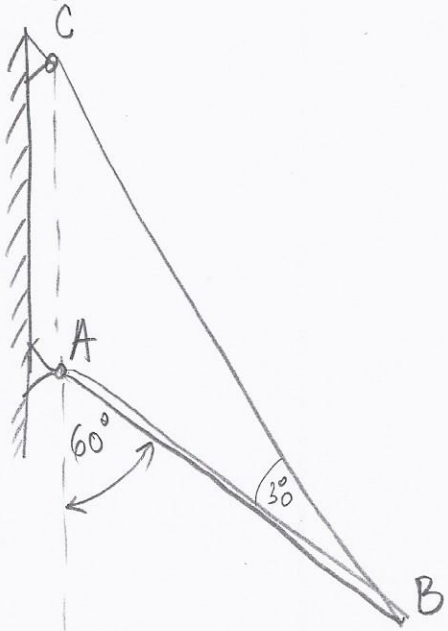
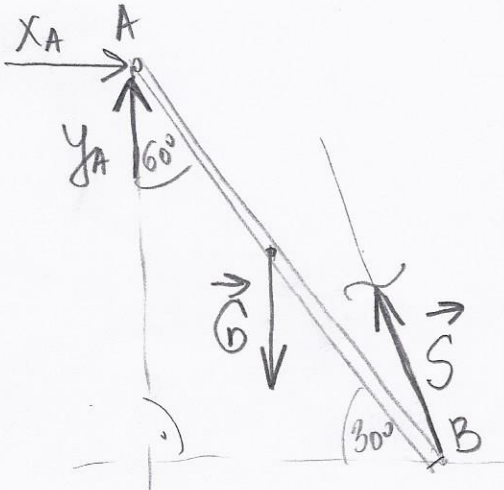
ZADACI ZA DOMAĆI

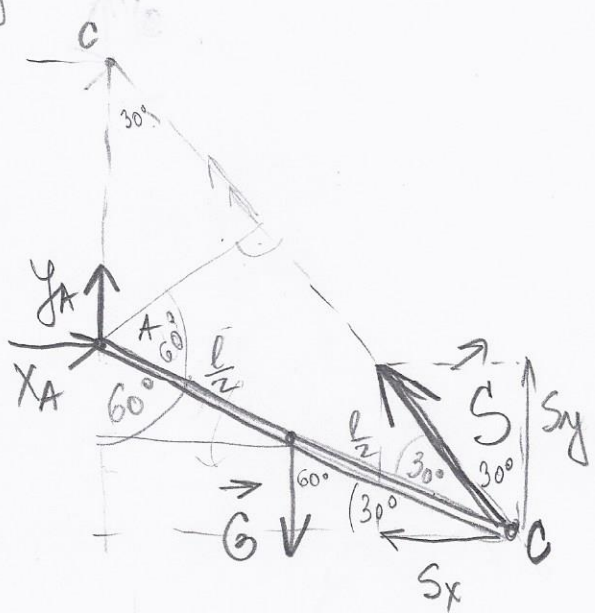
✓ZADATAK 1: Homogeni štap AB dužine L i težine $G = 2\,daN$ krajem A je zglobno vezan za vertikalni nepokretni zid, a krajem B je vezan za neistegljivo uže, koje je u tački C vezano za zid. U položaju prikazanom na slici, odrediti reakcije zgloba A i užeta BC. Težinu užeta zanemariti.



Rešenje:

Posmatrajmo štap AB i oslobodimo ga od veza.





Postavimo koordinatni sistem u tački A, pa postavimo 3 j-ne za uslove ravnoteže.

(1) $\sum X_i = 0 \Rightarrow X_A - S\cos 60° = 0$

(2) $\sum Y_i = 0 \Rightarrow Y_A - G + S\cos 30° = 0$

(3) $\sum M_A^{Fi} = 0 \Rightarrow -G \cdot \frac{l}{2}\cos 30° + S \cdot l \cos 60° = 0$

Iz j-ne (3) $\rightarrow$ $G \cdot \frac{l}{2} \cdot \cos 30 = S \cdot l \cos 60°$

$$S = G \cdot \frac{l}{2} \cdot \frac{\cos 30°}{l \cos 60°}$$

$$S = \frac{G}{2} \cdot \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$\boxed{S = \frac{G\sqrt{3}}{2}} \Rightarrow S = \frac{2 daN \sqrt{3}}{2} = \boxed{\sqrt{3} \cdot 10\ N}$$

Iz j-ne (1) $\rightarrow$ $X_A = S\cos 60°$

$$\boxed{X_A = \frac{G\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{G\sqrt{3}}{4} = \frac{2 \cdot 10^3 N \cdot \sqrt{3}}{4} = \frac{\sqrt{3}}{2} \cdot 10^3 N}$$

Iz j-ne (2) $\rightarrow$ $Y_A = G - S\cos 30°$

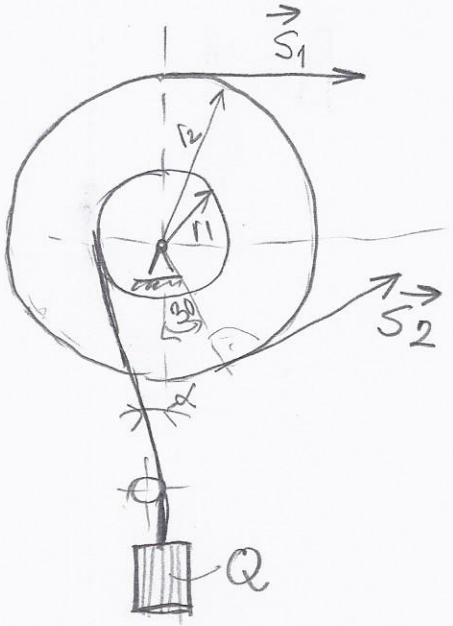
$$Y_A = G - \frac{G\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$Y_A = G - \frac{3G}{4}$$

$$Y_A = \frac{4G - 3G}{4}$$

$$\boxed{Y_A = \frac{1}{4}G} = \frac{1}{4} \cdot 2 \cdot 10^3 N = \boxed{\frac{1}{2} \cdot 10^3 N}$$

ZADATAK 2: Prenosnik prikazan na slici služi za dizanje tereta težine Q posredstvom remena. Kolici se teret može dizati pomoću ovog uređaja i kolica je reakcija u tački A, ako je zadato:
$S_1 = 350\,kN$; $S_2 = 150\,kN$; $r_1 = 0{,}5\,m$; $r_2 = 1{,}5\,m$, $\alpha = 15°$, $\beta = 30°$?



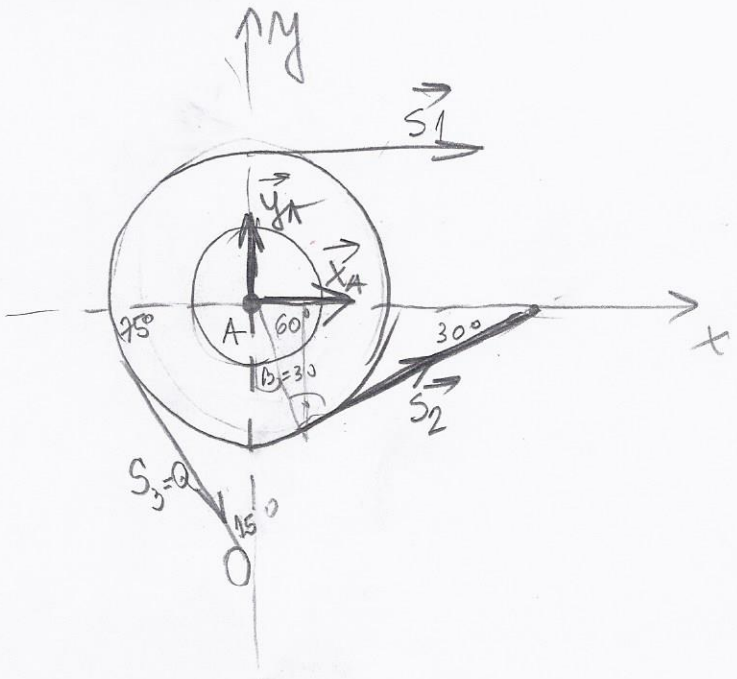
Slika 1

Rešenje:

Oslobađanjem od veza dobijamo sledeće slike i j-ne:

$S_3 = 0$

Slika 2



Slika 3

Projektujemo j-ne u koordinatnom sistemu Oxy:

(1) $\sum X_i = 0 \rightarrow X_A + S_1 + S_2 \cos 30^\circ + Q \cdot \sin 15 = 0$

(2) $\sum Y_i = 0 \rightarrow Y_A - S_3 \cos 15 + S_2 \sin 30^\circ = 0$ $\quad \{ \boxed{S_3 = Q} \}$

(3) $\sum M_A^{\vec{F_i}} = 0 \rightarrow - S_1 \cdot r_2 + S_2 \cdot r_2 + Q \cdot r_1 = 0$

Iz j-ne (3) $\rightarrow$ $Q \cdot r_1 = S_1 r_2 - S_2 \cdot r_2$

$$Q = \frac{S_1 r_2 - S_2 r_2}{r_1}$$

$$Q = \frac{350\,kN \cdot 1{,}5\,m - 150\,kN \cdot 1{,}5\,m}{0{,}5\,m}$$

$$Q = \frac{450\,kNm}{0{,}5\,m}$$

$$\boxed{Q = 600\,kN}$$

Iz j-ne (1) $\rightarrow$ $X_A = -S_1 - S_2 \cdot \cos 30^\circ - Q \sin 15$

$$X_A = -350\,kN - 150\,kN \cdot \frac{\sqrt{3}}{2} - 600\,kN \cdot 0{,}259$$

$$\boxed{X_A = -635{,}04\,kN} \approx 635\,kN \text{ (promenili smer)}$$

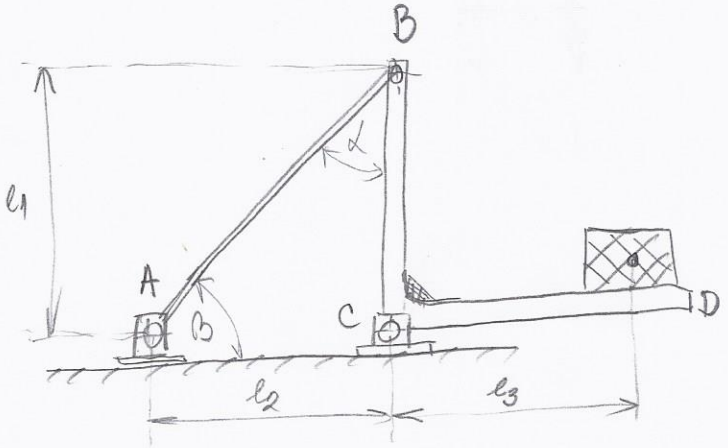
Iz j-ne (2) $\rightarrow$ $Y_A = S_3 \cos 15^\circ - S_2 \sin 30^\circ$

$$Y_A = 600\,kN \cdot 0{,}966 - 150\,kN \cdot \frac{1}{2}$$

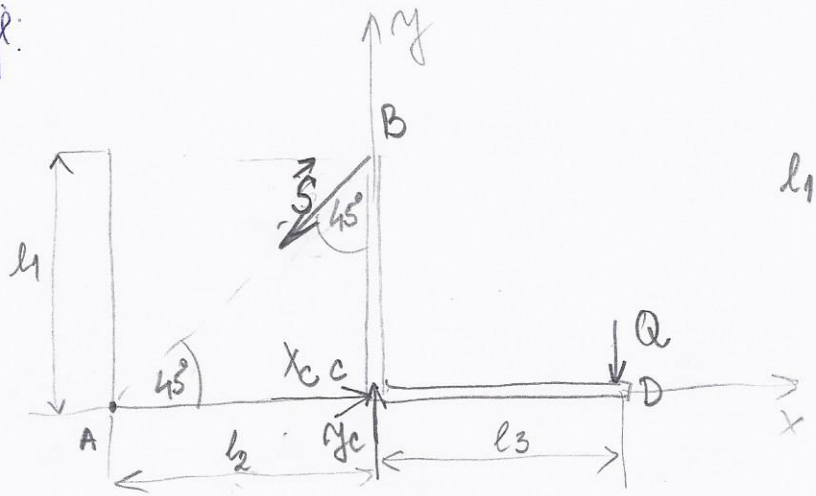
$$Y_A = 504{,}60\,kN$$

$$\boxed{Y_A \approx 505\,kN}$$

Zadatak 3: Platforma prema slici vezana je zglobno osovinicom u tački C i lakim štapom AB u tački B. Odrediti intenzitet reakcije zglobne veze (silu F_C) i silu u štapu, ako je maksimalna težina tereta koju platforma treba da nosi: $Q_{max} = 20\,kN$. Poznate su dužine: $l_1 = l_2 = 2\,m$ i $l_3 = 1{,}5\,m$



Rešenje:



$l_1 = l_2 \Rightarrow \alpha = \beta = 45°$

(1) $\sum X_i = 0 \rightarrow X_C - S\cos 45 = 0$

(2) $\sum Y_i = 0 \rightarrow Y_C - Q - S\cos 45° = 0$

(3) $\sum M_C^{Fi} = 0 \rightarrow +S\cos 45° \cdot l_1 - Q \cdot l_3 = 0$

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iz (3) $\rightarrow$ $l_1 S\cos 45 = Q \cdot l_3$

$S \cdot \frac{\sqrt{2}}{2} \cdot 2\,m = 20\,kN \cdot 1{,}5\,m$

$S = \frac{30\,kNm\sqrt{2}}{2\,m} = 15\sqrt{2}\,kN$

Iz j-ne (1) $\rightarrow$ $X_c = S \cdot \frac{\sqrt{2}}{2}$

$X_c = 15\sqrt{2} \cdot \frac{\sqrt{2}}{2}$ KN

$\boxed{X_c = 15\,kN}$

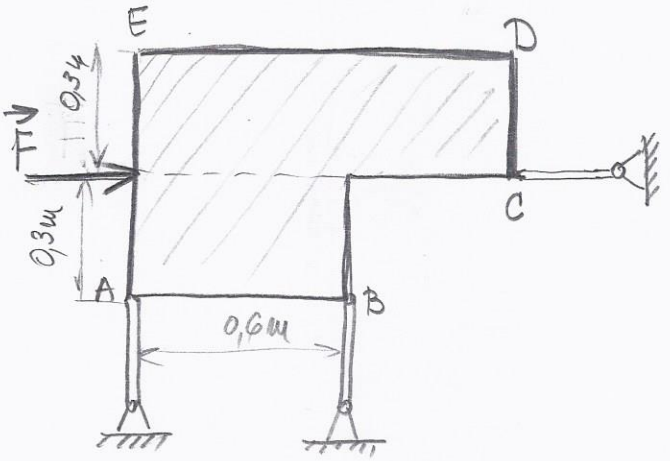
Iz j-ne (2) $\Rightarrow$ $Y_c = Q + S\cos 45°$

$Y_c = 20kN + 15\sqrt{2} \cdot \frac{\sqrt{2}}{2}$ KN

$Y_c = 20kN + 15\,kN$

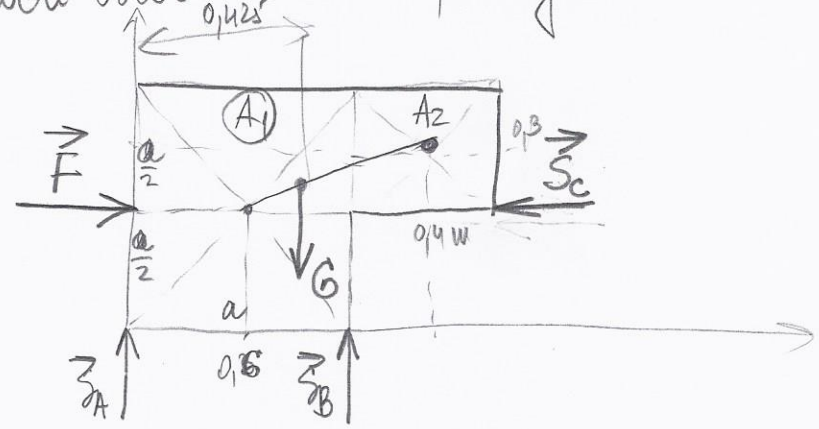
$\boxed{Y_c = 35kN}$

Zadatak 4: Homogena ploča ABCDE konstantne debljine prema slici opterećena je silom $F = 10\,kN$, a sopstvena težina ploče $G = 6\,kN$. Analitički odrediti sile u štapovima 1, 2 i 3.



Rešenje:

Ako homogenu ploču oslobodimo veza, dobijamo:



Koordinate težišta, gde ćemo smestiti silu G, odredićemo prema jednačini:

$$x_c = \frac{\sum A_i x_i}{A} \; ; \; y_c = \frac{\sum A_i y_i}{A}$$

$$A_1 = a^2 = (0{,}6m)^2 = 0{,}36\,m^2$$

$$A_2 = a \cdot b = 0{,}4 \cdot 0{,}3 = 0{,}12\,m^2$$

U koordinatnom sistemu xAy:

$x_{c1} = 0{,}3\,m$ | $x_{c2} = 0{,}8\,m$

$y_{c1} = 0{,}3\,m$ | $y_{c2} = 0{,}3 + \frac{0{,}3}{2} = 0{,}45\,m$

$$x_c = \frac{A_1 \cdot x_{c1} + A_2 \cdot x_{c2}}{A_1 + A_2} = \frac{0{,}3\,m \cdot 0{,}36\,m^2 + 0{,}12\,m^2 \cdot 0{,}8}{0{,}48\,m^2}$$

$$x_c = \frac{0{,}204\,m^3}{0{,}48\,m^2} = 0{,}425\,m$$

$$y_c = \frac{A_1 \cdot y_1 + A_2 \cdot y_2}{A_1 + A_2} = \frac{0{,}36\,m^2 \cdot 0{,}3\,m + 0{,}12\,m^2 \cdot 0{,}45\,m}{0{,}48\,m^2} = \frac{0{,}162\,m^3}{0{,}48\,m^2}$$

$$y_c = 0{,}3375$$

Sada pišemo j-ne za ravnotežu:

(1) $\sum X_i = 0 \Rightarrow F - S_C = 0$

(2) $\sum Y_i = 0 \Rightarrow S_A + S_B - G = 0$

(3) $\sum M_A^{\vec{F}} = 0 \Rightarrow -F \cdot 0{,}3\,m - G \cdot 0{,}425\,m + S_C \cdot 0{,}3\,m + S_B \cdot 0{,}6\,m = 0$

Iz j-ne (1) $\Rightarrow S_C = F \Rightarrow S_C = 10\,kN$

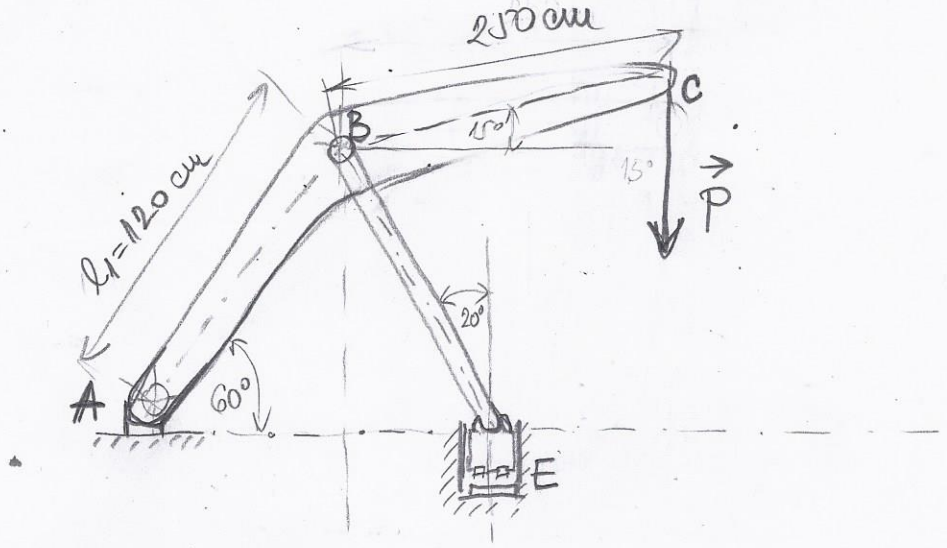
Iz j-ne (3) $\Rightarrow S_B \cdot 0{,}6\,m = F \cdot 0{,}3\,m + G \cdot 0{,}425 - S_C \cdot 0{,}3\,m$

$$S_B = \frac{10\,kN \cdot 0{,}3\,m + 6\,kN \cdot 0{,}425\,m - 10\,kN \cdot 0{,}3\,m}{0{,}6\,m}$$

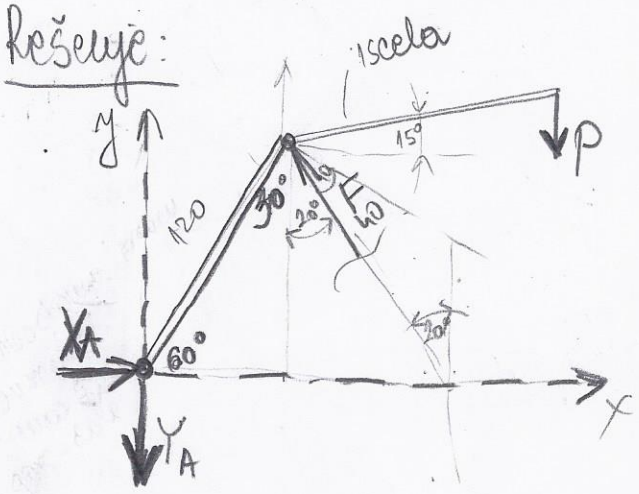
$$S_B = 4{,}25\,kN$$

Iz j-ne (2) $\Rightarrow S_A = G - S_B \Rightarrow S_A = 1{,}75\,kN$

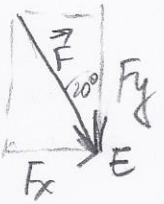
Zadatak 5: Presa prikazana na slici služi za zatvaranje malih konzervi. Ako je veličina sile $P = 250\,kN$, odrediti veličinu vertikalne sile (radnu silu) koja deluje na poklopac konzerve E i reakcija u osloncu A. Težinu elementa D zanemariti.



Rešenje:



Na poklopac deluje sila:



$$(1)\ \sum X_i = 0 \rightarrow -F\sin 20^\circ + X_A = 0$$

$$(2)\ \sum Y_i = 0 \rightarrow -Y_A + F\cos 20^\circ - P = 0$$

$$(3)\ \sum M_A^{\vec{F}_i} = 0 \rightarrow F\cos 40^\circ \cdot l_1 - P(l_2\cos 15^\circ + l_1\cos 60^\circ) = 0$$

$$12\ \text{j-ne}(3) \rightarrow F\cos 40^\circ \cdot l_1 = P\cdot(l_2\cos 15^\circ + l_1\cos 60^\circ)$$

$$F = \frac{P(l_2\cos 15^\circ + l_1\cos 60^\circ)}{\cos 40^\circ \cdot l_1}$$

$$F = \frac{250\,kN \cdot (2{,}5m \cdot 0{,}966 + 1{,}2m \cdot \frac{1}{2})}{0{,}766 \cdot 1{,}2m}$$

$$F = \frac{250\,kN \cdot 3{,}015}{0{,}9192\,m}$$

$$\boxed{F = 820{,}006\,kN}$$

Iz j-ne (2) $\rightarrow$ $Y_A = P - F\cos 20^\circ$

$Y_A = 250\,kN - F \cdot 0{,}9397$

$$\boxed{Y_A = -520{,}55\,kN}$$

Iz j-ne (1) $\rightarrow$ $X_A = F \sin 20^\circ$

$X_A = 820\,kN \cdot 0{,}342$

$$\boxed{X_A = 280{,}44\,kN}$$