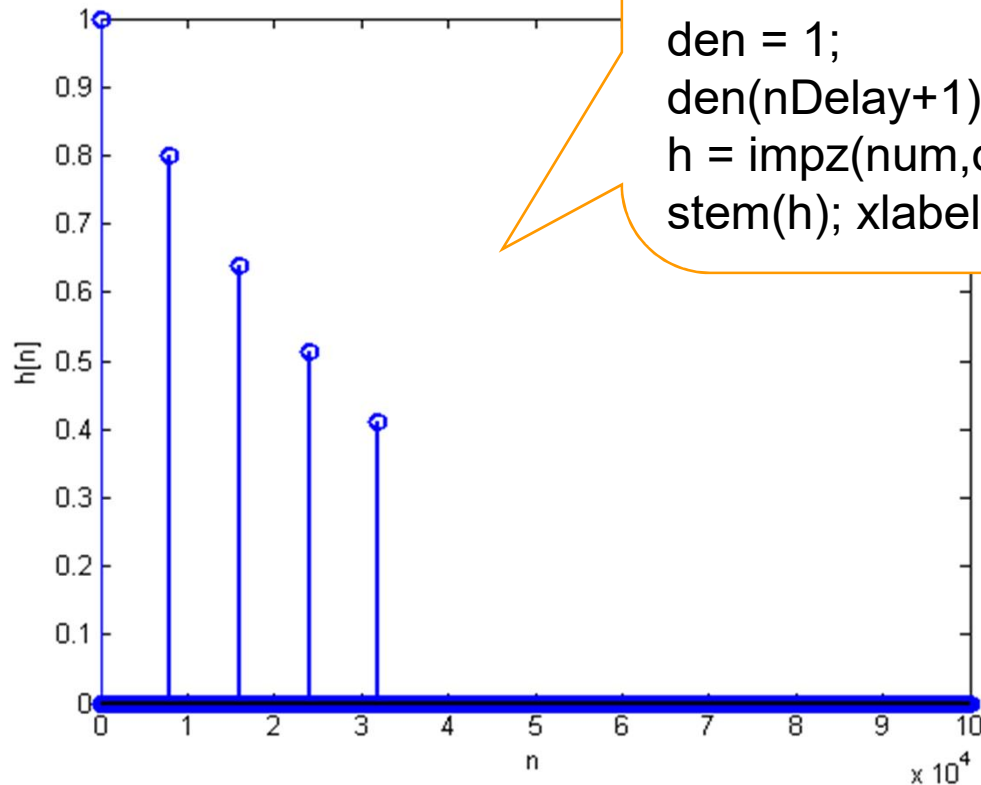


R= 8000 odbiraka
 $\alpha = 0.8$
N = 5

Procesiranje signala

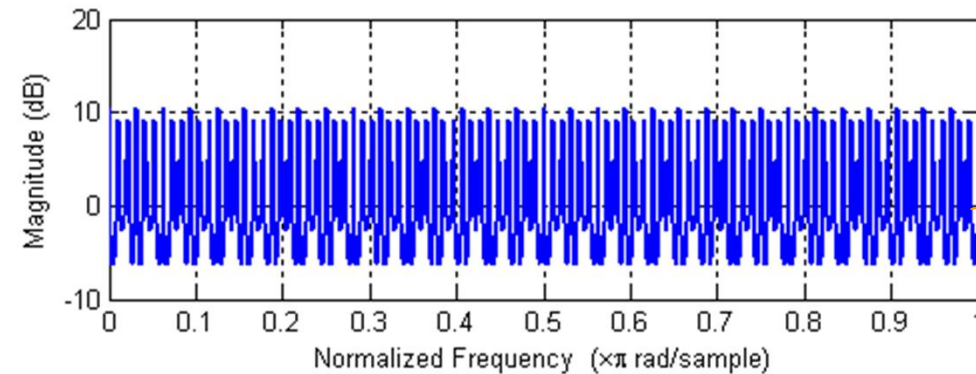
Program_m_echo (4)

```
num = 1;  
if nEcho > 0  
    num(nEcho*nDelay+1) = -aGain^nEcho;  
end  
den = 1;  
den(nDelay+1) = -aGain;  
h = impz(num,den,100000);  
stem(h); xlabel('n'); ylabel('h[n]')
```

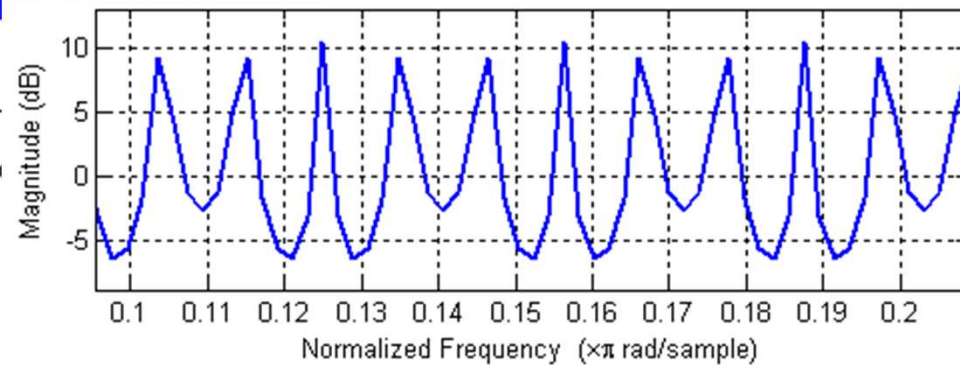
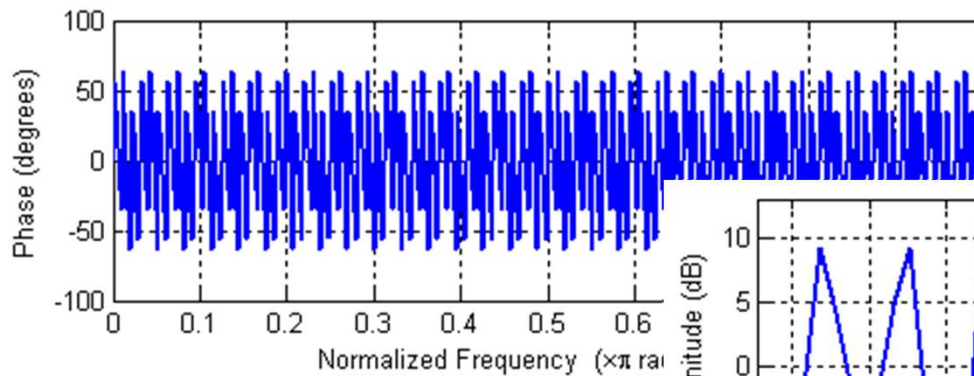


Procesiranje signala

Program_m_echo (5)



```
figure  
freqz(num,den);
```



Sadržaj

- Procesiranje muzičkih signala
- Sinteza digitalnih muzičkih signala
- Kompresija signala
- Trans-multiplekseri
- Discretni višetonski prenos digitalnih podataka
- Konvertori sa oversamplingom

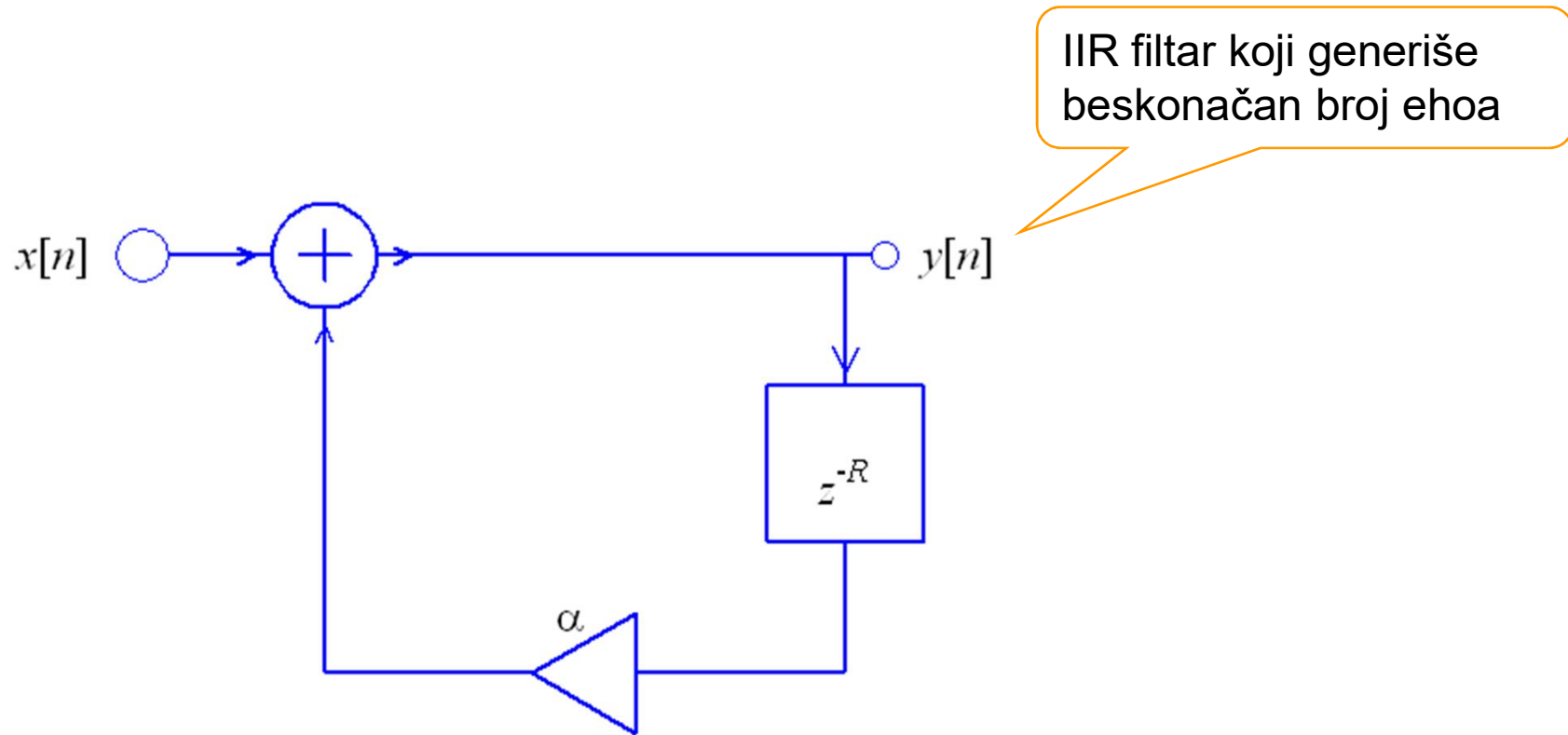
Reverberacija

- **Reverberacija** je niz uzastopnih sve slabijih refleksija koje prate direktan zvuk i koji se ne mogu međusobno razlučiti
- Echo ili odjek se čuju kao izdvojene refleksije – razlika od reverberacije
- Vreme reverberacije je vreme potrebno da akustička energija opadne po isključivanju izvora na milioniti deo prvobitne vrednosti (60 dB)
- Nagib krive opadanja zvuka u dB po jedinici vremena je konstantan
- Zavisi od veličine prostorije i ukupne apsorpcije
- Muzika na otvorenom prostoru, kada nema reverberacije, zvuči *suvo* i *prazno* a orkestar deluje *razbijeno*

Reverberacija i refleksija

- U zatvorenom prostoru, zvuk koji dolazi do slušaoca se sastoji od
 - (1) **direktnog zvuka**,
 - (2) **prve refleksije** i
 - (3) **reverberacije**
- Prve refleksije su one koje učestvuju u prvih 5 dB (10 dB) pada nivoa
- Zvuk koji se proizvodi u studiju ne zvuči ***prirodno***
- Digitalni filter se koristi da bi zvuk zvučao prirodno

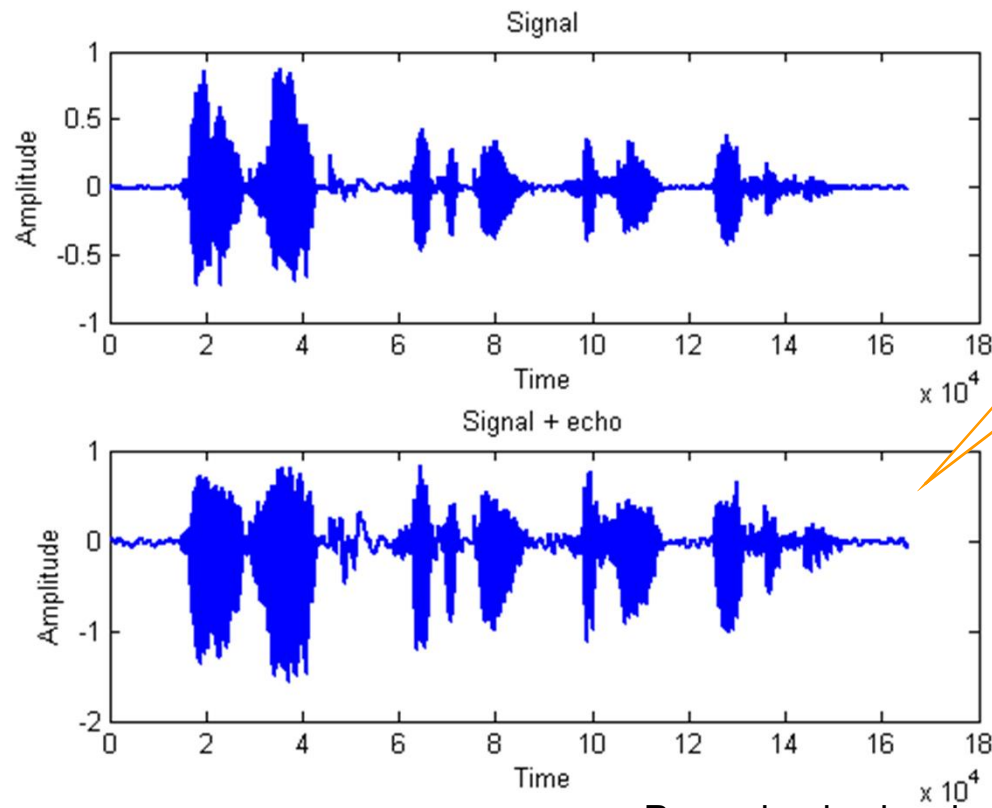
Beskonačan broj ehoa (1)



Beskonačan broj ehoa (2)

```
[x,fs,nbits] = wavread('dsp01.wav');  
wavplay(x,fs);  
nDelay = 10;  
aGain = 0.8;  
nReverb = nDelay*(log(1/1000)/log(aGain));  
num = 1;  
den = 1;  
den(nDelay+1) = -aGain;  
y = filter(num,den,x);  
wavplay(y,fs);  
subplot(2,1,1)  
plot(1:length(x),x)  
xlabel('Time'); ylabel('Amplitude')  
title('Signal')  
subplot(2,1,2)  
plot(1:length(y),y)  
xlabel('Time'); ylabel('Amplitude')  
title('Signal + echo')
```

Beskonačan broj ehoa (3)

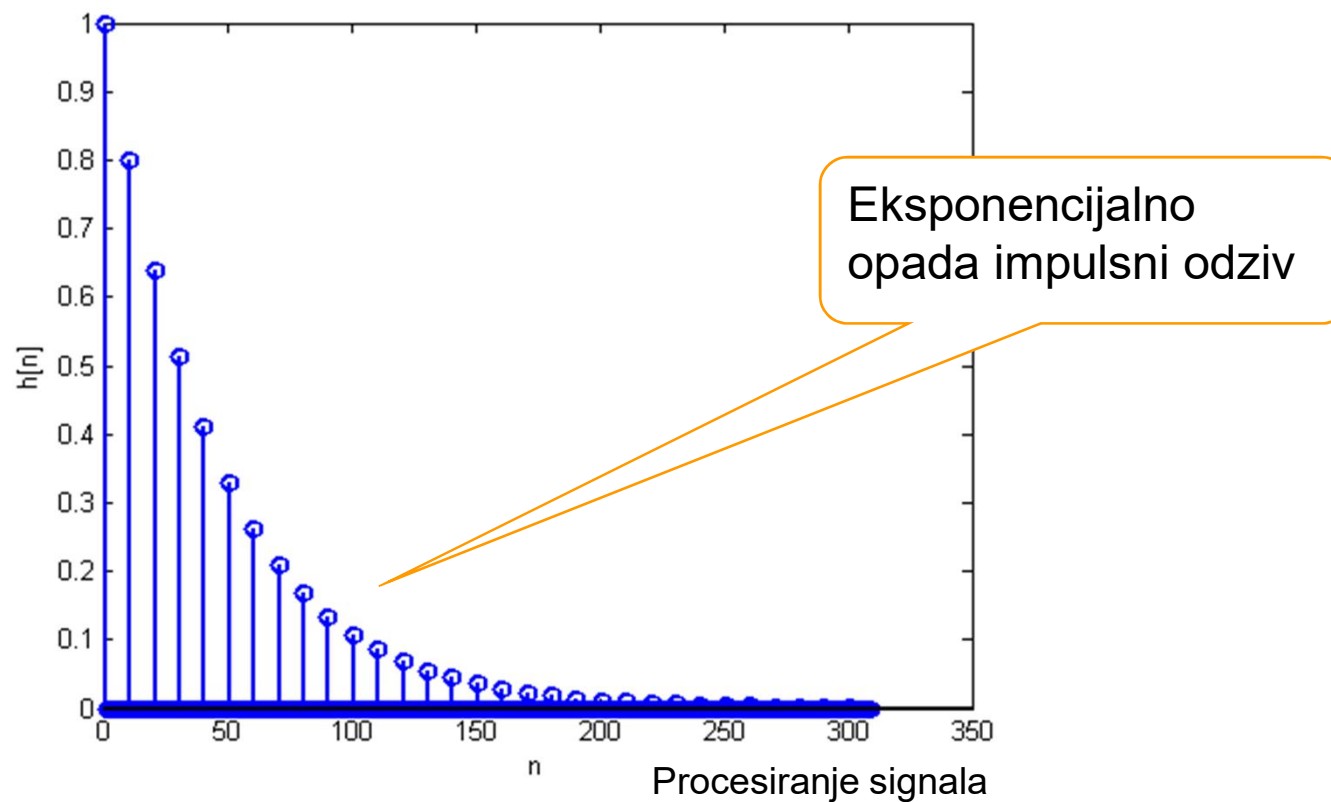


R= 10 odbiraka
 $\alpha = 0.8$

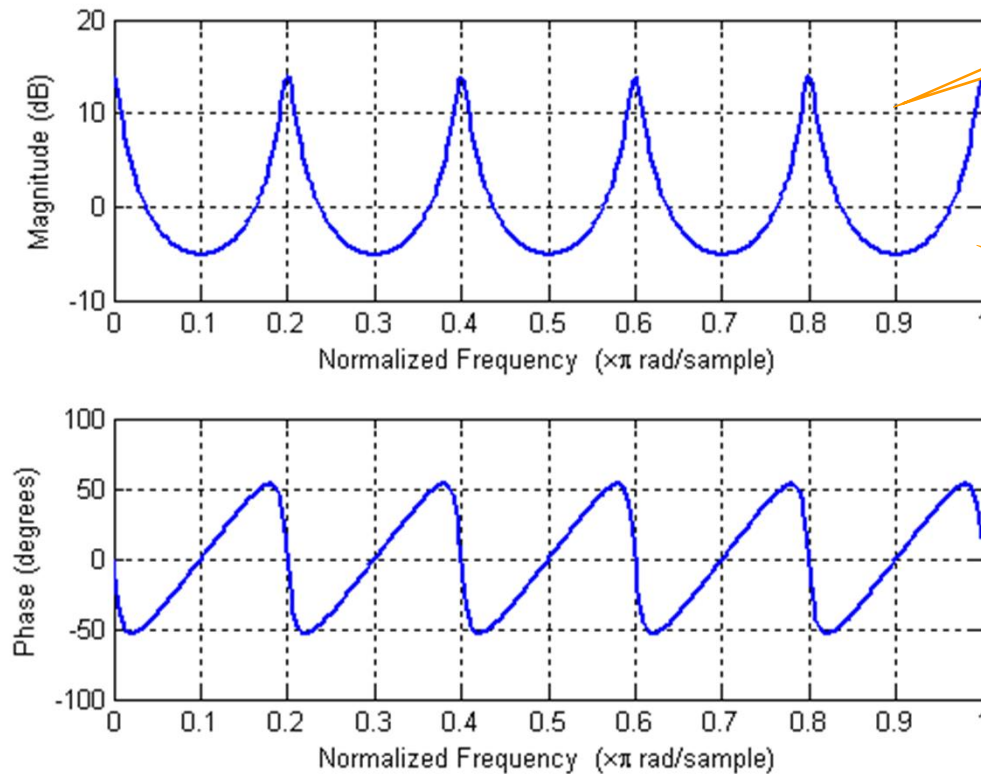
Prirodan zvuk

Procesiranje signala

Beskonačan broj ehoa (4)



Beskonačan broj ehoa (5)



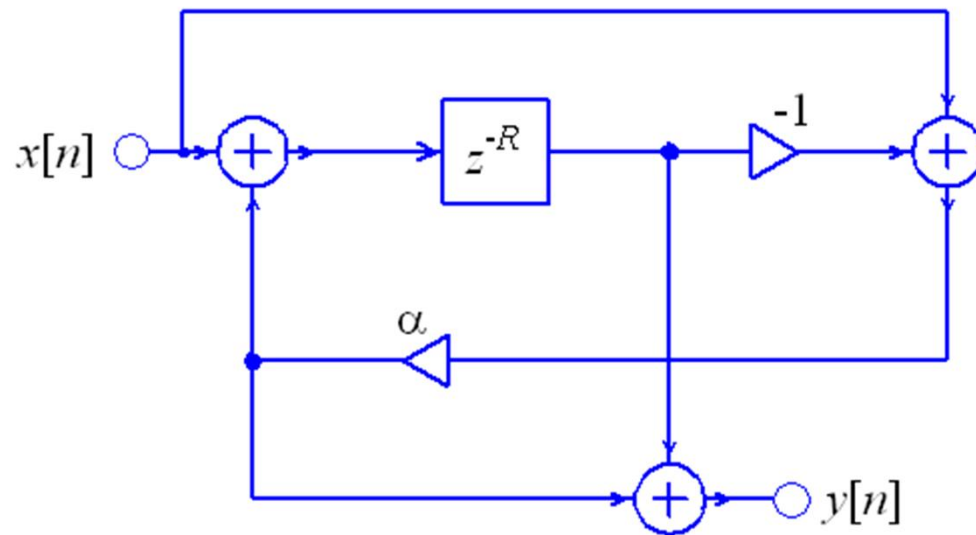
Frekvencijska
karakteristika

Spektar je obojen,
amplitudska karakteristika
nije ravna

Procesiranje signala

Allpass reverberator (1)

IIR allpass reverberator

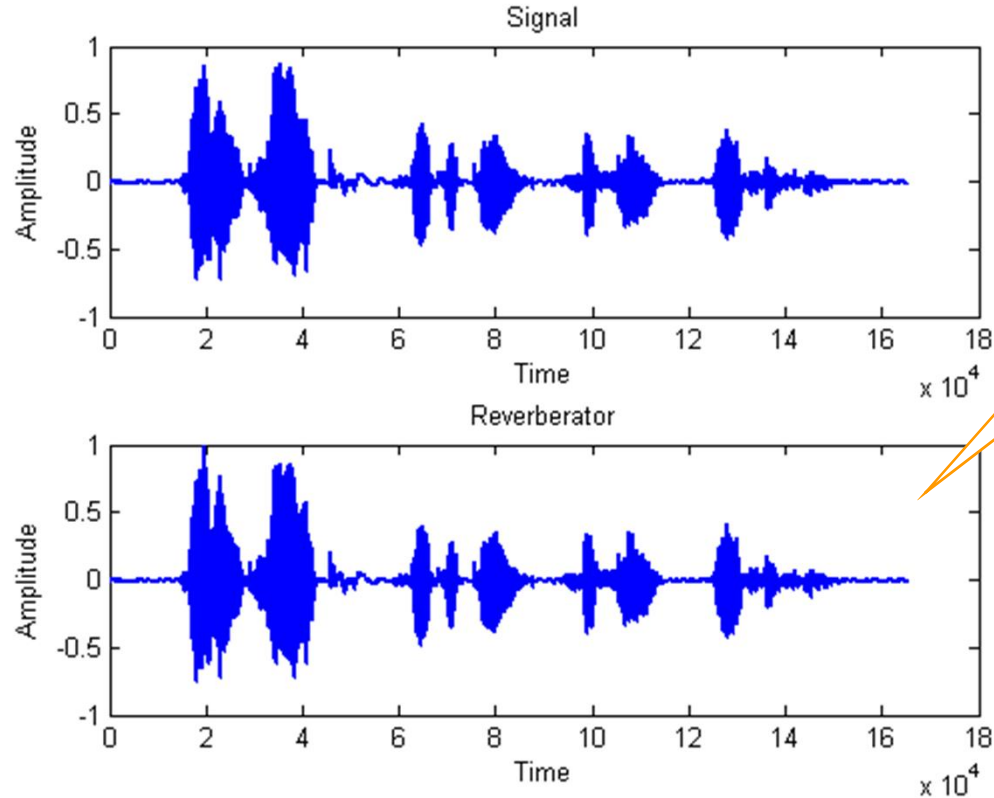


Allpass reverberator (2)

```
[x,fs,nbits] = wavread('dsp01.wav');  
wavplay(x,fs);  
nDelay = 4;  
aGain = 0.8;  
nReverb = nDelay*(log(1/1000)/log(aGain));  
num = aGain;  
num(nDelay+1) = 1;  
den = 1;  
den(nDelay+1) = aGain;  
y = filter(num,den,x);  
wavplay(y,fs);  
subplot(2,1,1)  
plot(1:length(x),x)  
xlabel('Time'); ylabel('Amplitude')  
title('Signal')  
subplot(2,1,2)  
plot(1:length(y),y)  
xlabel('Time'); ylabel('Amplitude')  
title('Reverberator')
```

Procesiranje signala

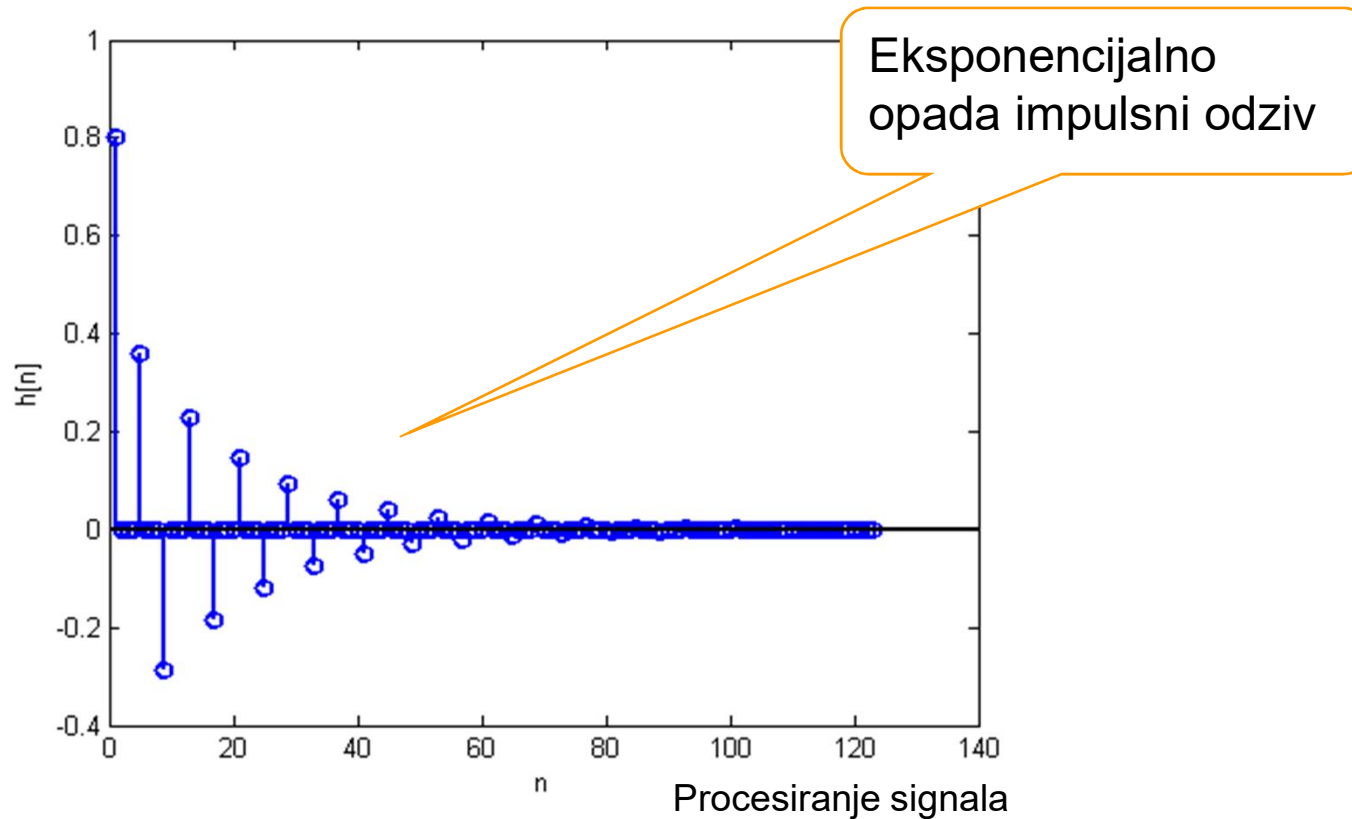
Allpass reverberator (3)



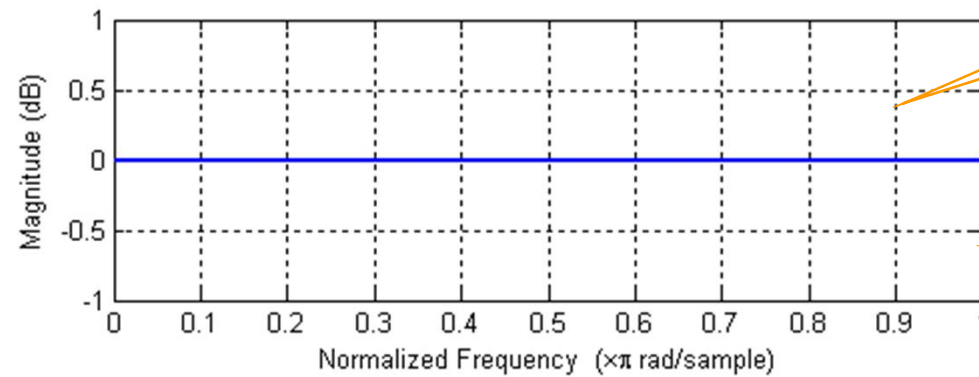
R= 4 odbiraka
 $\alpha = 0.8$

Procesiranje signala

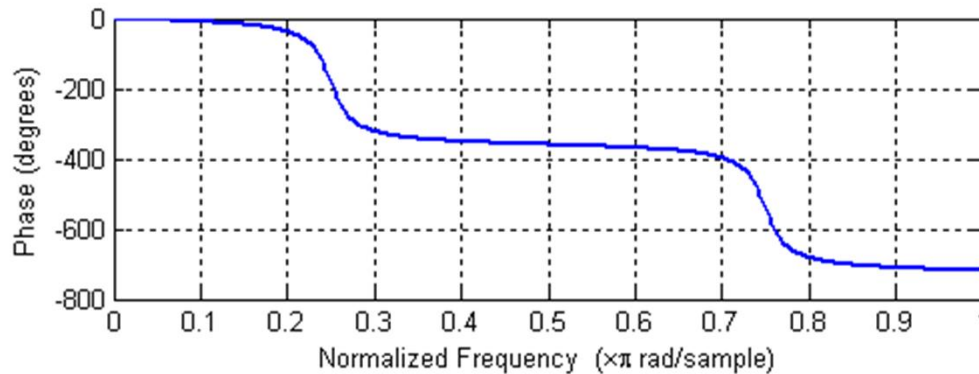
Allpass reverberator (4)



Allpass reverberator (5)



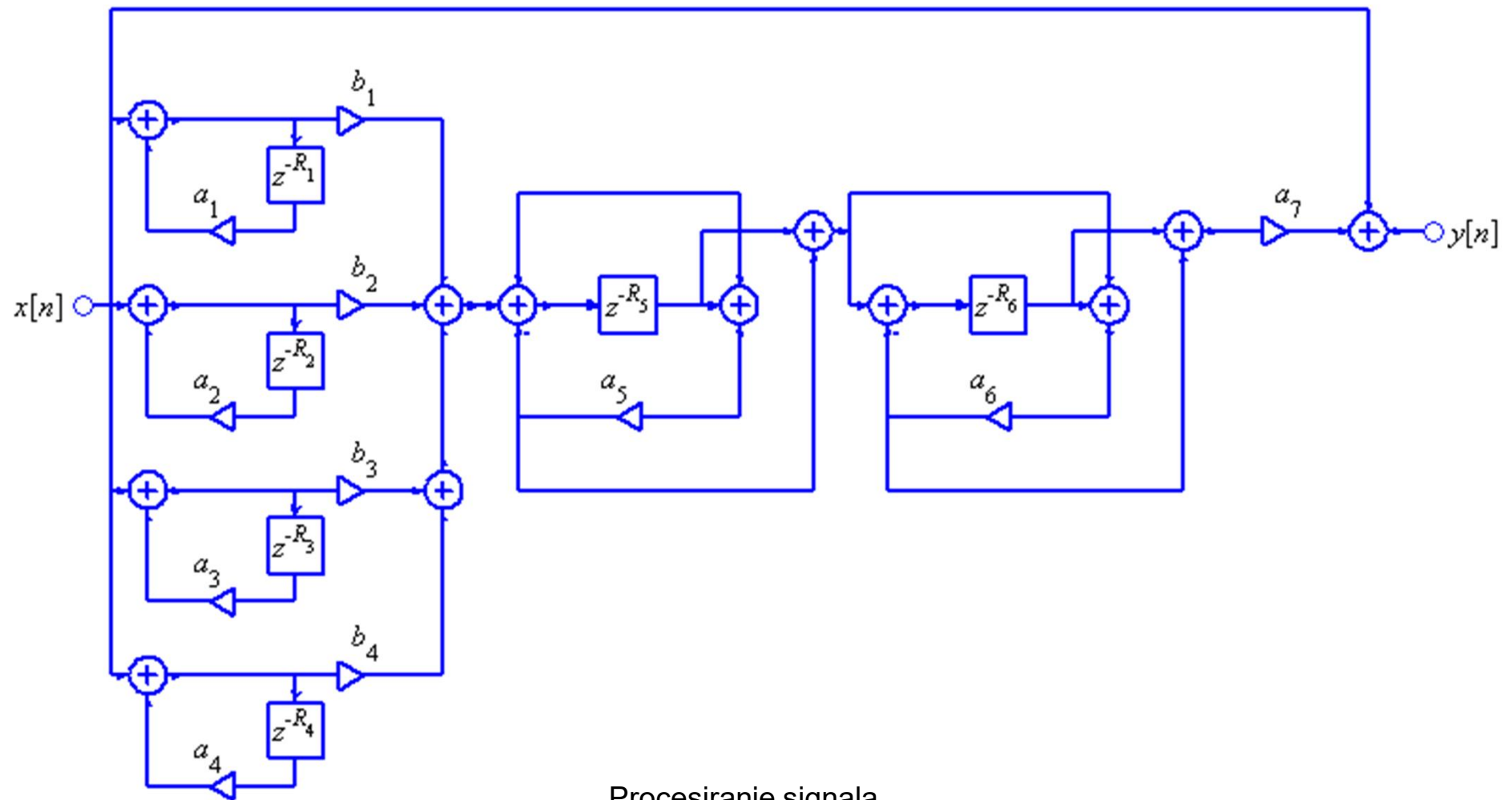
Frekvencijska
karakteristika



Spektar je ravan,
amplitudska karakteristika je
ravna

Procesiranje signala

Eho + allpass reverberator (1)



Procesiranje signala

Eho + allpass reverberator (2)

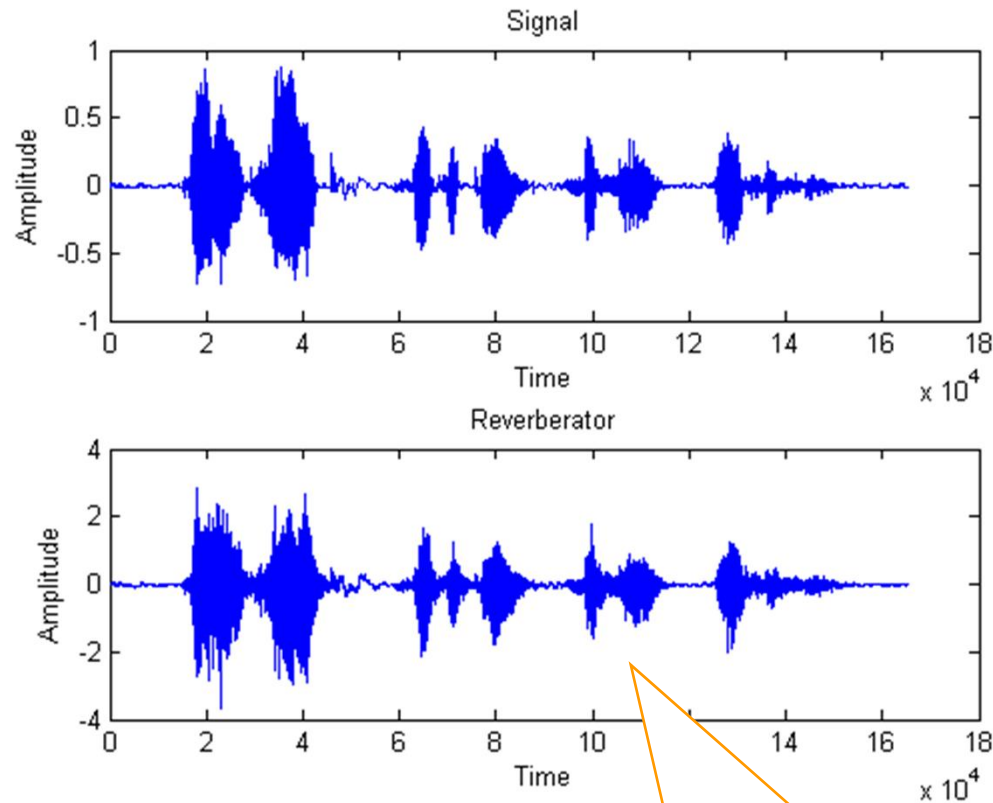
```
a = [0.6 0.4 0.2 0.1 0.7 0.6 0.8];  
R = [700 900 600 400 450 390];  
[x,fs,nbits] = wavread('dsp01.wav');  
wavplay(x,fs);  
pause (0.1)  
  
d1 = multiecho(x, R(1), a(1), 0);  
d2 = multiecho(x, R(2), a(2), 0);  
d3 = multiecho(x, R(3), a(3), 0);  
d4 = multiecho(x, R(4), a(4), 0);  
d_IIR = d1 + d2 + d3 + d4; %output of IIR echo  
generators  
d_ALL1 = alpas(d_IIR, R(5), a(5));  
d_ALL2 = alpas(d_ALL1, R(6), a(6));  
y = x + a(7)*d_ALL2;
```

Eho + allpass reverberator (3)

```
x = 0*x; x(1)=1;
d1 = multiecho(x, R(1), a(1), 0);
d2 = multiecho(x, R(2), a(2), 0);
d3 = multiecho(x, R(3), a(3), 0);
d4 = multiecho(x, R(4), a(4), 0);
d_IIR = d1 + d2 + d3 + d4; %output of IIR echo
generators
d_ALL1 = alpas(d_IIR, R(5), a(5));
d_ALL2 = alpas(d_ALL1, R(6), a(6));
y = x + a(7)*d_ALL2;

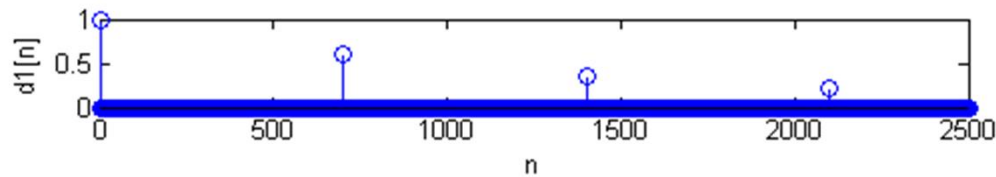
freqz(y,1);
```

Eho + allpass reverberator (4)

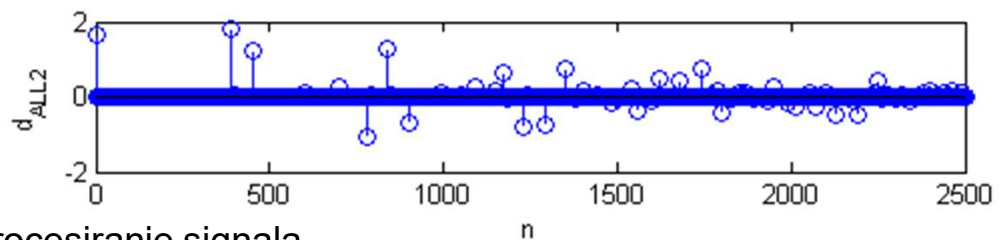
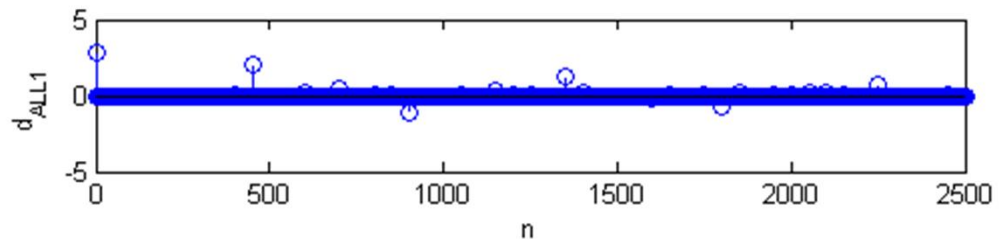
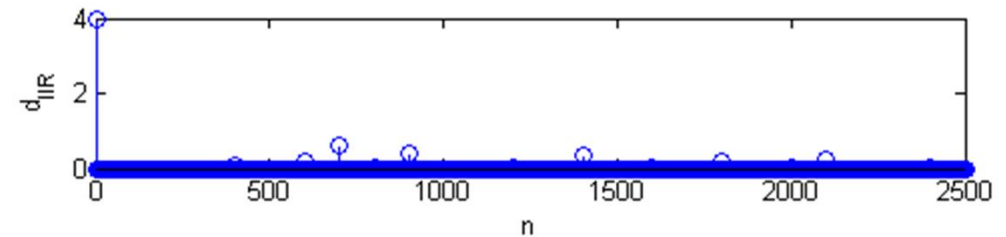
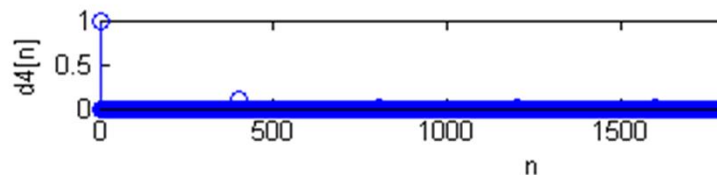
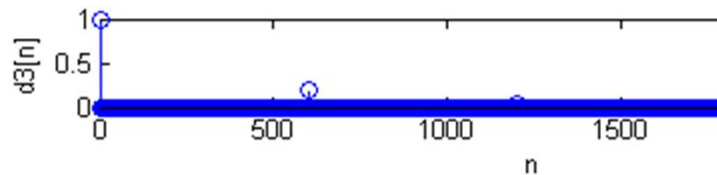
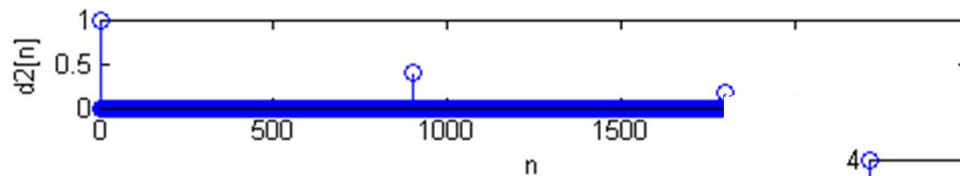


```
a = [0.6  0.4  0.2  0.1  0.7  0.6  0.8];  
R = [700  900  600  400  450  390];  
Procesiranje signala
```

Eho + allpass reverberator (5)

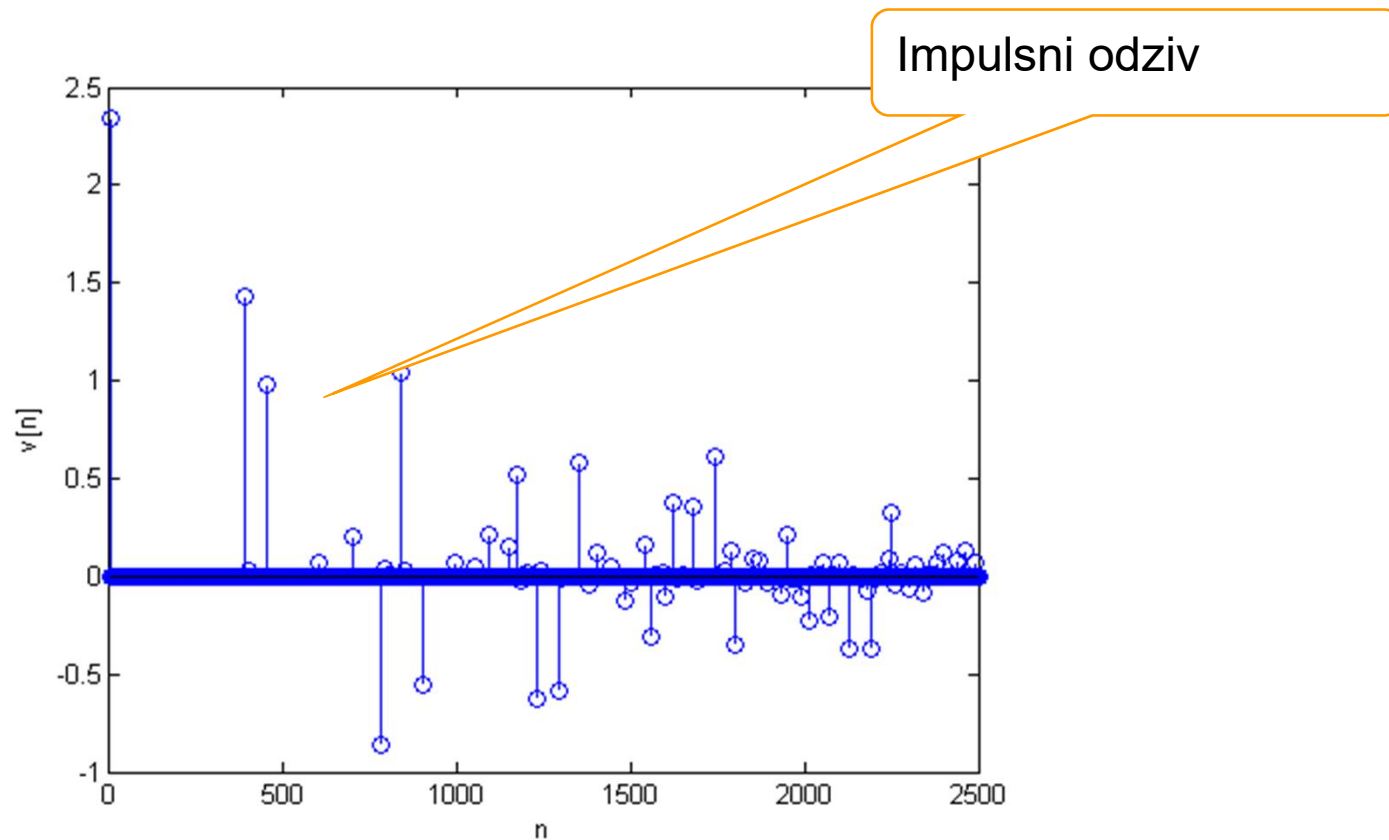


Eksponencijalno
opada impulsni odziv

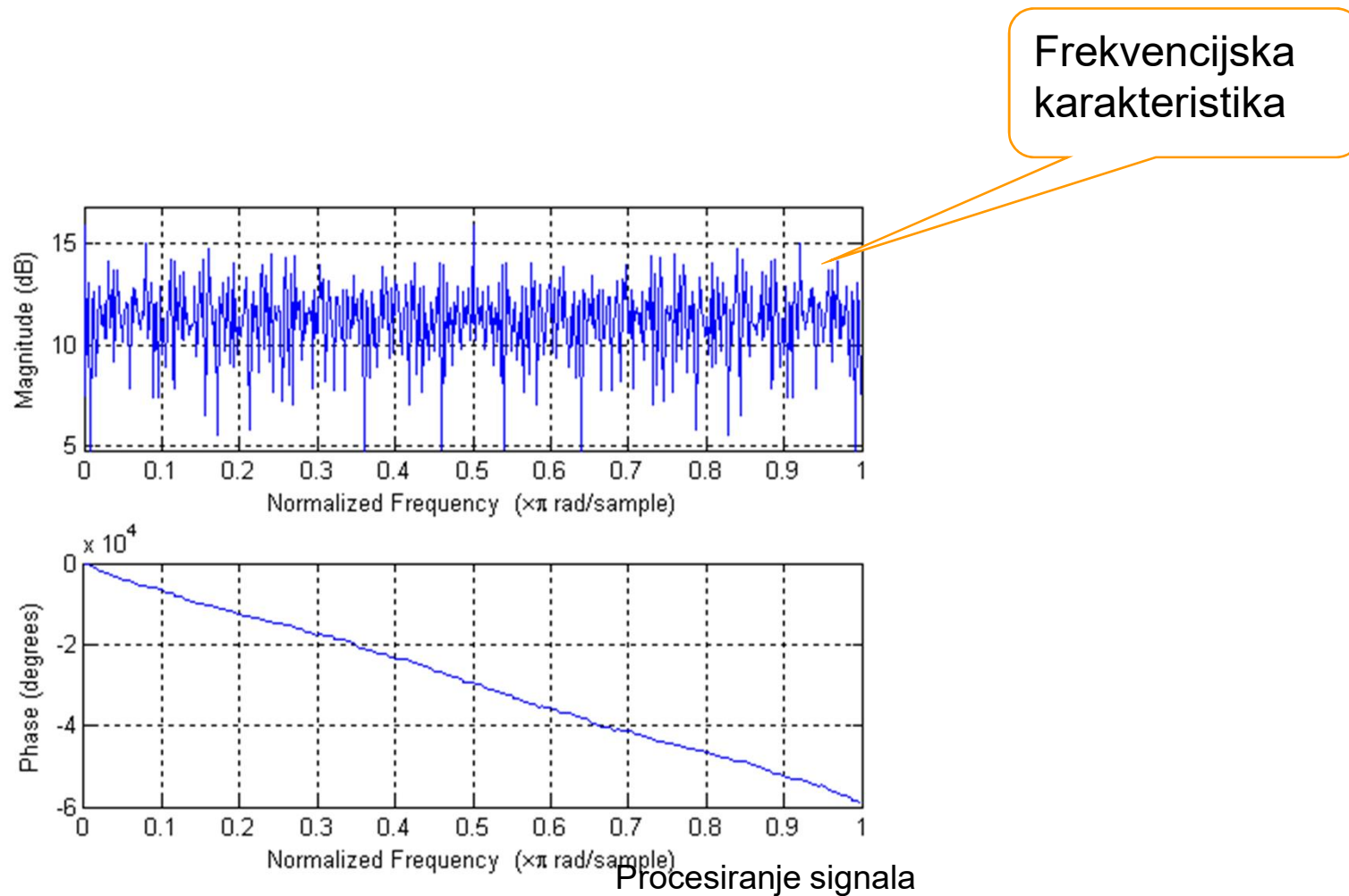


Procesiranje signala

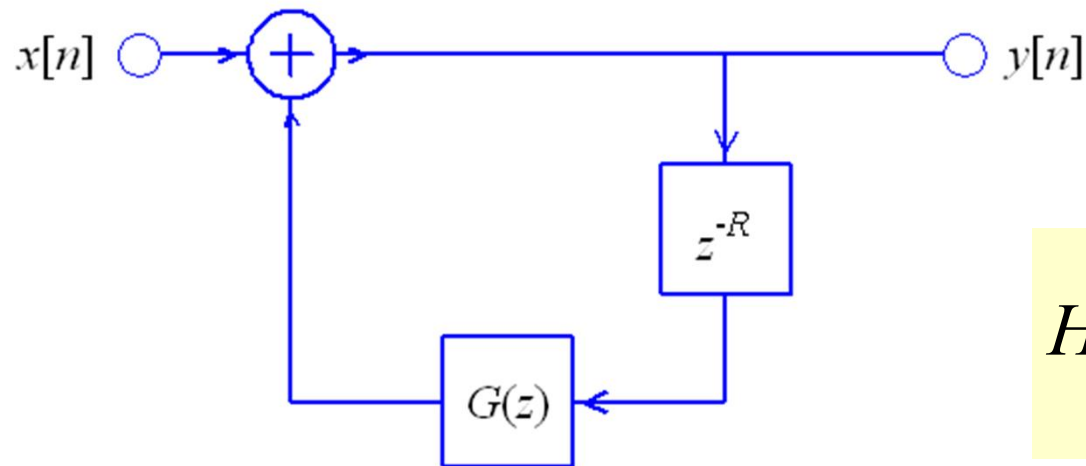
Eho + allpass reverberator (6)



Eho + allpass reverberator (7)



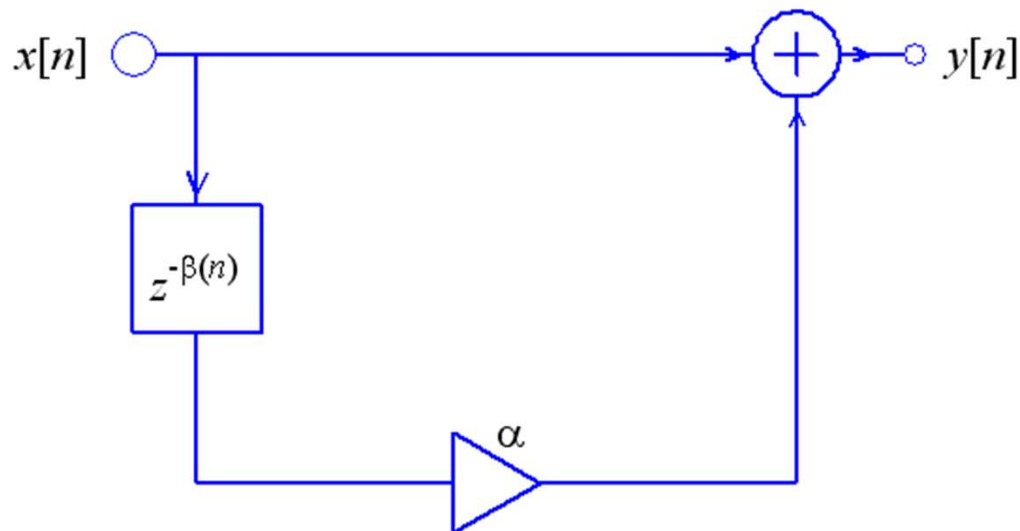
Lowpass reverberator



$$H(z) = \frac{1}{1 - z^{-R}G(z)}$$

Flanging efekt

- Flanging efekt se pravi tako što se isti signal dovodi na dva izvora a zatim menja vreme kašnjenja između dva signala
- Promena vremena kašnjenja je kontinualna od 0 do R
- Pošto kašnjenje ne mora da bude ceo broj, primenjuje se interpolacija za izračunavanje vrednosti signala



$$y[n] = x[n] + \alpha x[n - \beta(n)]$$

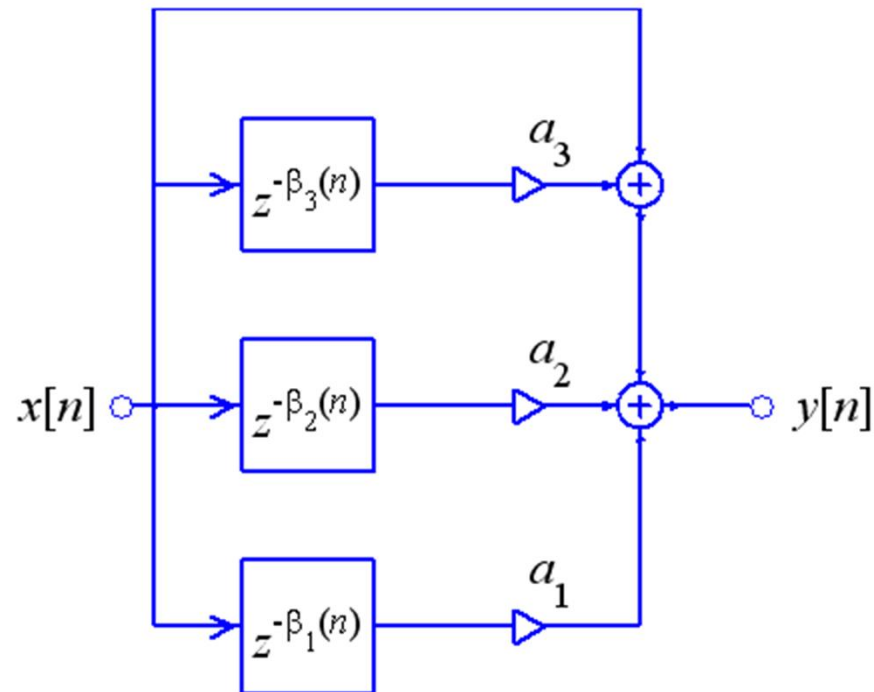
$$\beta(n) = \frac{R}{2} (1 - \cos(\omega_0 n))$$

Flanging efekat

```
clear all, close all, clc
[x,fs,nbits] = wavread('dsp01.wav');
wavplay(x,fs);
y = flang(x,1000,0.5,2*pi*6,fs);
wavplay(y,fs);
subplot(2,1,1)
plot(1:length(x),x)
xlabel('Time'); ylabel('Amplitude')
title('Signal')
subplot(2,1,2)
plot(1:length(y),y)
xlabel('Time'); ylabel('Amplitude')
title('Flang')
```

Chorus efekt

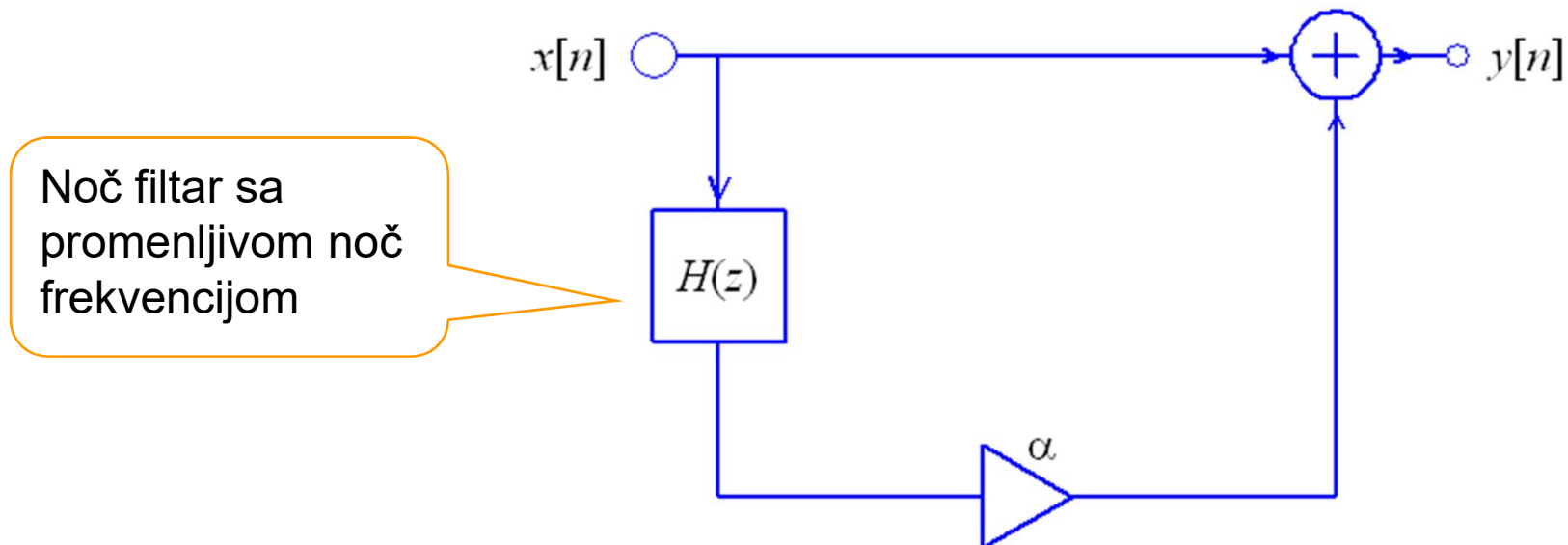
- Chorus efekt se dobija kada nekoliko muzičara svira isti instrument u isto vreme ali sa malim razlikama u amplitudi i vremenskom razlikom
- Ovaj efekt može da proizvede jedan muzičar pomoću digitalnog filtra, na slici, kao da svira 4 muzičara
- Kašnjenje je kontinualno, slučajno i sporo promenljivo



Procesiranje signala

Phasing efekat

- Phasing efekat se proizvodi kada se signal propusti uskopojasni noč filter (ima beskonačno slabljenje na jednoj učestanosti) a zatim dodaje originalnom signalu
- Noč učestanost i propusni opseg se sporo menjaju



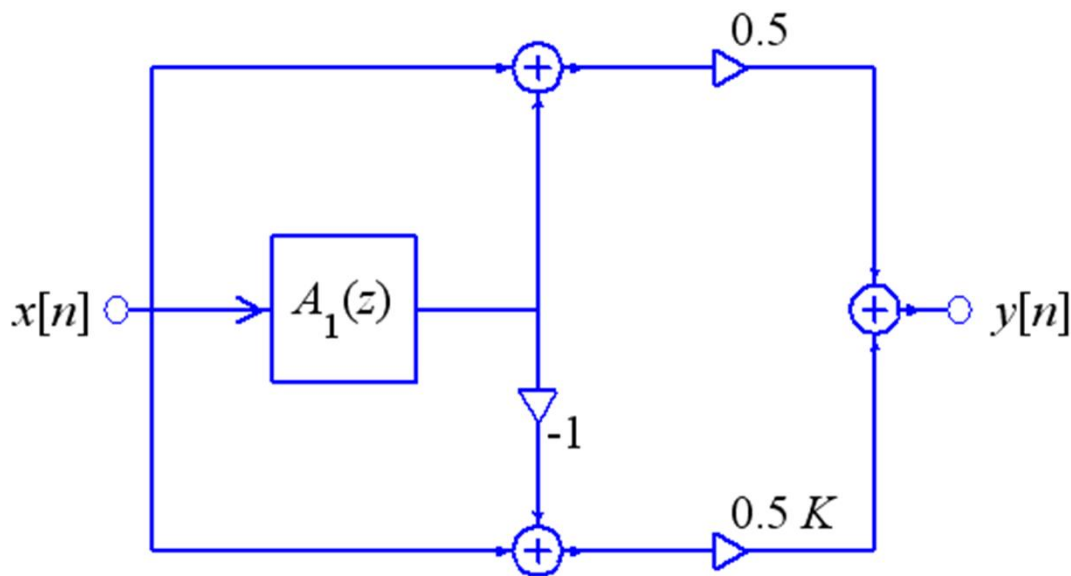
Procesiranje signala

Ekvilajzer

- Ekvilajzer modifikuje frekvencijsku karakteristiku tako da pojačava ili slabi sinusne signale na određenim učestanostima
- (a) filtri propusnici svih učestanosti (allpass),
(b) izlazni signali allpass filtara se množe konstantom,
(c) sabiraju se signali sa ulaznim signalom

Ekvilajzer I reda LP

$$G_{LP}(z) = \frac{K}{2}(1 - A_1(z)) + \frac{1}{2}(1 + A_1(z))$$



$$A_1(z) = \frac{a_C - z^{-1}}{1 - a_C z^{-1}}$$

$$a_C = \frac{K - \tan(\omega_C T / 2)}{K + \tan(\omega_C T / 2)}$$

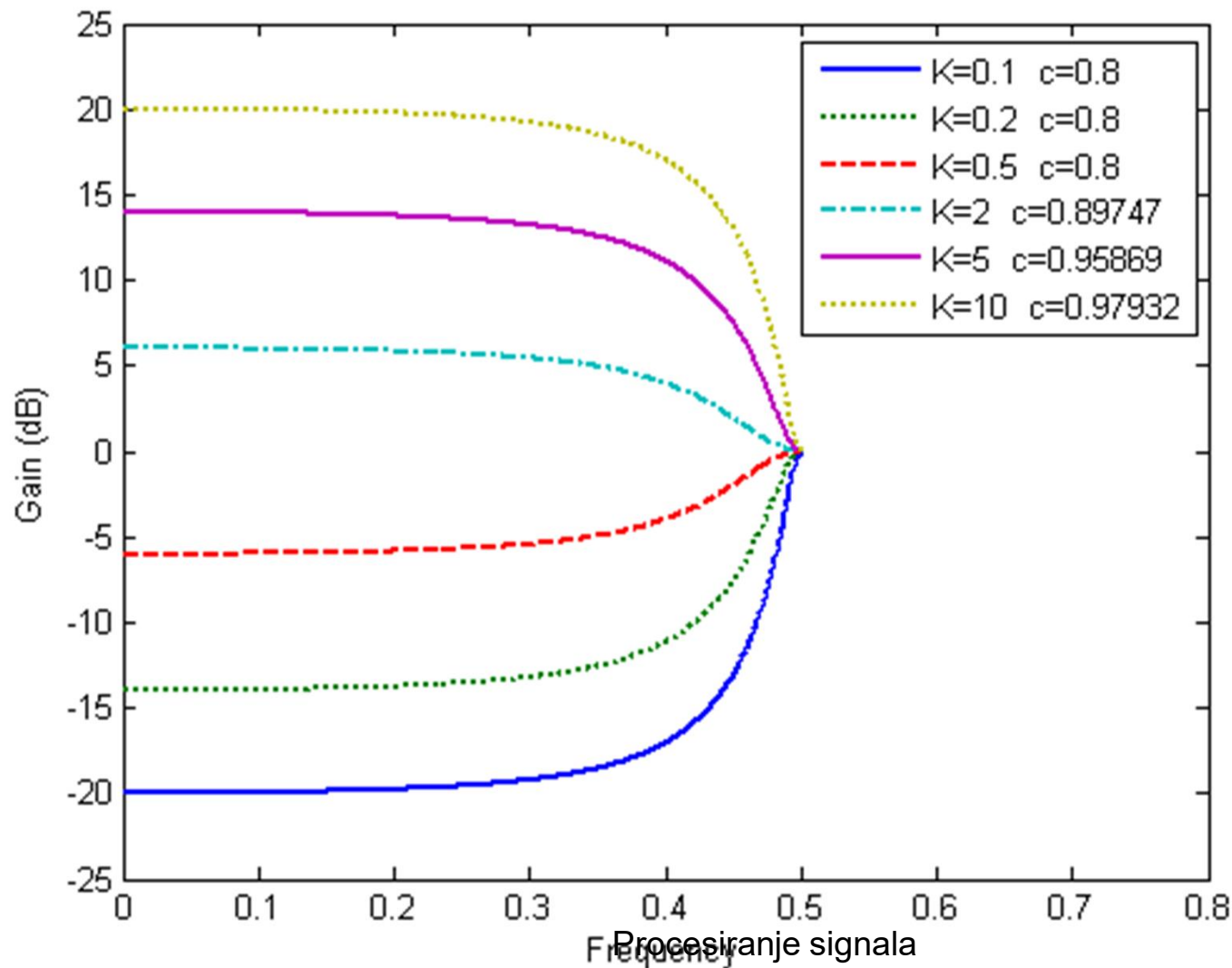
Ekvilajzer I reda LP - kod

```
c = 0.8;
wc = pi*c;
f = 0:0.001:0.5;
T = 1;

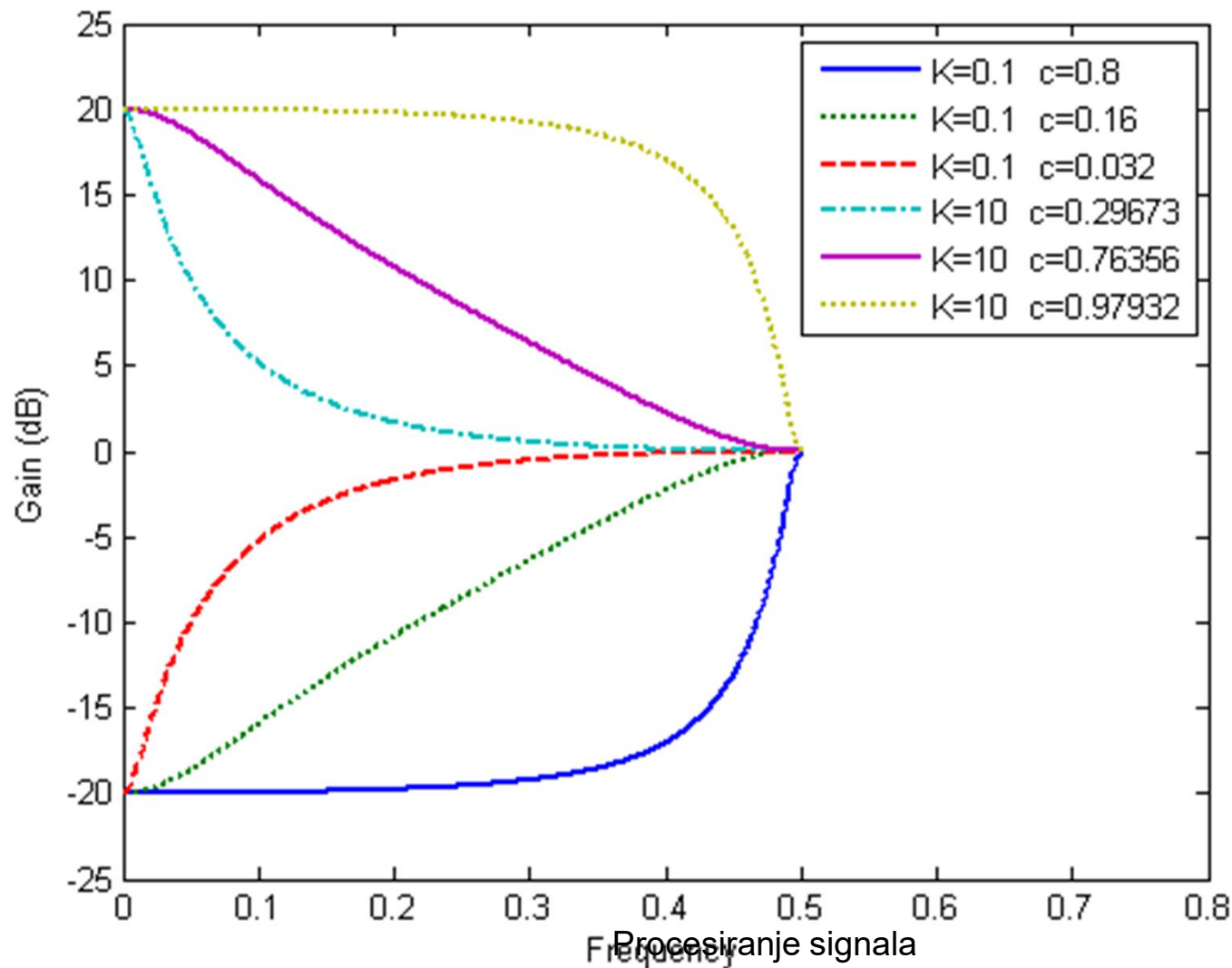
K1 = 1/10; c1 = c;
[numGlp11,denGlp11] = filter_g1_lp(wc,K1,T)
h1 = freqz(numGlp11,denGlp11,2*pi*f);
ah1 = 20*log10(abs(h1));

plot(f,ah1,'-',f,ah2,':',f,ah3,'--',f,ah4,'-.')
xlabel('Frequency'), ylabel('Gain (dB)')
axis([0 0.8 -25 25])
legend(['K=' num2str(K1) ' c=' num2str(c1)],...
```

Ekvilajzer I reda LP (2)

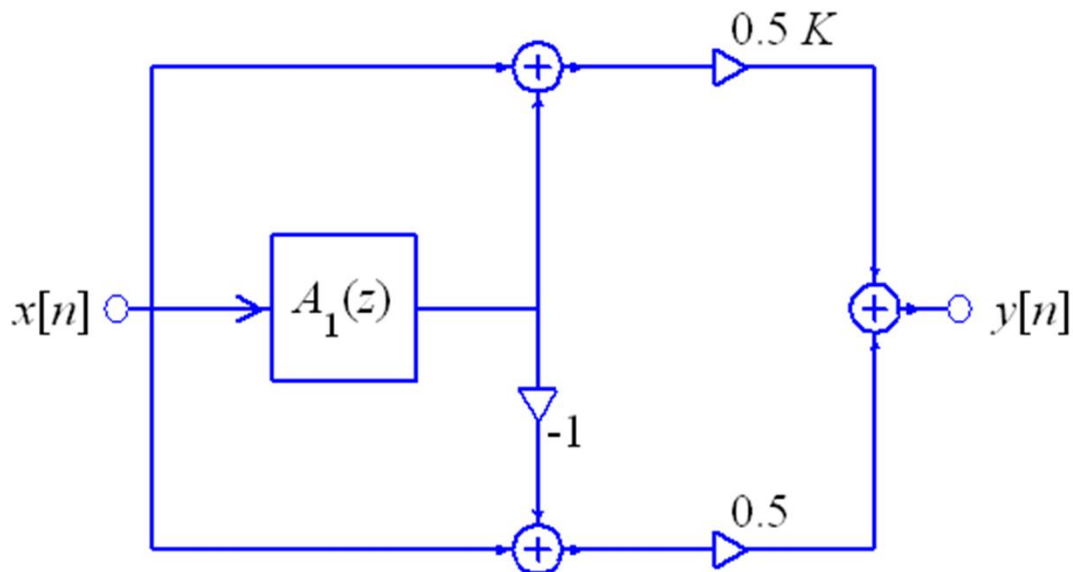


Ekvilajzer I reda LP (3)



Ekvilajzer I reda HP

$$G_{HP}(z) = \frac{1}{2}(1 - A_1(z)) + \frac{K}{2}(1 + A_1(z))$$



$$A_1(z) = \frac{a_C - z^{-1}}{1 - a_C z^{-1}}$$

$$a_C = \frac{1 - K \tan(\omega_C T / 2)}{1 + K \tan(\omega_C T / 2)}$$

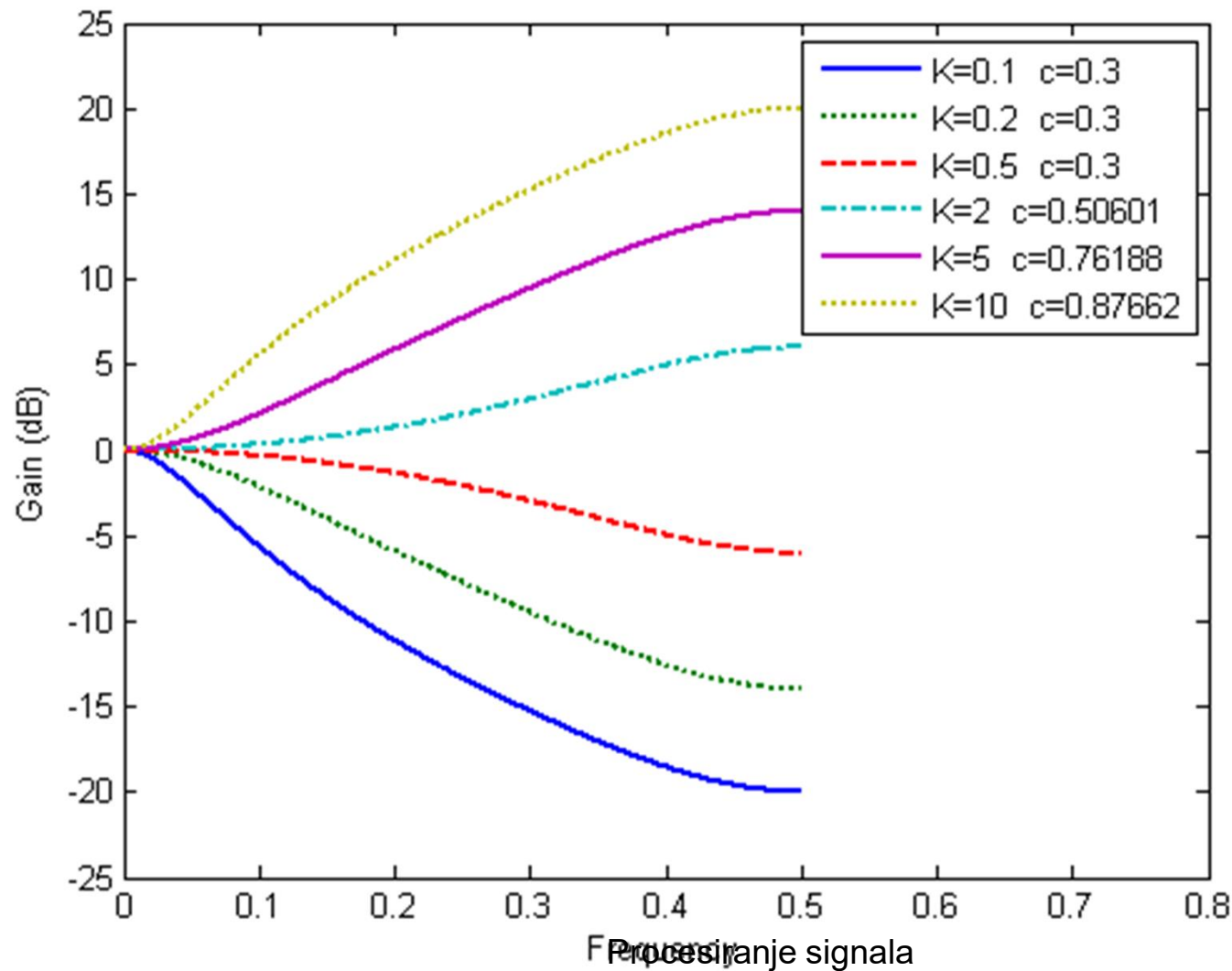
Ekvilajzer I reda HP - kod

```
c = 0.3;
wc = pi*c;
f = 0:0.001:0.5;
T = 1;

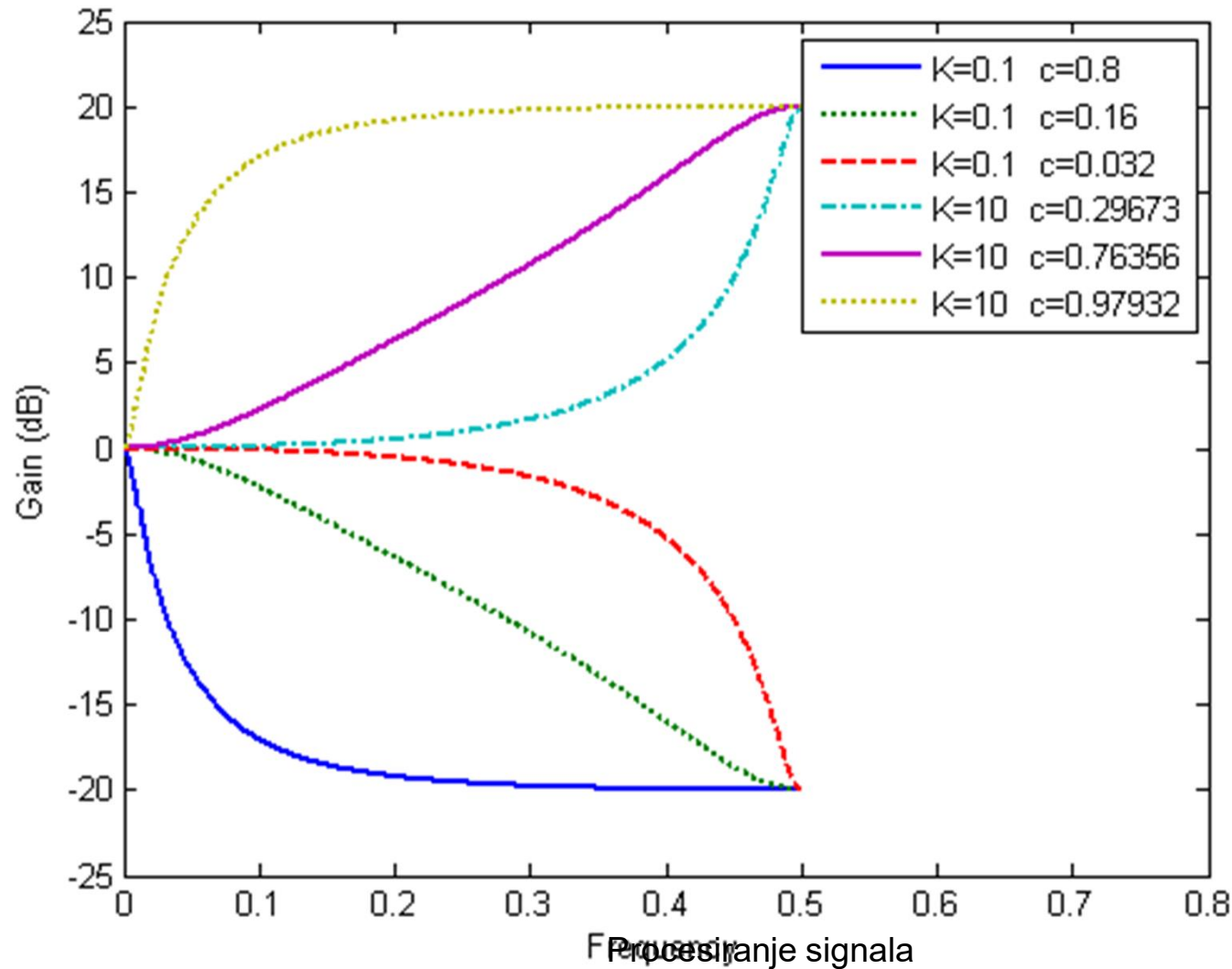
K1 = 1/10; c1 = c;
[numGlp11,denGlp11] = filter_g1_hp(wc,K1,T)
h1 = freqz(numGlp11,denGlp11,2*pi*f);
ah1 = 20*log10(abs(h1));

plot(f,ah1,'-',f,ah2,':',f,ah3,'--',f,ah4,'-.')
xlabel('Frequency'), ylabel('Gain (dB)')
axis([0 0.8 -25 25])
legend(['K=' num2str(K1) ' c=' num2str(c1)],...
```

Ekvilajzer I reda HP (2)



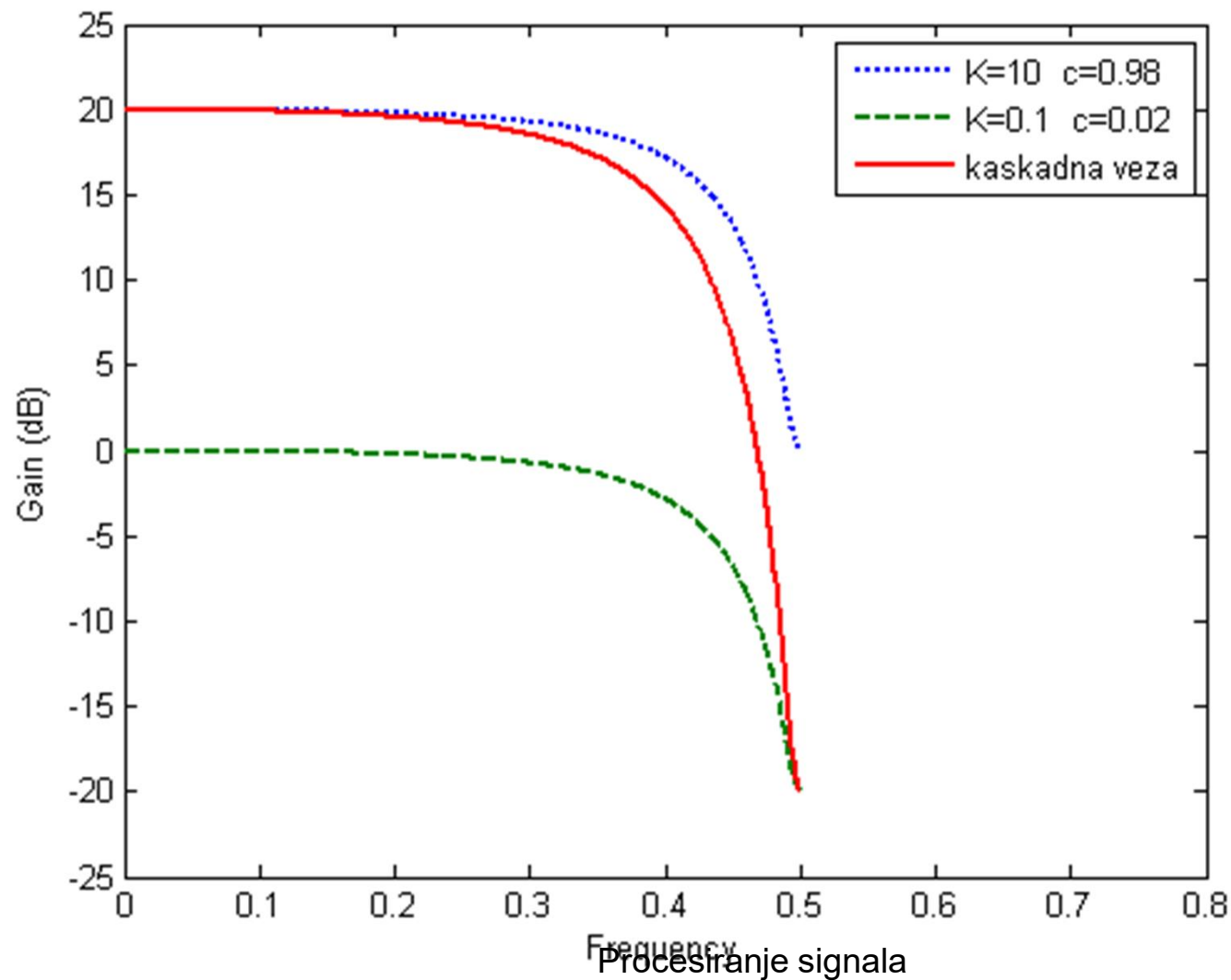
Ekvilajzer I reda HP (3)



Ekvilajzer I reda LP + HP

```
c = 0.98;
wc = pi*c;
f = 0:0.001:0.5;
T = 1;
K1 = 10; c1 = c;
[numGlp11,denGlp11] = filter_g1_lp(wc,K1,T)
h1 = freqz(numGlp11,denGlp11,2*pi*f);
ah1 = 20*log10(abs(h1));
K2 = 1/10; c2 = 0.02;
wc = pi*c2;
[numGlp12,denGlp12] = filter_g1_hp(wc,K2,T)
h2 = freqz(numGlp12,denGlp12,2*pi*f);
ah2 = 20*log10(abs(h2));
numGlp13 = conv(numGlp11,numGlp12);
denGlp13 = conv(denGlp11,denGlp12);
h3 = freqz(numGlp13,denGlp13,2*pi*f);
ah3 = 20*log10(abs(h3));
```

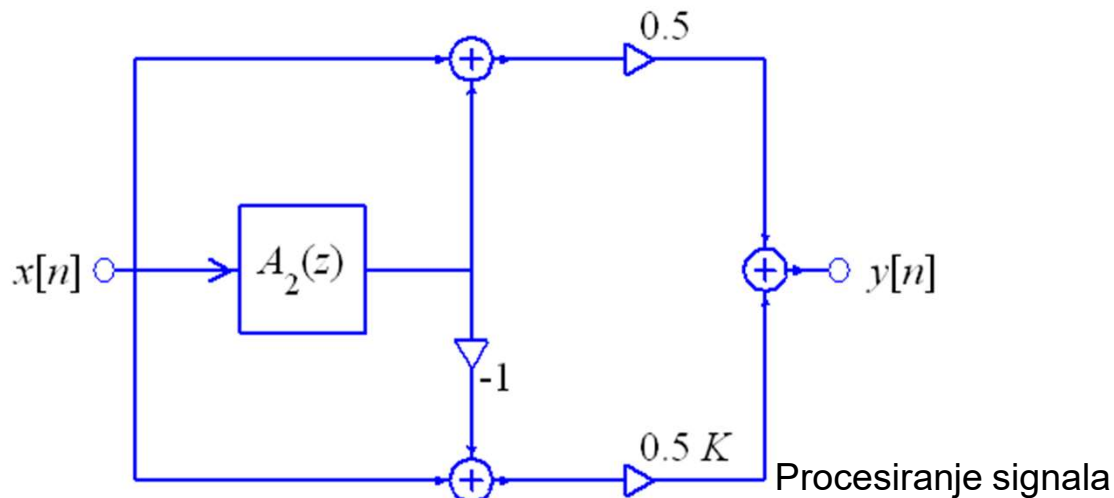
Ekvilajzer I reda LP + HP



Ekvilajzer II reda LP

$$G_{LP}(z) = \frac{K}{2} (1 - A_2(z)) + \frac{1}{2} (1 + A_2(z))$$

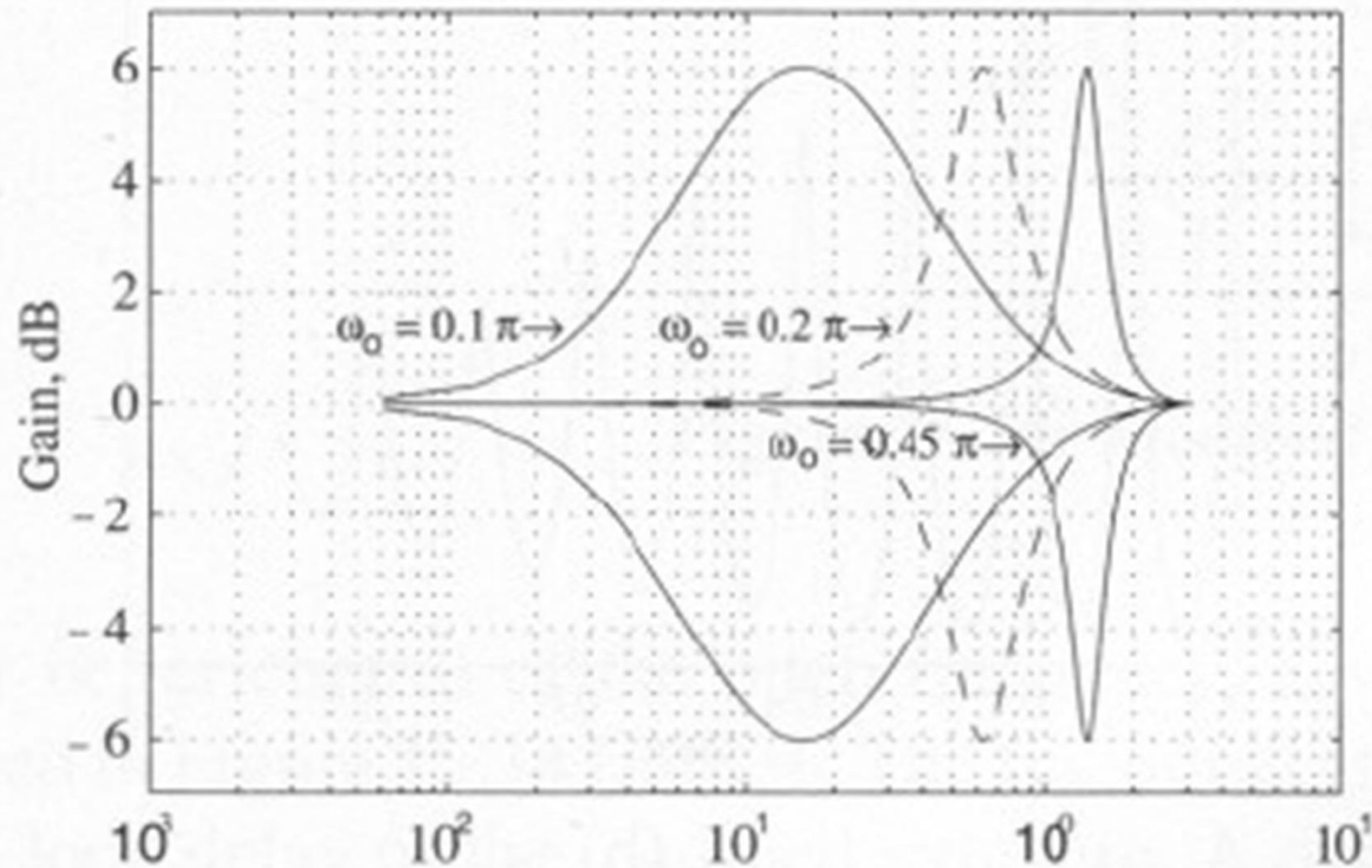
$$A_2(z) = \frac{a_C - \beta(1 + a_C)z^{-1} + z^{-2}}{1 - \beta(1 + a_C)z^{-1} + a_C z^{-2}}$$



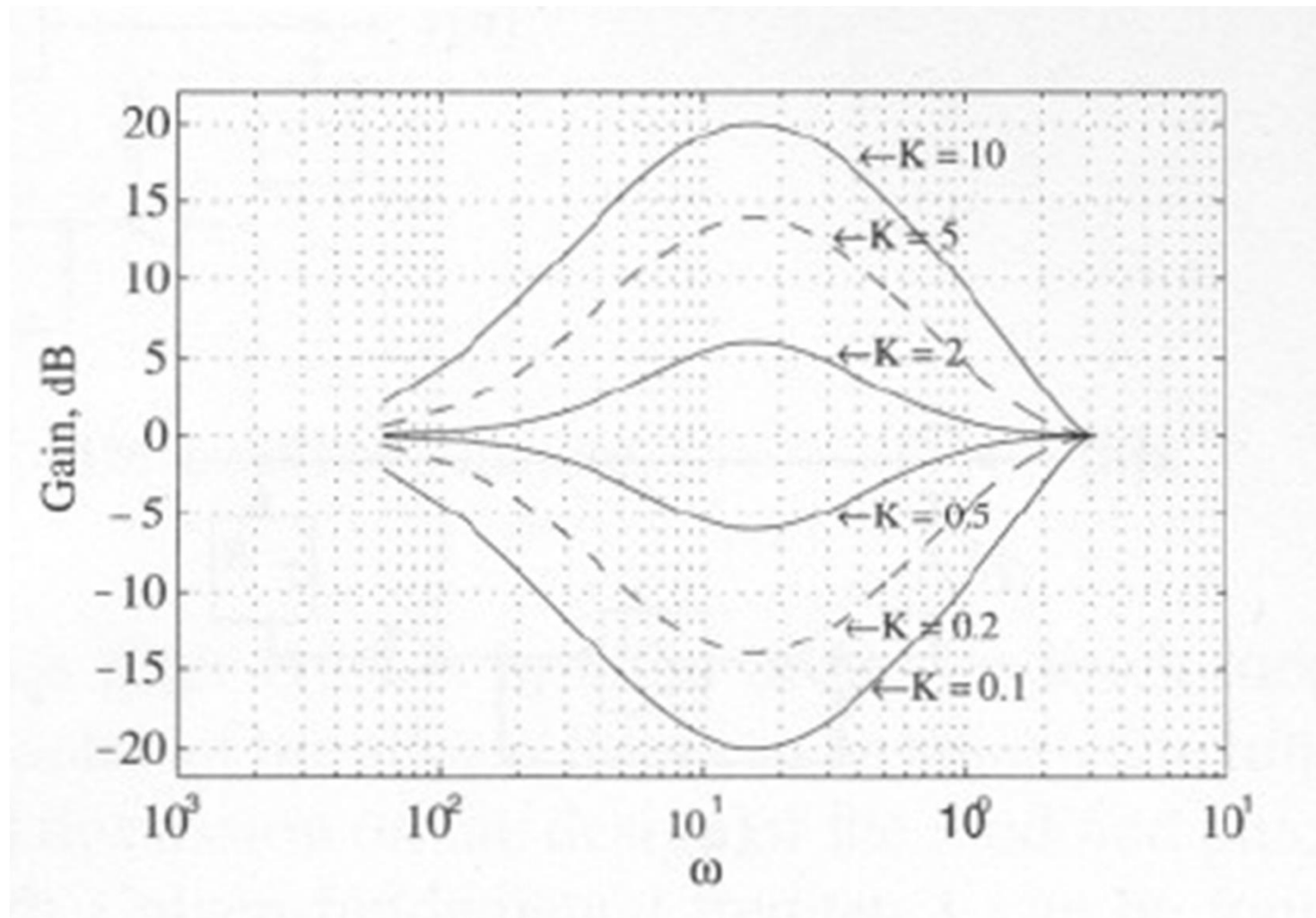
$$\beta = \cos(\omega_0)$$

$$a_C = \frac{K - \tan(B_C T / 2)}{K + \tan(B_C T / 2)}$$

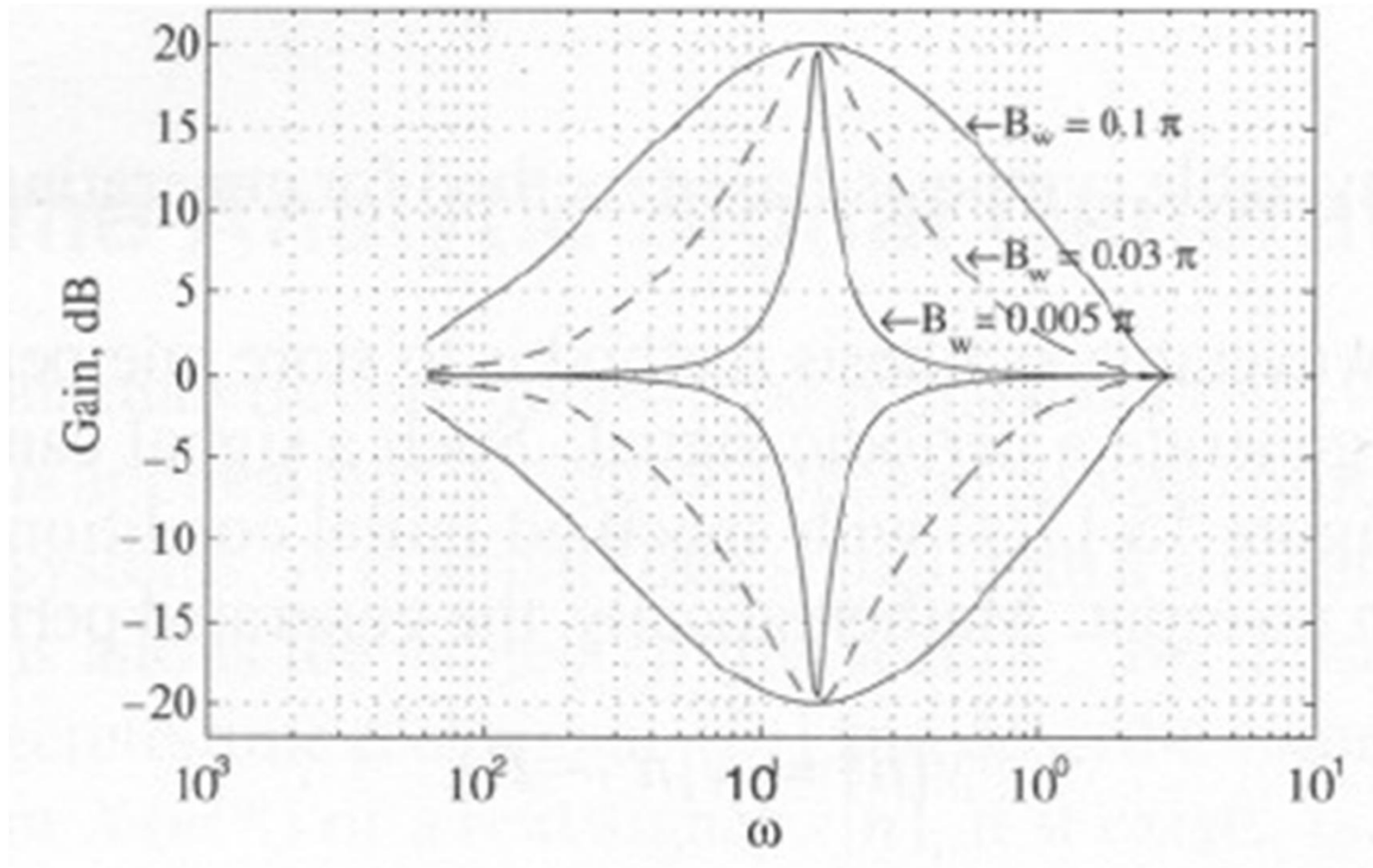
Ekvilajzer II reda (2)



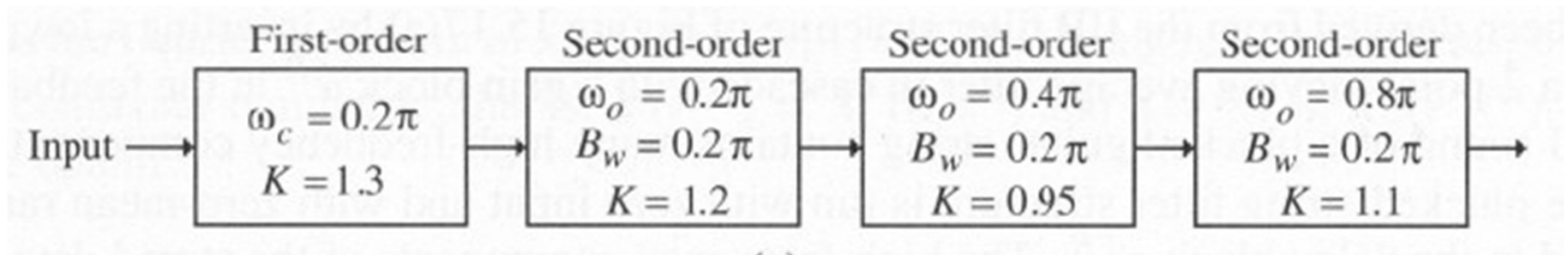
Ekvilajzer II reda (3)



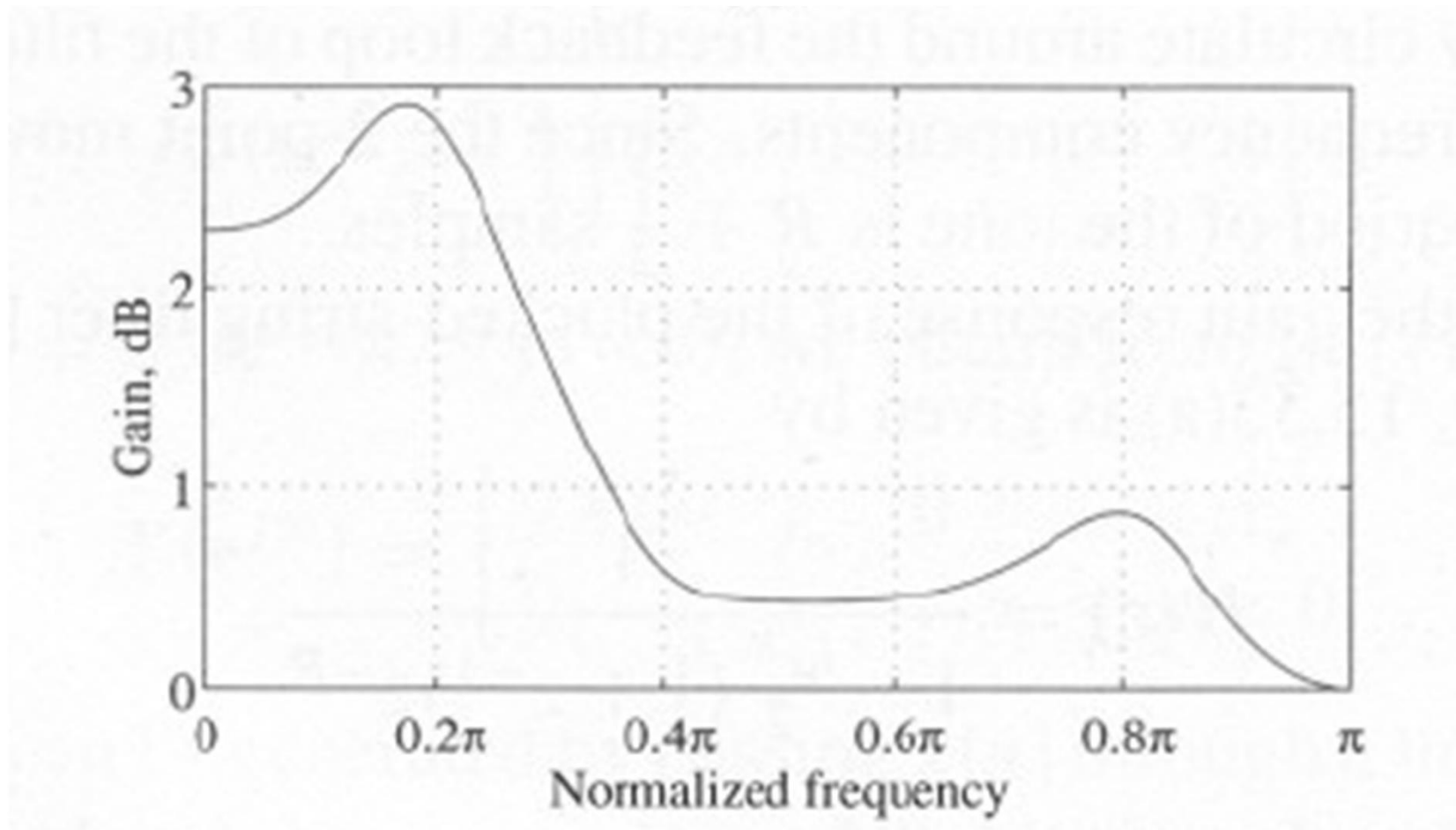
Ekvilajzer II reda (4)



Ekvilajzer višeg reda



Ekvilajzer višeg reda (2)

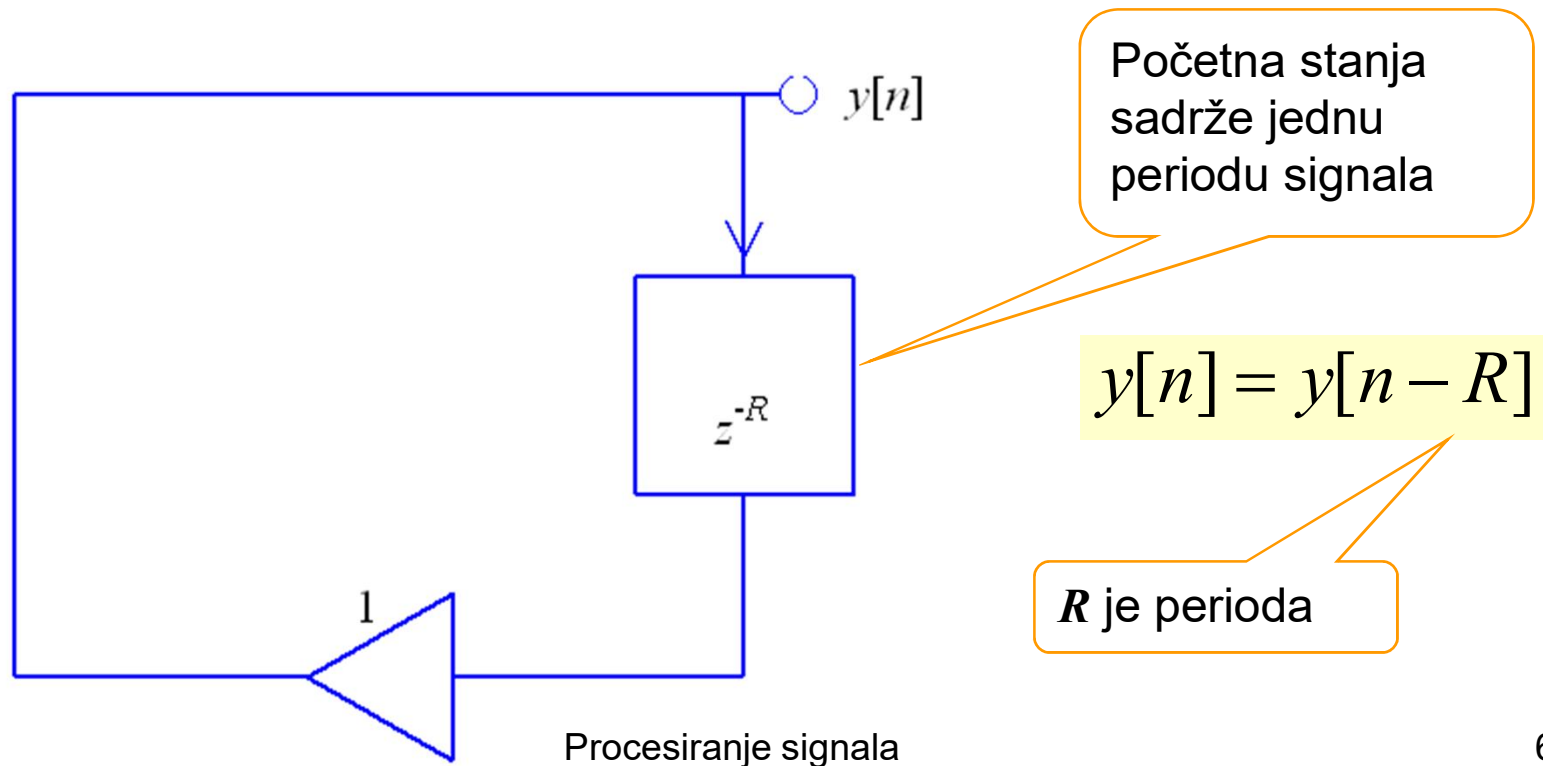


Sinteza muzičkih signala

- Korišćenjem tabela
- Modelovanjem spektra
- Nelinearne metode
- Modelovanjem fizičkih pojava

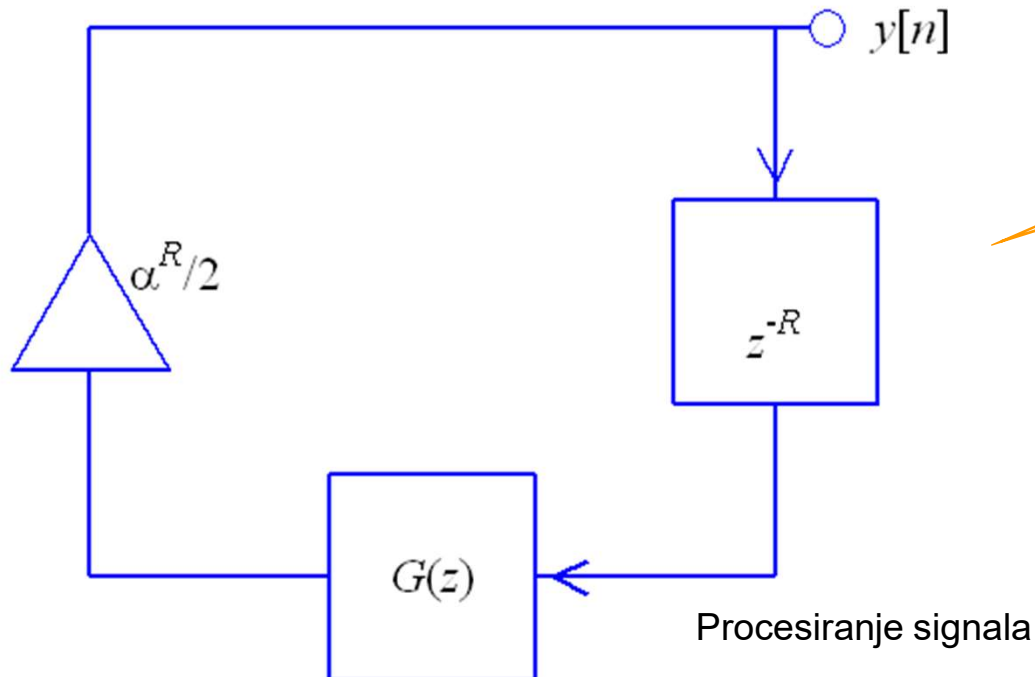
Sinteza korišćenjem tabela

- U tabeli se čuva jedna perioda željenog muzičkog signala
- Ponavlja se isčitavanjem iz tabele periodičnog signala



Sinteza korišćenjem tabela

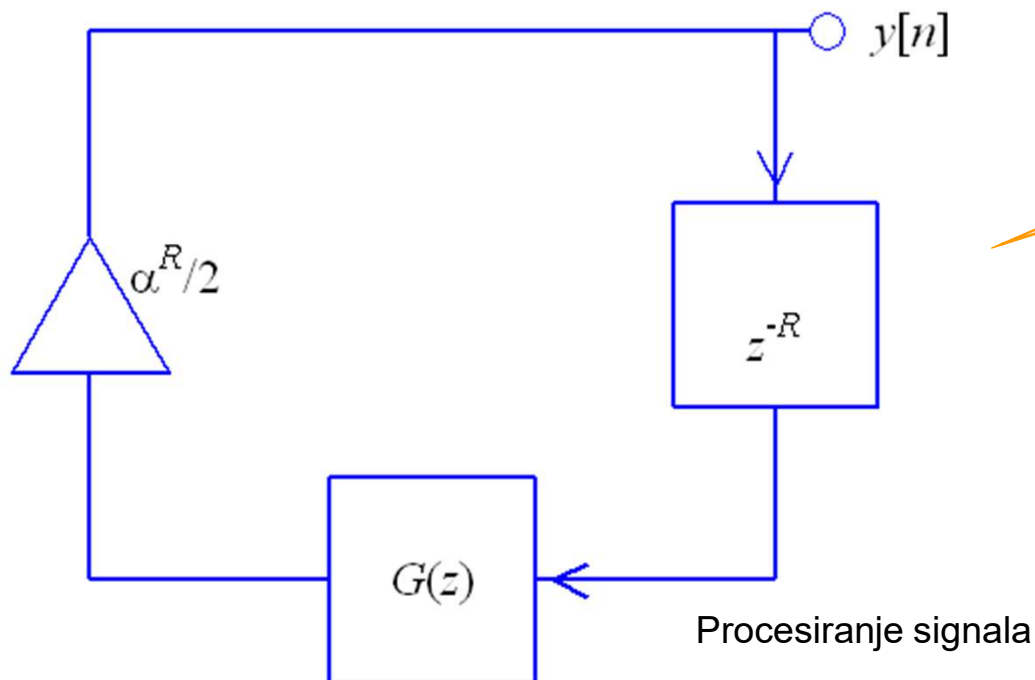
$$y[n] = \frac{\alpha^R}{2} [y[n-R] + y[n-R-1]]$$



Početna stanja
sadrže jednu
periodu signala

Sinteza korišćenjem lowpass filtra

$$y[n] = \frac{\alpha^R}{2} [y[n-R] + y[n-R-1]]$$

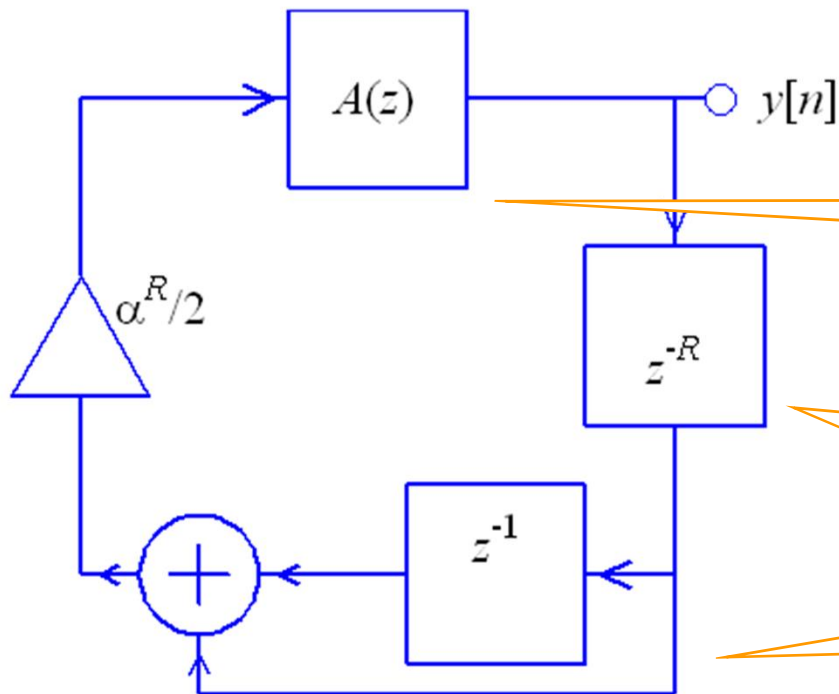


Početna stanja
sadrže jednu
periodu signala

Procesiranje signala

Plucked-string filter

$$y[n] = \frac{\alpha^R}{2} [y[n-R] + y[n-R-1]]$$



Početni zvuk gitare
sadrži više
visokofrekvencijskih
komponenti

Allpass filter za kontrolu tona

Srednja vrednost u
registru je 0.

Grupno kašnjenje je $1/2$

Procesiranje signala

Transmultiplekseri - FDM

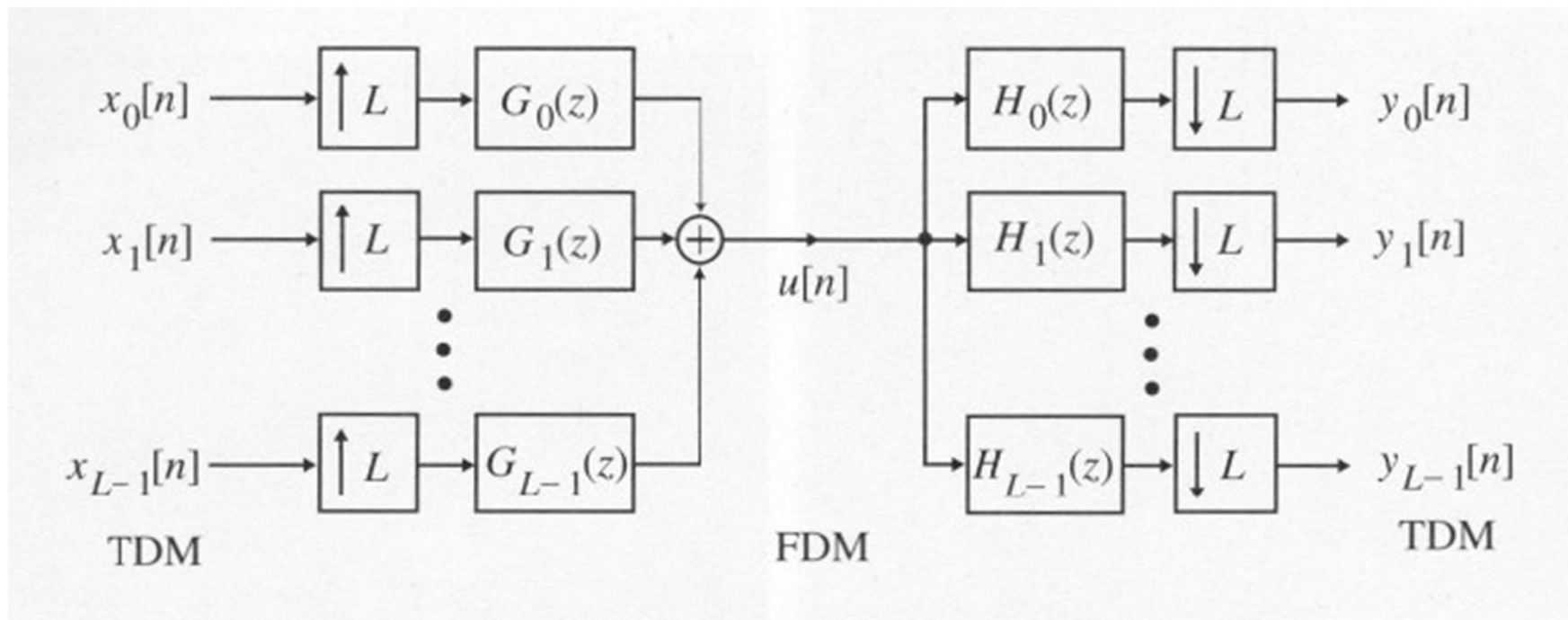
- FDM - Frequency Division Multiplex
- Više analognih signala se moduliše modulacijom SSB (single-side-band) a zatim se zajedno istovremeno prenosi jednim širokopojasnim kanalom
- U prijemu, pojedini signali se razdvajaju filterima propusnicima opsega a zatim se demodulišu

TDM

- TDM – Time Division Multiplex
- Analogni signal se digitalizuje, odbirci se pakuju redom iz svakog kanala i zajedno prenose
- U prijemu, prvo se izdvajaju odbirci koji pripadaju jednom kanalu

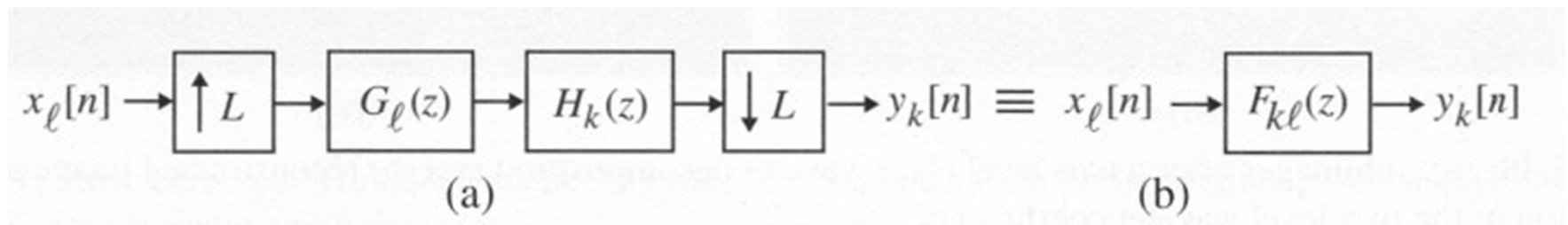
Transmultiplexeri

- Transmultiplexer je struktura sa više ulaza i više izlaza



Ulazno-izlazne relacije

- Tipičan put jednog ulaznog signla do izlaznog signala



$$Y_k = \sum_{l=0}^{L-1} F_{kl}(z) X_l(z), \quad 0 \leq k \leq L-1$$

Ulazno-izlazne relacije

$$\mathbf{Y}(z) = [Y_0(z) \quad Y_1(z) \quad \cdots \quad Y_{L-1}(z)]^t$$

$$\mathbf{X}(z) = [X_0(z) \quad X_1(z) \quad \cdots \quad X_{L-1}(z)]^t$$

$$\mathbf{Y}(z) = \mathbf{F}(z)\mathbf{X}(z)$$

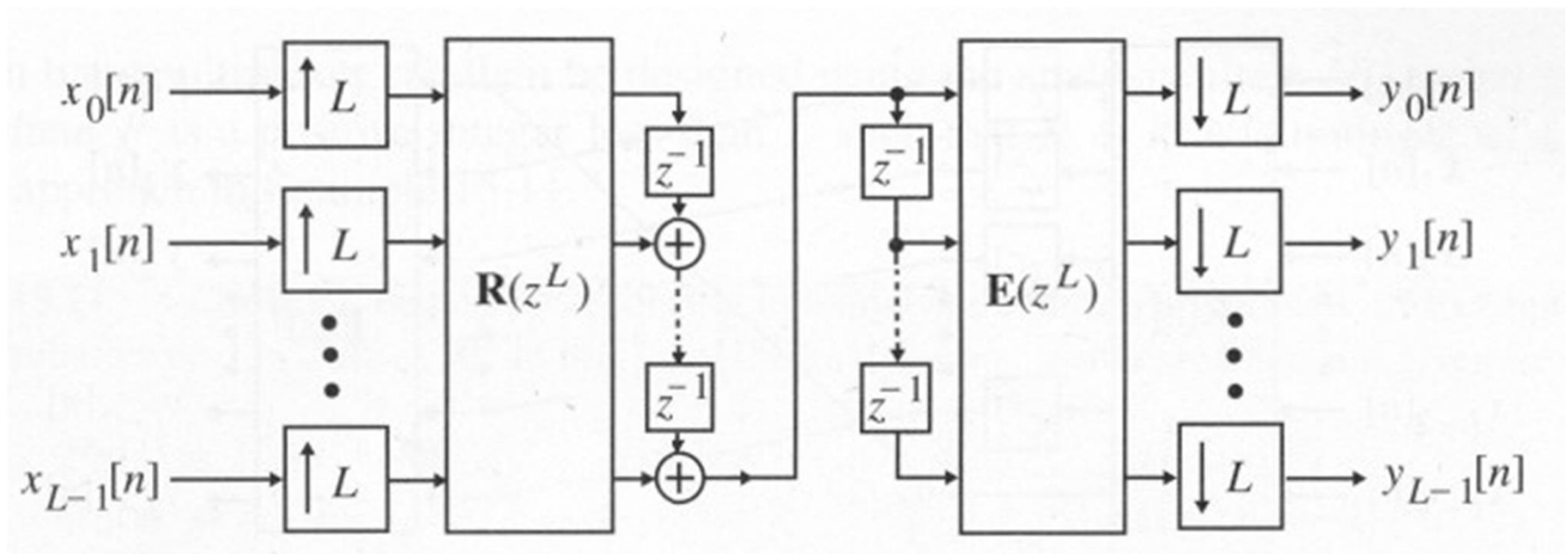
Nema preslušavanja

$$Y_k(z) = F_{kk}(z)X_k(z), \quad 0 \leq k \leq L-1$$

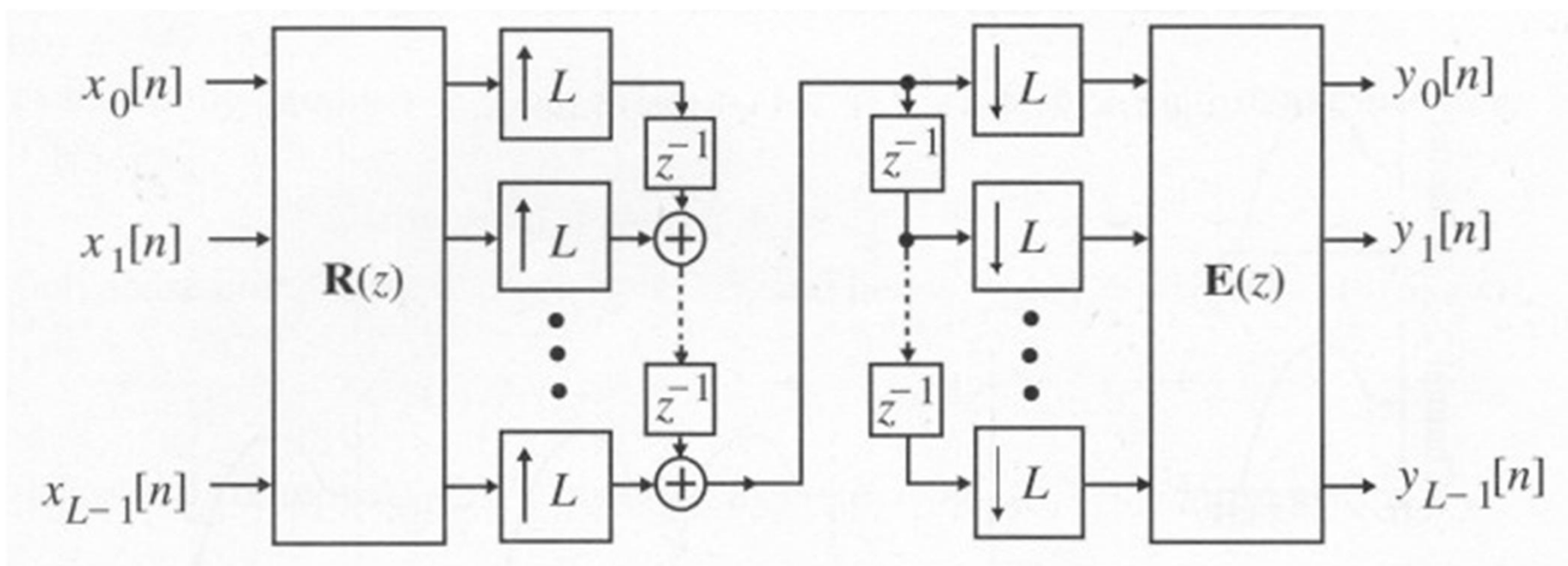
$$Y_k(z) = a_k z^{-n_k}, \quad 0 \leq k \leq L-1$$

Perfektna rekonstrukcija

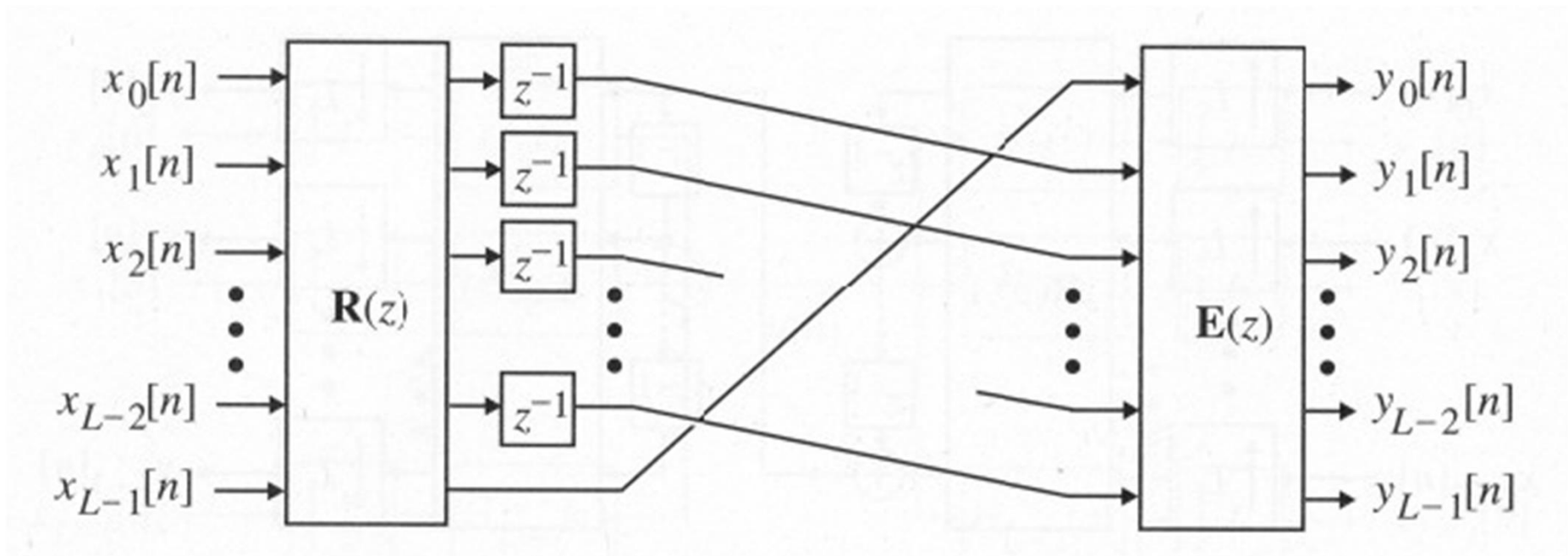
Polifazna reprezentacija L -kanalnog transmultipleksera



Efikasna realizacija L -kanalnog transmultipleksera



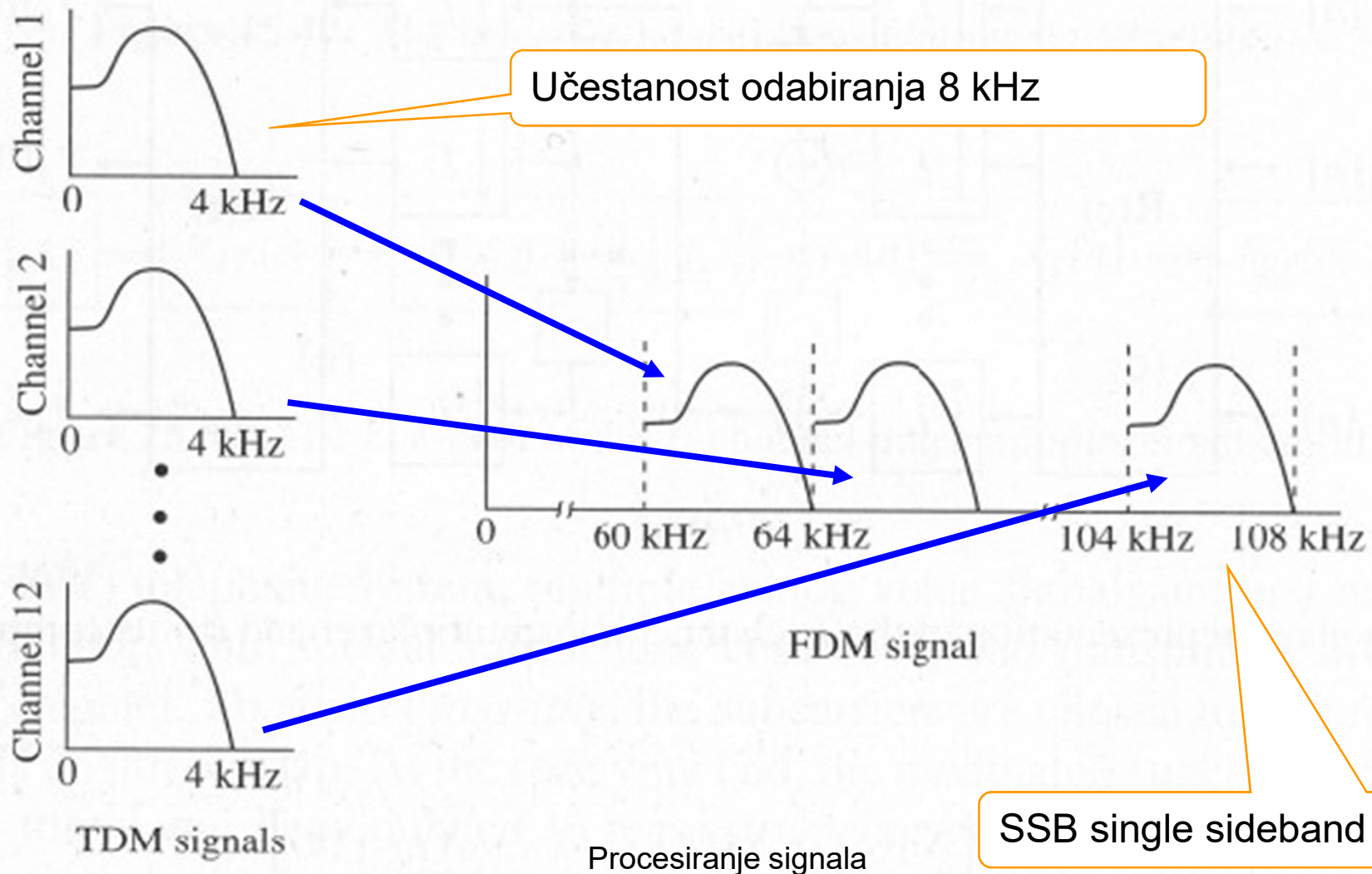
Ekvivalentna realizacija L -kanalnog transmultiplexera



$$\mathbf{F}(z) = \mathbf{E}(z) \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ z^{-1} \mathbf{I}_{L-1} & \mathbf{0} \end{bmatrix} \mathbf{R}(z)$$

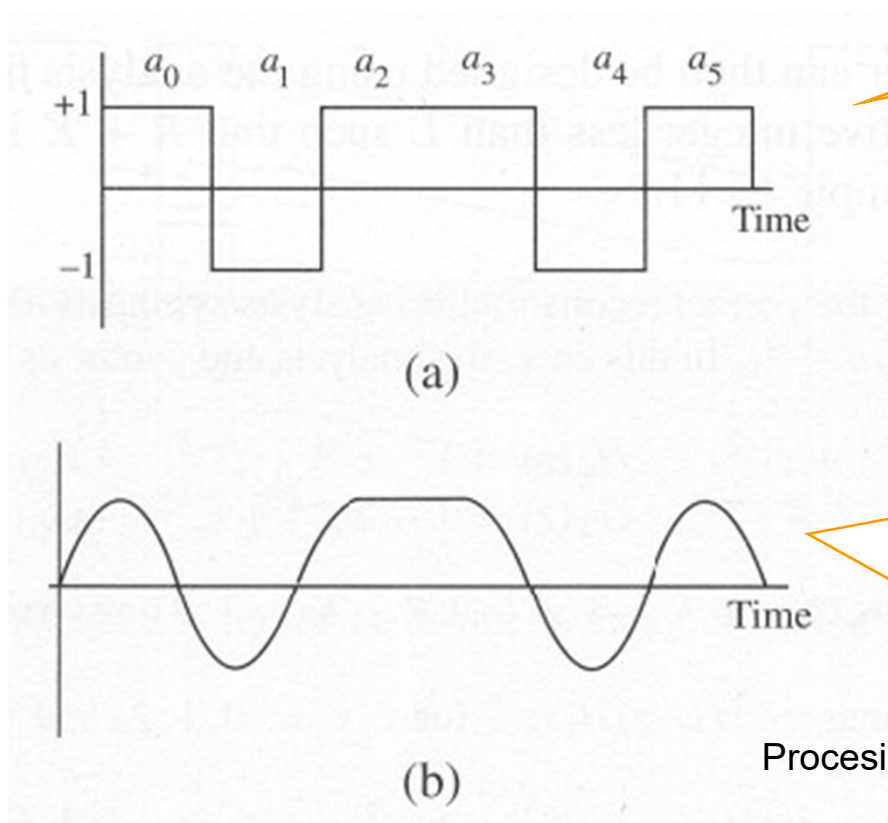
Procesiranje signala

Spektri TDM i FDM



Prenos binarnih digitalnih podataka

- Binarni podaci mogu da se prenose serijski signalima koji imaju vrednosti $+1$ i -1



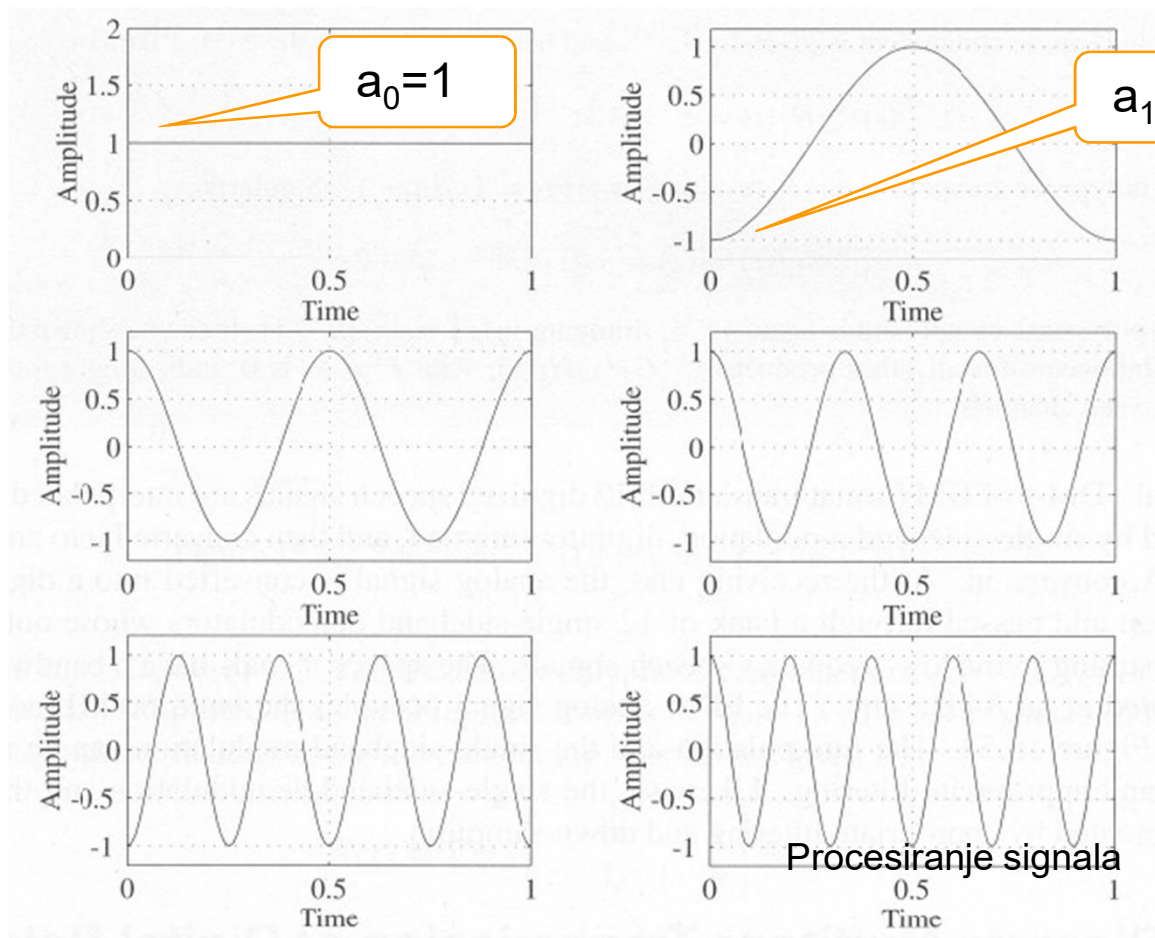
Poslat signal

Primljen signal je drugačiji zbog nesavršenosti kanala i ograničenog frekvencijskog opsega. Potrebna složena procedura ekvilizacije funkcije prenosa kanala.

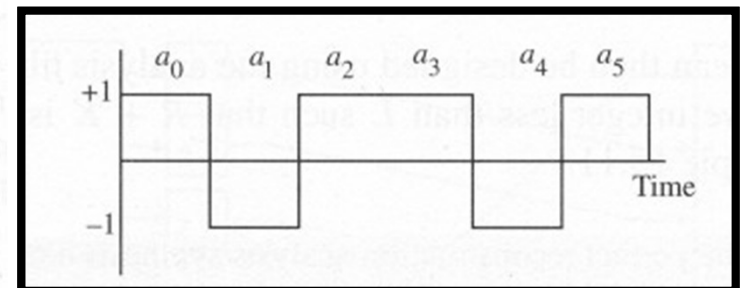
Procesiranje signala

Višetonski prenos digitalnih podataka

- N binarnih cifara se moduliše svojim nosiocem i sabira
- Prijemnik – koherentni demodulatori



DMT
Discrete
Multitone
Transmission



DMT - discrete multitone transmission

$$\{a_k[n]\}, \{b_k[n]\}, \quad 0 \leq k \leq M-1$$

Realni signali (sekvence)
učestanost odabiranja F_T

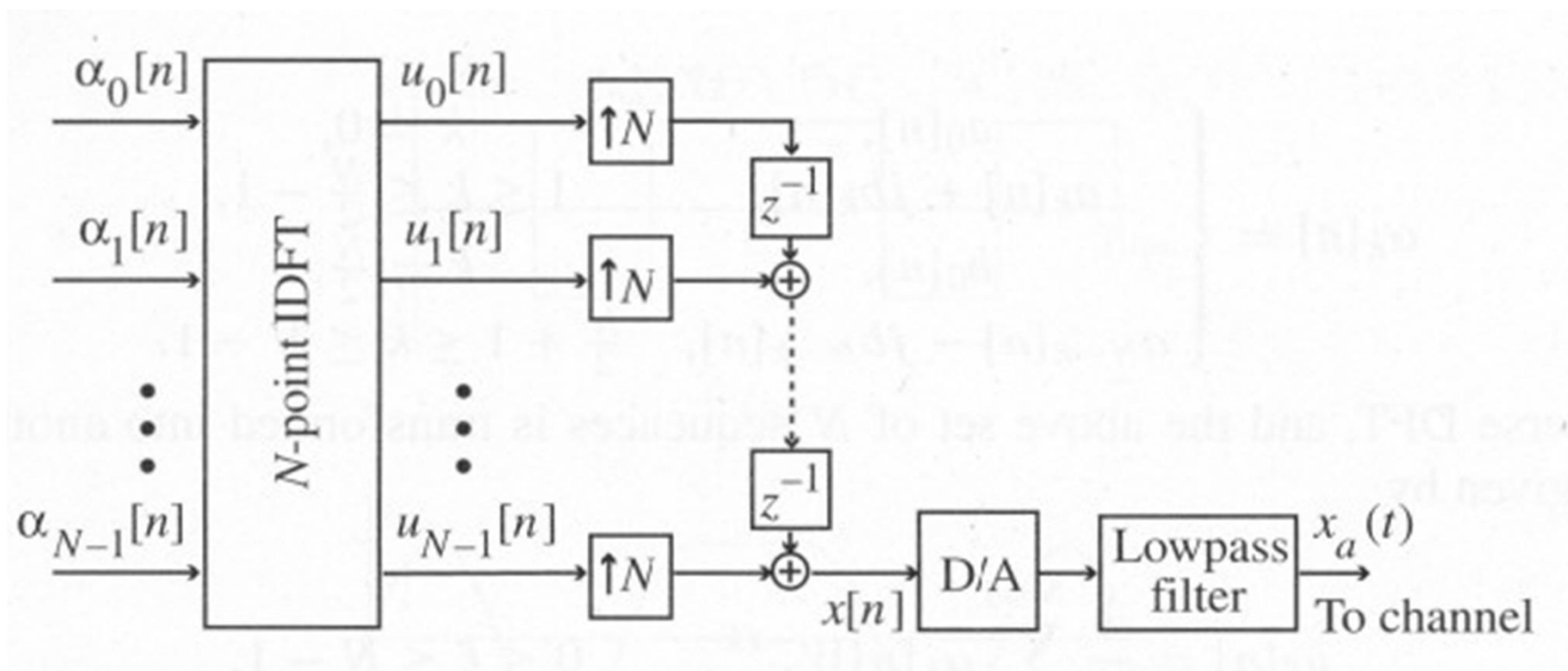
$$\alpha_k[n] = \begin{cases} a_0[n] & k = 0 \\ a_k[n] + jb_k[n] & 1 \leq k \leq N/2 - 1 \\ b_0[n] & k = N/2 \\ a_k[n] - jb_k[n] & N/2 + 1 \leq k \leq N - 1 \end{cases} \quad N = 2M$$

IDFT

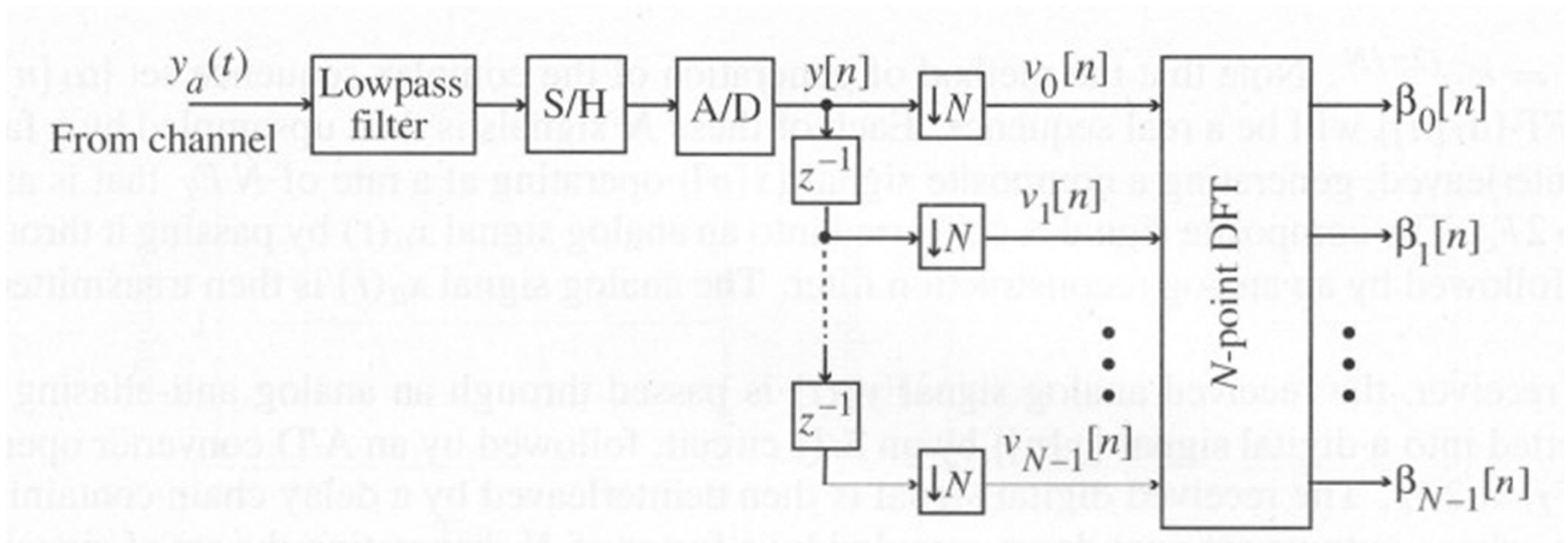
$$u_l[n] = \frac{1}{N} \sum_{k=0}^{N-1} \alpha_k[n] W_N^{-lk}, \quad 0 \leq l \leq N-1$$

$$W_N = e^{-j2\pi/N}$$

DMT - predajnik



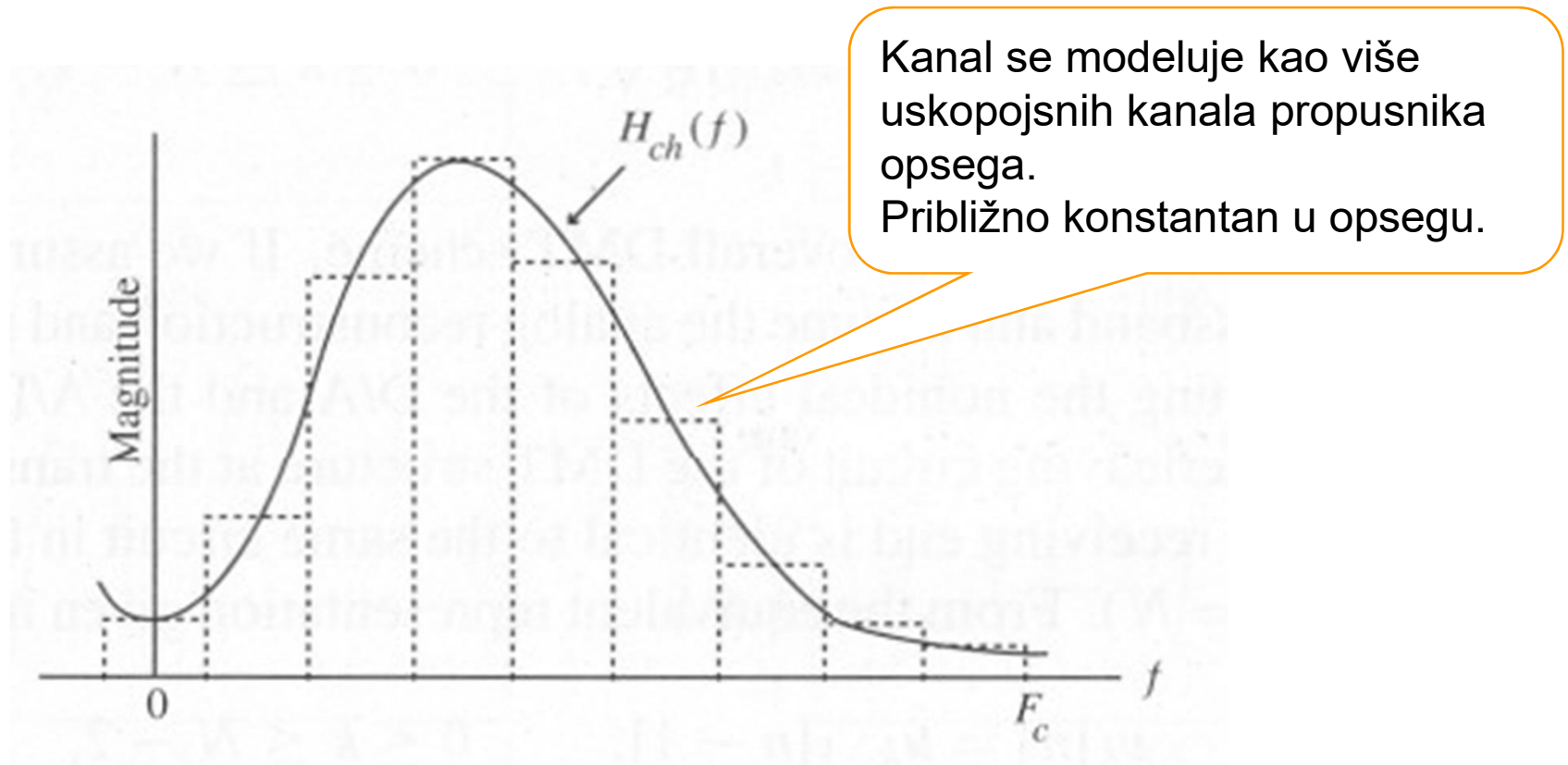
DMT - prijemnik



$$\beta_k[n] = \alpha_{k-1}[n], \quad 1 \leq k \leq N-1$$

$$\beta_0[n] = \alpha_{N-1}[n]$$

Karakteristika prenosnog kanala



Oversampling AD konverzija

- Učestanost odabiranja treba da bude 2 puta veća od najviše učestanosti korisnog signala
- Pre AD konverzije mora da se koristi filter propusnik niskih učestanosti čija je učestanost granice propusnog opsega jednaka najvišoj učestanosti korisnog signala, učestanost granice nepropusnog opsega neznatno veća od granice propusnog opsega
- Analogni filter mora da bude visokog reda sa kvalitetnim komponentama (visoka cena)
- Filter sa oštrom karakteristikom unosi fazna izobličenja u propusnom opsegu
- Alternativa – učestanost odabiranja je znatno veća, filter nema strmu karakteristiku, digitalnim filtrom se realizuje oštra karakteristika, smanji se učestanost odbiranja

Analiza šuma (1)

- AD konvertor sa b bita i učestanost odabiranja F_T
- Full-scale peack-to-peack ulazni signal (napon) je R_{FS}
- Najmanji opseg vrednosti koji se predstavlja binarno ΔV

$$\Delta V = \frac{R_{FS}}{2^b - 1} \approx \frac{R_{FS}}{2^b}$$

Analiza šuma (2)

- Snaga kvantizacionog šuma je σ_e^2
- Podrazumeva se uniformna distribucija greške u opsegu između $-\Delta V/2$ i $\Delta V/2$
- Gustina šuma je snaga šuma po jedinici frekvencijskog opsega (*noise density*)
- Snaga (ukupna) u korisnom opsegu (*in-band noise power*)

$$\sigma_e^2 = \frac{(\Delta V)^2}{12}$$

$$P_{e,n} = \frac{\sigma_e^2}{F_T / 2} = \frac{(\Delta V)^2}{6F_T}$$

$$P_{\text{total}} = \frac{\sigma_e^2}{F_T / 2} = \frac{\left(R_{FS} / 2^b\right)^2 / 12}{12} \frac{F_m}{F_T / 2}$$

Analiza šuma (3)

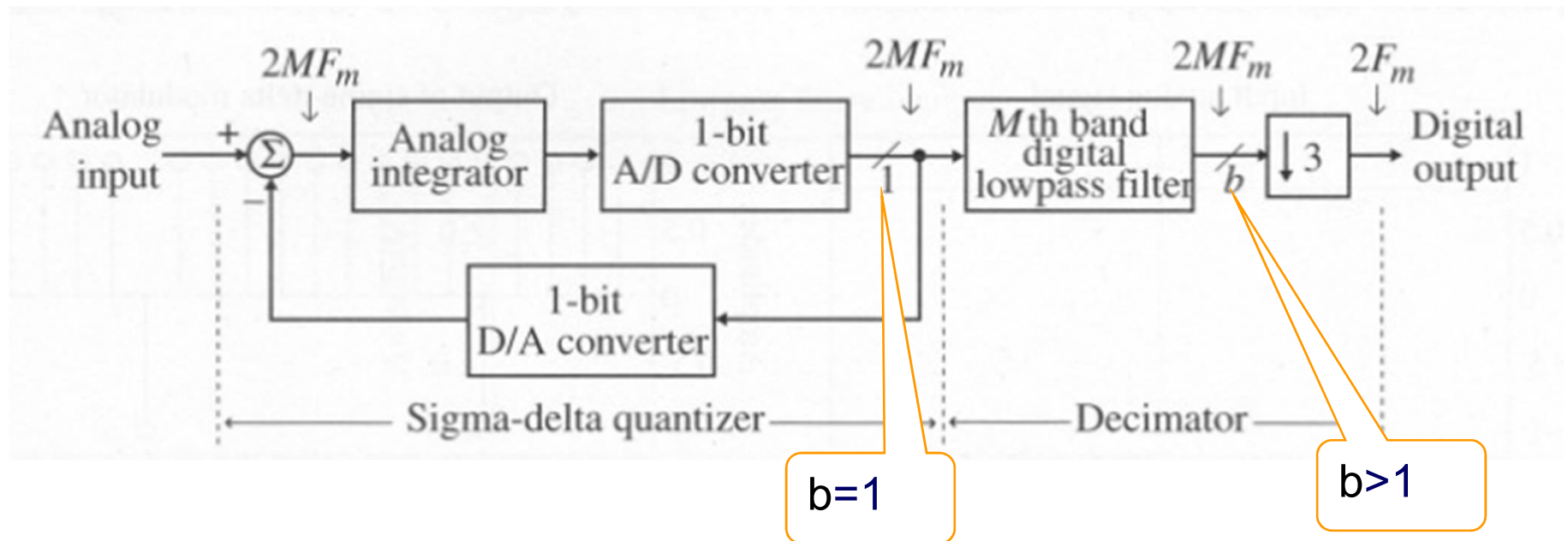
- Za učestanost odabiranja $F_T=2F_m$, broj bita β
- Za učestanost odabiranja $F_T=2MF_m$, broj bita je b
- Oversampling ratio (OSR) $M=F_T/2F_m$
- Koristi se idealni filter propusnik niskih učestanosti
- Koliko bita treba manje za veće M ?

$$\beta = b + \frac{1}{2} \log_2 M$$

*Brži a manje tačan
AD konvertor –
ekonomičnije rešenje*

*$M=1000$, $b=8$
isto kao da se koristi
 $M=1$, $b=13$*

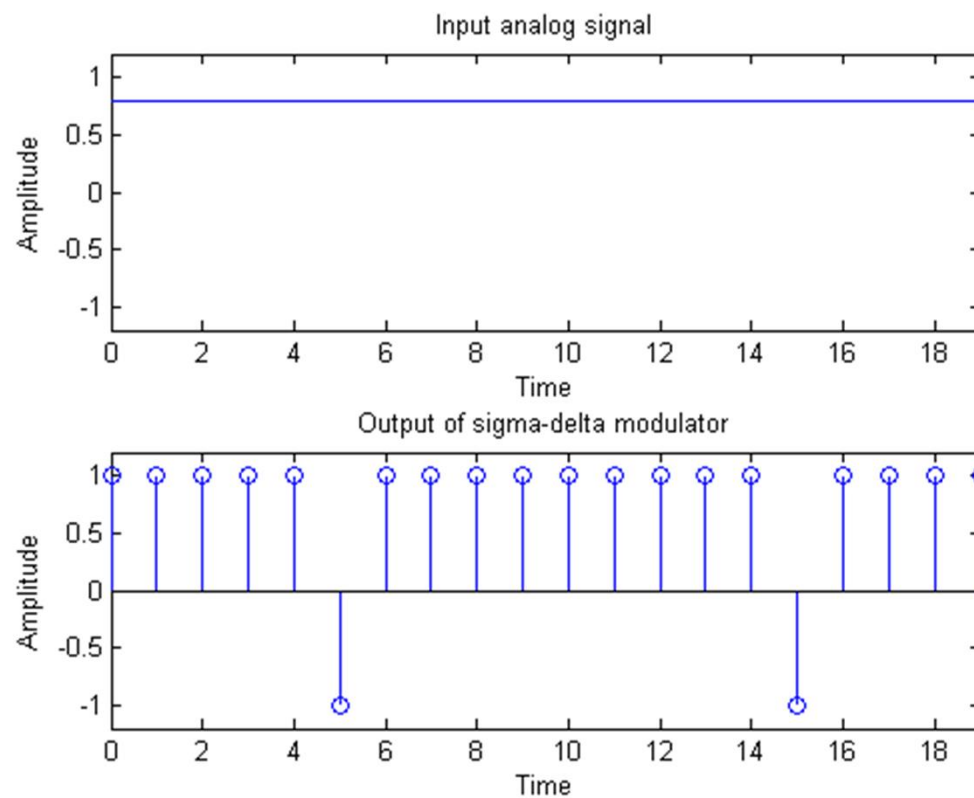
Sigma-delta ($\Sigma\Delta$) AD konvertor



Sigma-delta konvertor (1)

```
N = 20;  
n = 1:1:N;  
m = n-1;  
A = 0.8;  
x = A*ones(1,N) ;  
subplot(2,1,1) ,  
plot(m,x) ;  
subplot(2,1,2)  
y = zeros(1,N+1) ;  
v0 = 0;  
    for k = 2:1:N+1;  
        v1 = x(k-1) - y(k-1) + v0;  
        y(k) = sign(v1) ;  
        v0 = v1;  
    end  
yn = y(2:N+1) ;  
stem(m, yn) ;
```

Sigma-delta konvertor (2)



Procesiranje signala

Sigma-delta konvertor (3)

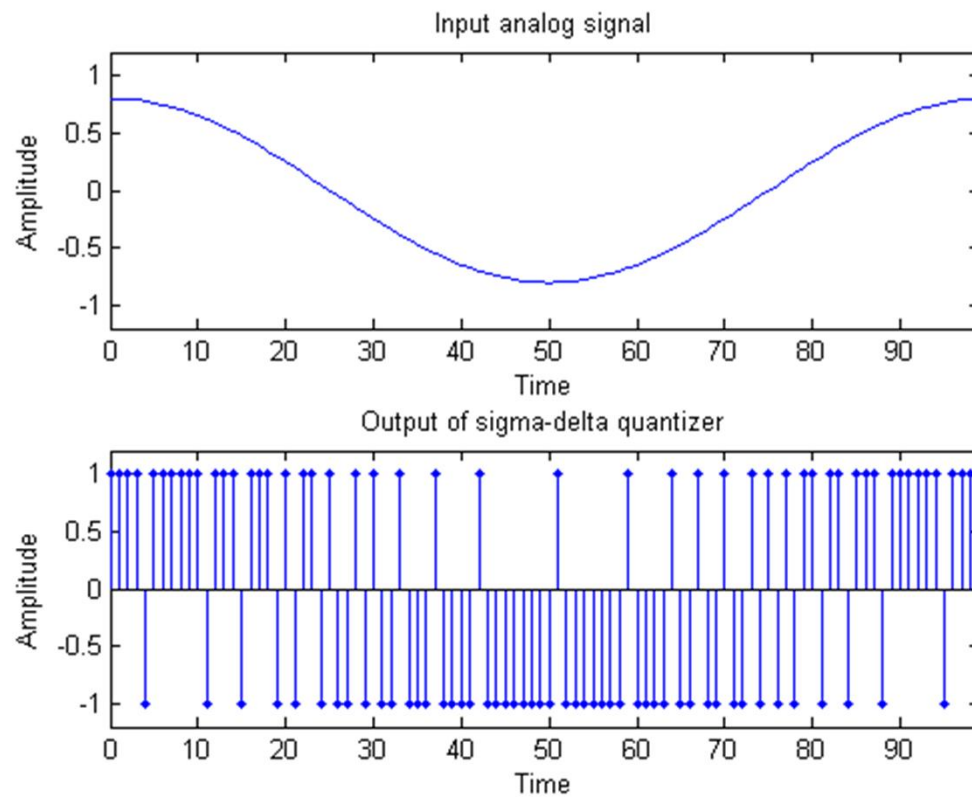
```
wo = 2*pi*0.01;
sequence = ');
N = 100;
n = 1:1:N;
m = n-1;
A = 0.8;
x = A*cos(wo*m) ;

. . .

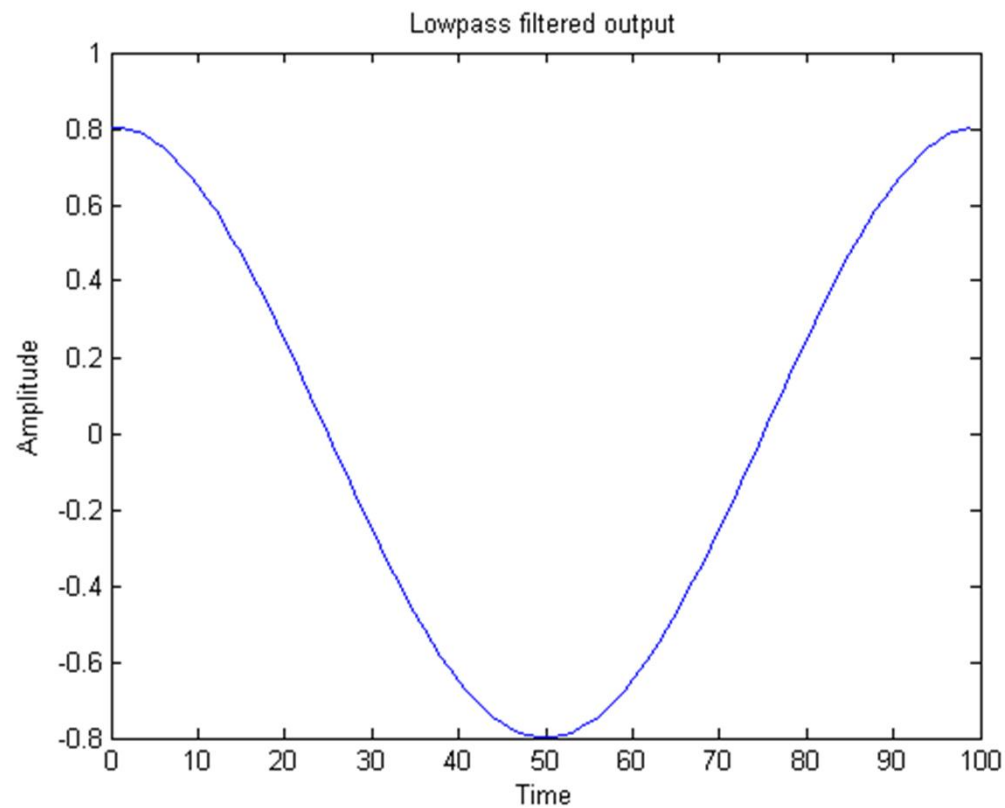
figure
H = [1 1 0.5 zeros(1,N-5) 0.5 1];
YF = Y.*H;
out = ifft(YF);
axis([0 N-1 -1.2 1.2]);
plot(m,out);
xlabel('Time'); ylabel('Amplitude');
title('Lowpass filtered output');
```

Procesiranje signala

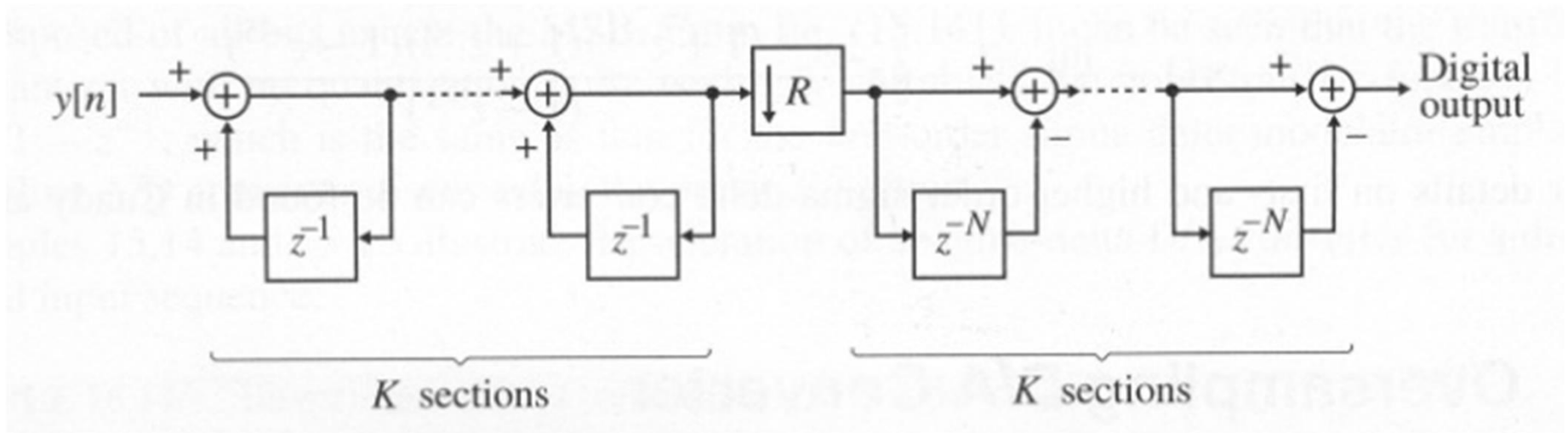
Sigma-delta konvertor (4)



Sigma-delta konvertor (5)



CIC decimator



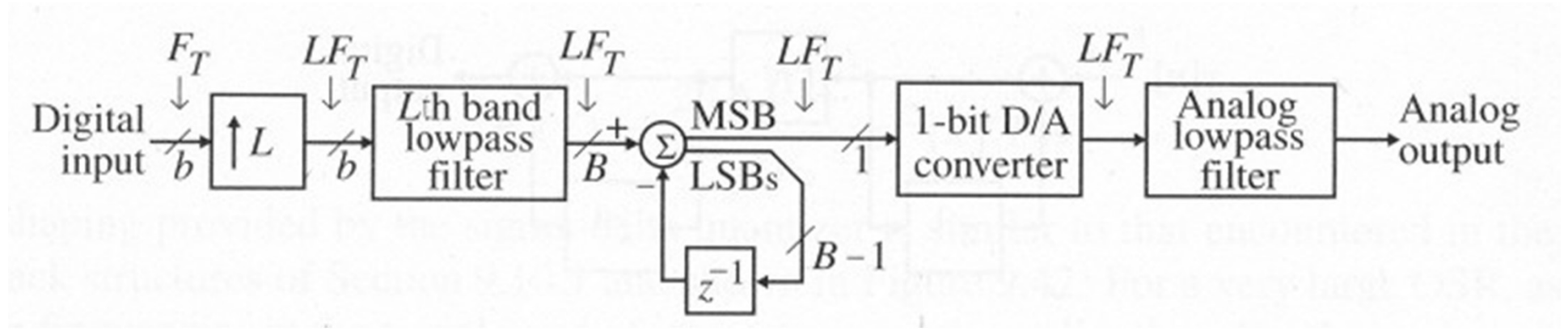
$$H(z) = \left(\frac{1 - z^{-RN}}{1 - z^{-1}} \right)^K$$

Procesiranje signala

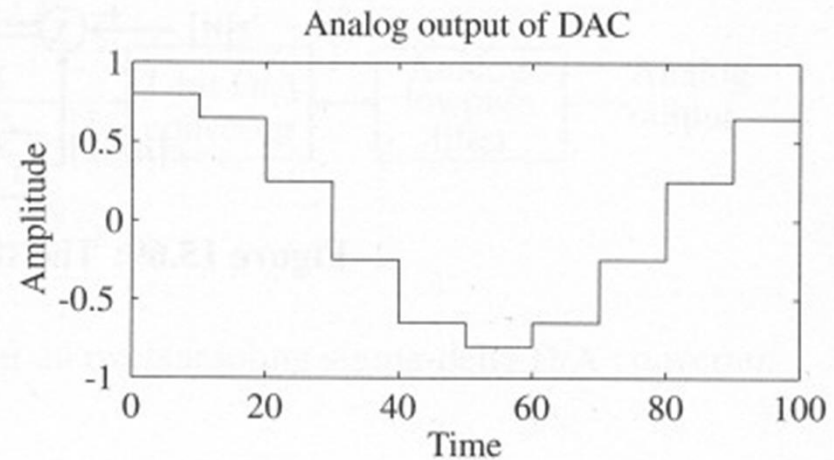
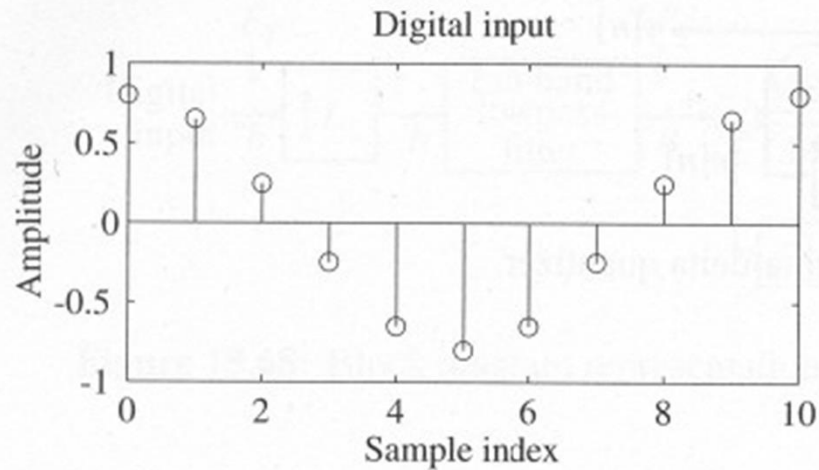
Oversampling DA konverzija

- DA konverzija se sastoji od konverzije u kontinualni signal i analognog filtra
- Ako je učestanost odabiranja blizu dvostrukoj najvećoj učestanosti korisnog signala, filter mora da ima oštru karakteristiku, da bude visokog reda, od skupih komponenti
- Da se značajno poveća učestanost odabiranja, digitalnim filtrom potisnu neželjene komponente, upotrebi ekonomičan analogni filter (niskog reda od komponenti koje nisu skupe) koji ima široku prelaznu zonu

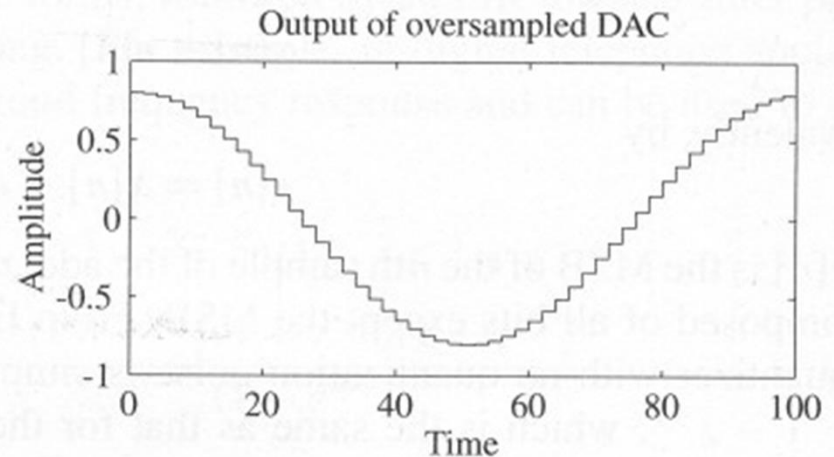
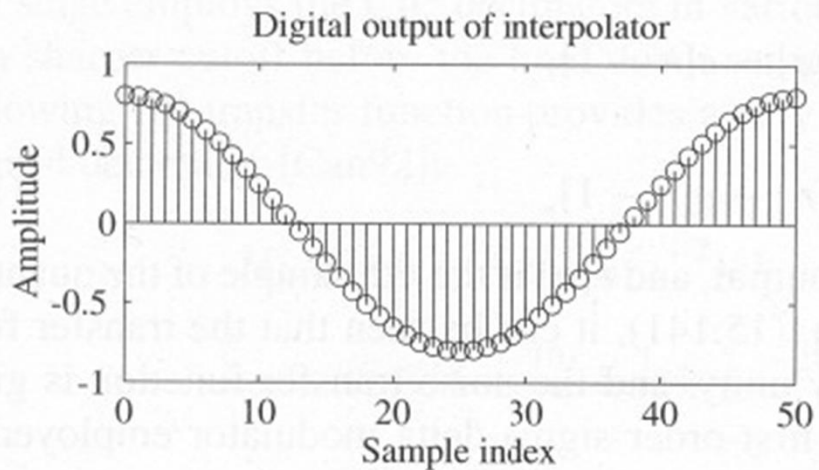
Realizacija DA konverzije



Ulazni i izlazni signal DA konvertora

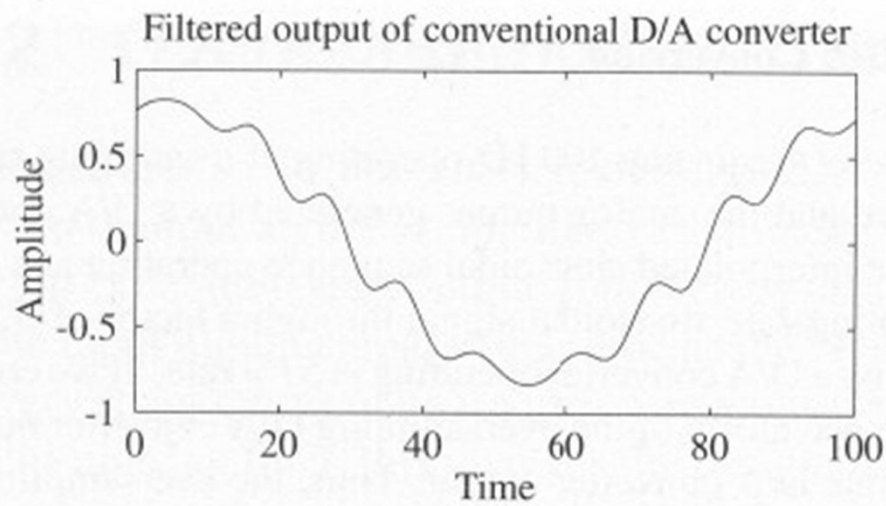


(a)

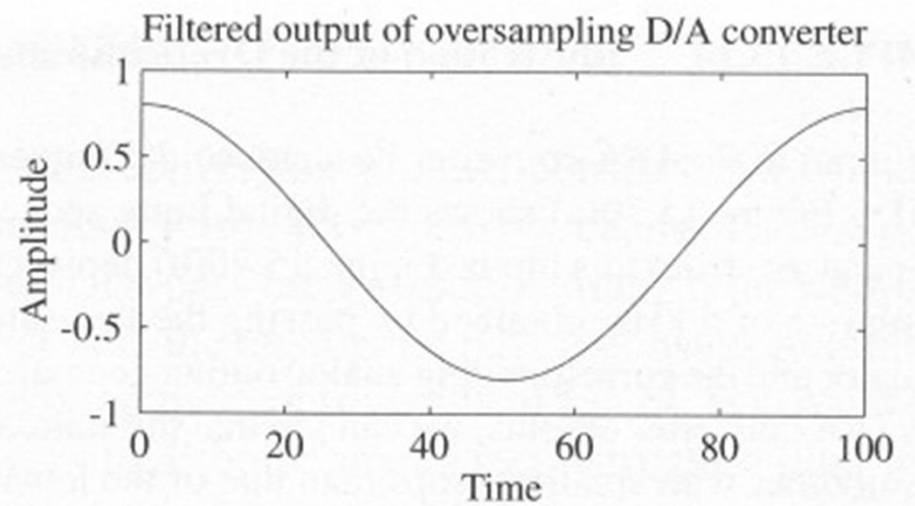


Procesiranje signala

Filtrirani signal



(a)



(b)

Profesor dr Miroslav Lutovac

mlutovac@viser.edu.rs

Ova prezentacija je nekomercijalna.

Slajdovi mogu da sadrže materijale preuzete sa Interneta, stručne i naučne građe, koji su zaštićeni Zakonom o autorskim i srodnim pravima.

Ova prezentacija se može koristiti samo privremeno tokom usmenog izlaganja nastavnika u cilju informisanja i upućivanja studenata na dalji stručni, istraživački i naučni rad i u druge svrhe se ne sme koristiti –

Član 44 - Dozvoljeno je bez dozvole autora i bez plaćanja autorske naknade za nekomercijalne svrhe nastave:

(1) javno izvođenje ili predstavljanje objavljenih dela u obliku neposrednog poučavanja na nastavi;

- ZAKON O AUTORSKOM I SRODNIM PRAVIMA

("Sl. glasnik RS", br. 104/2009 i 99/2011)