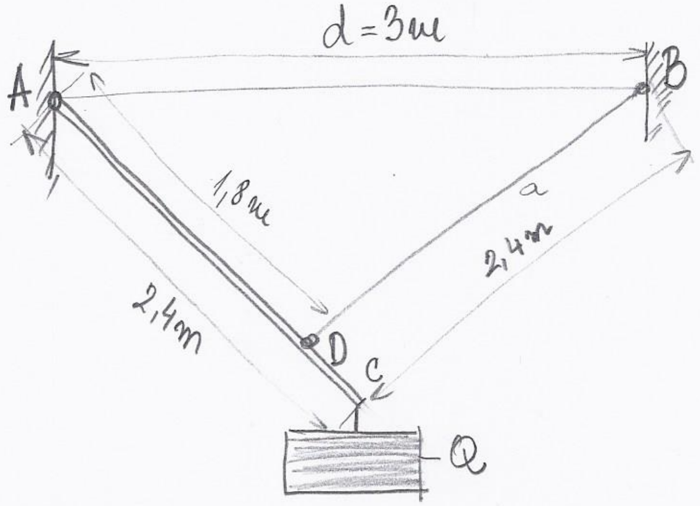
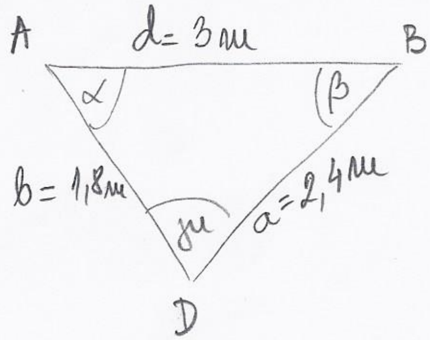
Zadatak 1: Homogeni štap AC, dužine AC = l = 2,4m i težine G, zglobno je vezan za vertikalni zid u tački A. Za kraj štapa, u tački C, privezan je teret težine Q = G. Štap u nagnutom položaju održava uže, zanemarljive težine, dužine BD = L = 2,4m, koje je vezano u tački D na rastojanju AD = 1,8m. Naći reakciju u zglobu A i silu u užetu. Sistem tela se nalazi u vertikalnoj ravni homogenog polja zemljine teže.



Rešenje:

Prvo moramo da odredimo uglove u trouglu Δ ABD. Za to ćemo primeniti kosinusnu teoremu, pa sinusnu:



$$a^2 = b^2 + d^2 - 2bd\cos\alpha$$

$$2bd\cos\alpha = b^2 + d^2 - a^2$$

$$\cos\alpha = \frac{b^2 + d^2 - a^2}{2bd}$$

$$\cos\alpha = \frac{1{,}8^2 + 3^2 - 2{,}4^2}{2 \cdot 1{,}8 \cdot 3}$$

$$\boxed{\cos\alpha = \frac{6{,}48}{10{,}80} = 0{,}6}$$

$$\sin\alpha = \sqrt{1 - \cos^2\alpha}$$

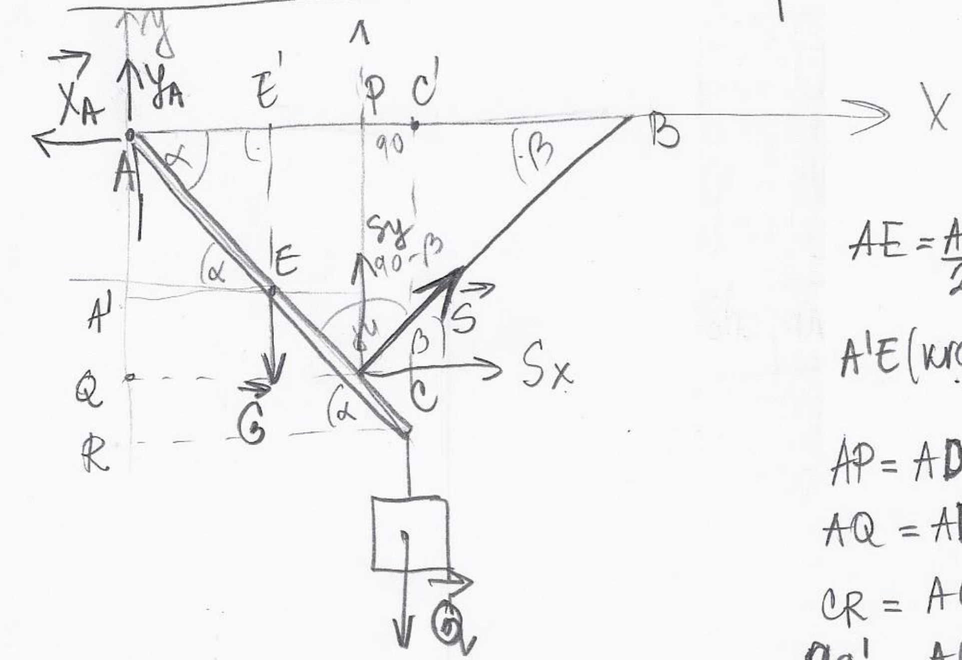
$$\sin\alpha = \sqrt{1 - 0{,}36}$$

$$\sin\alpha = \sqrt{0{,}64}$$

$$\boxed{\sin\alpha = 0{,}8}$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow \sin\beta = \frac{b\sin\alpha}{a} = \frac{1{,}8 \cdot 0{,}8}{2{,}4} = 0{,}6 \quad (\cos\beta = 0{,}8)$$

Oslobađamo se od veza kod štapa AC:



$$AE = \frac{AC}{2} = \frac{2{,}4m}{2} = 1{,}2m$$

$$A'E\,(\text{krak sile } G) = AE\cos\alpha = 1{,}2m\cdot 0{,}6 = 0{,}72m$$

$$AP = AD\cos\alpha = 1{,}8m\cdot 0{,}6 = 1{,}08m$$

$$AQ = AD\sin\alpha = 1{,}8m\cdot 0{,}8 = 1{,}44m$$

$$CR = AC\cos\alpha = 2{,}4m\cdot 0{,}6 = 1{,}44m$$

$$RC' = AC\sin\alpha = 2{,}4m\cdot 0{,}8 = 1{,}92m$$

$$AE' = AE\cos\alpha = 1{,}2m\cdot 0{,}6 = 0{,}72m$$

$$S_x = S\cos\beta$$

$$S_y = S\sin\beta$$

(1) $\sum X_i = 0 \Rightarrow -X_A + S\cos\beta = 0$

(2) $\sum Y_i = 0 \Rightarrow Y_A - G - Q + S\sin\beta = 0$

(3) $\sum M_A^{F_i} = 0 \Rightarrow -Q\cdot RC' - G\cdot AE' + S_y\cdot AP + S_x\cdot AQ = 0$

$$\sum M_A^{F_i} = 0 \Rightarrow -Q\cdot 1{,}92m - G\cdot 0{,}72m + S\sin\beta\cdot 1{,}08m + S\cos\beta\cdot 1{,}44m = 0$$

$$S(\sin\beta\cdot 1{,}08m + \cos\beta\cdot 1{,}44m) = Q\cdot 1{,}92m + G\cdot 0{,}72m$$

$$S\cdot(0{,}6\cdot 1{,}08m + 0{,}8\cdot 1{,}44m) = G\cdot 1{,}92m + G\cdot 0{,}72m$$

$$S\cdot(0{,}648m + 1{,}152m) = G(1{,}92m + 0{,}72m)$$

$$S\cdot 1{,}8m = G\cdot 2{,}64m$$

$$S = G\cdot\frac{2{,}64m}{1{,}80m} = G\cdot 1{,}47$$

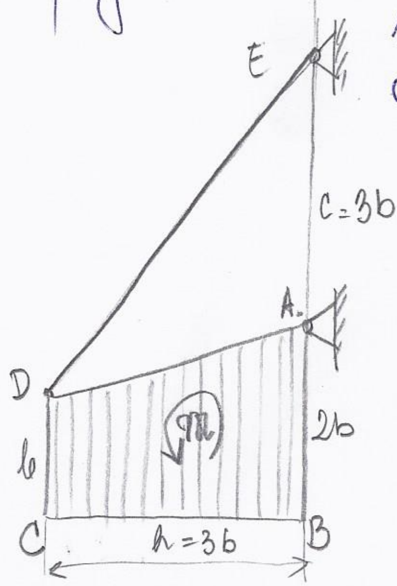
$$\boxed{S = 1{,}47G}$$

(1) $\Rightarrow X_A = S\cos\beta = 1{,}47\cdot G\cdot 0{,}8 = 1{,}176\cdot G$

(2) $\Rightarrow Y_A = G + G - S\sin\beta$

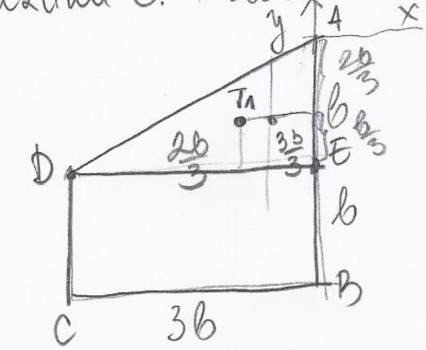
$$Y_A = 2G - 1{,}47\cdot G\cdot 0{,}6$$

Zadatak 2: Homogena ploča ABCD ima oblik trapeza sa stranicama $AB = 2b$; $CD = b$ i težinom G. Ploča je u temenu A vezana nepokretnim zglobom, a u tački D za ploču je vezano uže DE zanemarljive težine. Drugi kraj užeta je vezan za tačke E, koja se nalazi na vertikali iznad oslonca A, a na udaljenost $AE = c = 3b$. U ravni ploče deluje moment sprega $\mathfrak{M} = \frac{7}{15} Gb$. Ploča se nalazi u vertikalnoj ravni u polju teže. strana AB ($AB \parallel DC$). Visina trapeza je 3b. Naći silu u užetu i reakciju u zglobu A u f-ji od težine G



Rešenje:

Za početak, potrebno je odrediti težište trapeza ABCD, jer u njemu deluje težina G. Postavimo koordinatni sistem u tački A:



Površina Δ AED je: $\frac{1}{2} b \cdot 3b \Rightarrow P_\Delta = \frac{3b^2}{2}$

Površina $\square$ EBCD je: $b \cdot 3b \Rightarrow P_\square = 3b^2$

$$P_{ABCD} = \frac{3b^2}{2} + 3b^2 = \frac{9b^2}{2}$$

Težište Δ AED$_1$: $x_{T_1} = -\frac{3b}{3} = -b$

$y_{T_2} = -\frac{2b}{3}$

Težište $\square$:

$y_{T2} = -b - \frac{b}{2} = -\frac{3b}{2}$

$x_{T2} = -\frac{3b}{2}$

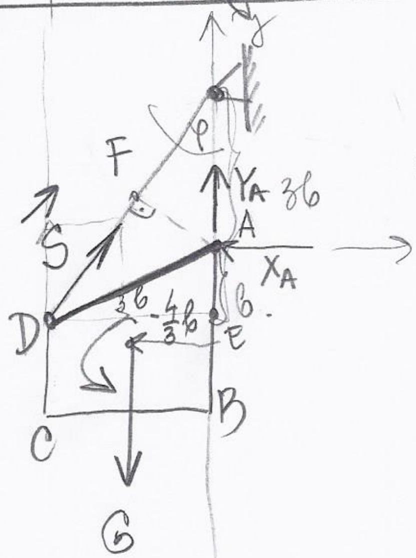
Koordinate težišta su:

$$\Sigma A_i x_i \quad -b \cdot \frac{3b^2}{2} - \frac{3b}{2} \cdot 3b^2 \qquad -\frac{3b^2}{2} \cdot \left(-\frac{2b}{2} - \frac{3b}{2}\right) = -\frac{3b^2}{2} \cdot 4b$$

②

$$\boxed{x_T = -\frac{\frac{12b^2}{2}\cdot b}{\frac{9b^2}{2}} = -\frac{12}{9}b = -\frac{4}{3}b}$$

Sada pišemo j-ne za određivanje ravnoteže:



(1) $\sum X_i = 0 \Rightarrow S\sin\varphi - X_A = 0$

(2) $\sum Y_i = 0 \Rightarrow S\cos\varphi + Y_A - G = 0$

(3) $\sum M_A^{Fi} = 0 \Rightarrow G\cdot\frac{4}{3}b - S\cdot\frac{9}{5}b + \mathfrak{M} = 0$

Iz (3) $\rightarrow$ $S\cdot\frac{9}{5}b = G\cdot\frac{4}{3}b + \mathfrak{M}$

$S\cdot\frac{9}{5}\cancel{b} = \frac{4}{3}\cancel{b}\cdot G + \frac{7}{15}G\cdot\cancel{b}$

$S = \frac{5G}{9}\cdot\left[\frac{4}{3} + \frac{7}{15}\right]$

$S = \frac{5}{9}G\cdot\frac{20+7}{15}$

$S = \frac{\cancel{27}}{\cancel{15}}\cdot\frac{\cancel{5}}{\cancel{9}}G$

$$\boxed{S = G}$$

Pomoć:

Ugao φ:

$tg\,\varphi = \frac{3b}{4b} = \frac{3}{4}$

$\sin\varphi = \frac{tg\,\varphi}{\sqrt{1+tg^2\varphi}} = \frac{3}{5}$

$\cos\varphi = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}}$

$$\boxed{\cos\varphi = \frac{4}{5}}$$

Krak sile S:

$AF = DF = c\sin\varphi = 3b\cdot\frac{3}{5} = \frac{9}{5}b$

Iz (1) $S\sin\varphi = X_A$

$X_A = G\cdot\sin\varphi$

$$\boxed{X_A = G\cdot\frac{3}{5}}$$

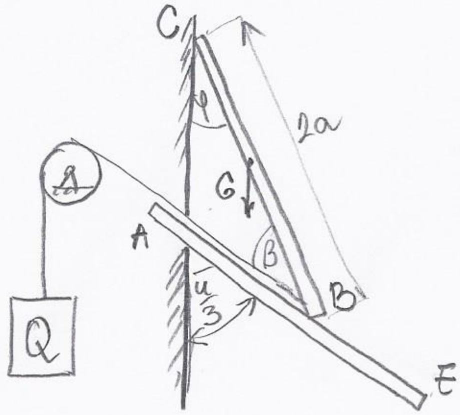
Iz (2) $Y_A = G - S\cos\varphi$

$Y_A = G - G\cdot\cos\varphi$

$Y_A = G\left(1-\frac{4}{5}\right)$

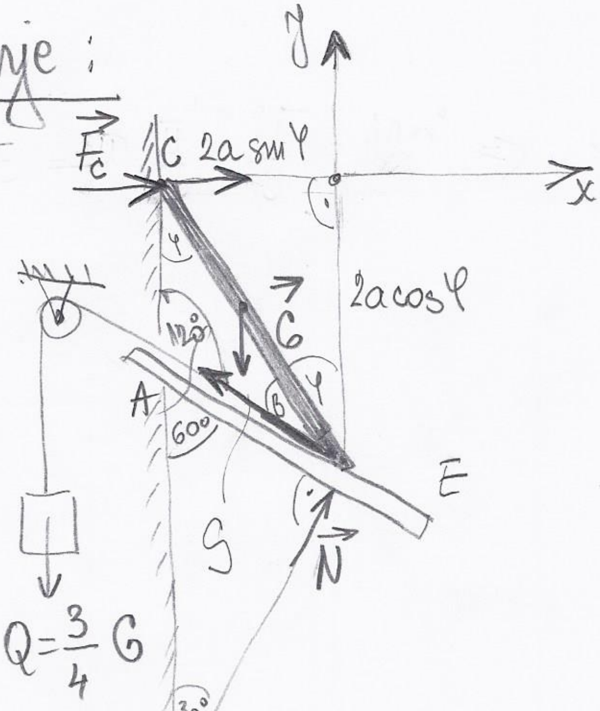
$$\boxed{Y_A = \frac{1}{5}G}$$

Zadatak 3: Homogeni štap BC, težine G i dužine 2a, oslanja se krajevima B i C na glatku konzolu AE i gladak vertikalni zid. U ravnotežnom stanju, štap održava teret Q, koji je vezan preko nerastegljivog užeta, paralelnog sa konzolom, za kraj B štapa. Uzimajući da je $Q = \frac{3}{4}G$, naći za ravnotežni položaj ugao α i reakcije u tačkama C i B.

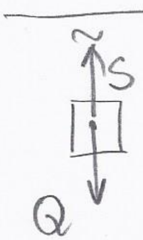


$$Q = \frac{3}{4}G$$
$$\overline{BC} = 2a$$
$$\alpha = \frac{\pi}{3} = 60^\circ$$

Rešenje :

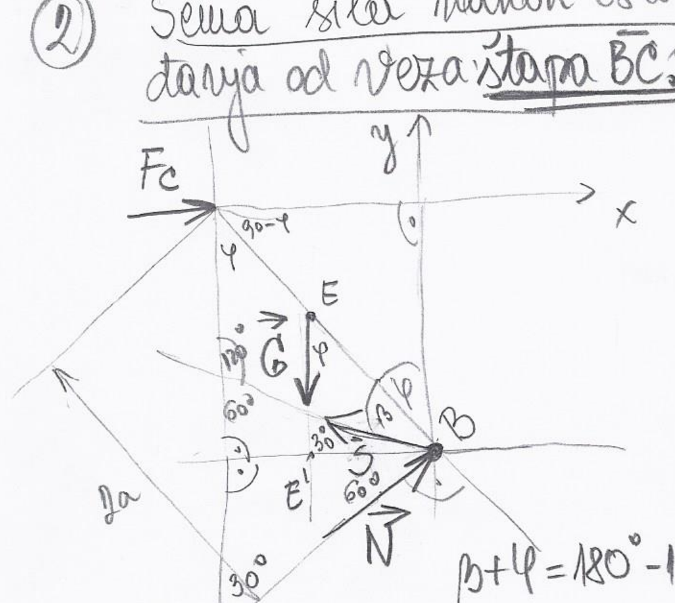


① Oslobađanje od veze tega Q:



$$\boxed{S = Q = \frac{3}{4}G}$$

② Šema sila nakon oslobađanja od veza štapa $\overline{BC}$:



$\measuredangle\ CAE = 180^\circ - 60^\circ = 120^\circ$

$\beta + \varphi = 180^\circ - 120^\circ =$

Formiramo j-ne ravnoteže:

(1) $\sum X_i = 0 \Rightarrow \quad -S \cdot \cos(90^\circ - (\beta + \varphi)) + N \cos 60^\circ + F_c = 0$

$-\frac{3}{4} G \cos 30^\circ + N \cos 60^\circ + F_c = 0$

(2) $\sum Y_i = 0 \Rightarrow \quad -G + N \cos 30^\circ + S \cos(\beta + \varphi) = 0$

$-G + N \cos 30^\circ + \frac{3}{4} G \cos 60^\circ = 0$

(3) $\sum M_B^{\vec{R}_i} = 0 \Rightarrow \quad -F_c \cdot 2a \cos\varphi + G \cdot \underbrace{a \cdot \sin\varphi}_{BE'} = 0$

Iz j-ne (3) $\rightarrow F_c \cdot 2\cancel{a} \cos\varphi = G \cancel{a} \sin\varphi$

$$\boxed{F_c = \frac{1}{2} G \cdot tg\varphi}$$

Iz j-ne (1) $\rightarrow N \cos 60^\circ = \frac{3}{4} G \cos 30^\circ - F_c$

$N = \frac{1}{\cos 60} \cdot \left[\frac{3}{4} G \cos 30^\circ - \frac{1}{2} G\, tg\varphi\right]$

$N = \frac{1}{\frac{1}{2}} \cdot \left[\frac{3}{4} G \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} G \cdot tg\varphi\right]$

$N = \cancel{2} \cdot \frac{3}{4} \cdot \frac{\sqrt{3}}{\cancel{2}} G - \cancel{2} \cdot \frac{1}{\cancel{2}} G\, tg\alpha$

$N = \frac{3\sqrt{3}}{4} G - G\, tg\alpha$

Iz j-ne (2) $\rightarrow$ $N\cos 30^\circ = G - \frac{3}{4}G \cdot \cos 60^\circ$

$$N \cdot \frac{\sqrt{3}}{2} = G\left(1 - \frac{3}{4}\cdot\frac{1}{2}\right)$$

$$N \cdot \frac{\sqrt{3}}{2} = G \cdot \left(\frac{8-3}{8}\right)$$

$$N \cdot \frac{\sqrt{3}}{2} = G \cdot \frac{5}{8}$$

$$N \cdot \frac{\sqrt{3}}{\cancel{2}} = \cdot \frac{5}{4}$$

$$N = \frac{5}{4}\cdot\frac{\sqrt{3}}{3}G$$

$$\boxed{N = \frac{5\sqrt{3}}{12}G}$$

Izjednačavanjem N dobijenog iz j-ne 1 i N dobijenog iz j-ne 2 dobijamo φ:

$$\frac{3\sqrt{3}}{\cancel{4}}G - G\,tg\alpha = \frac{5\sqrt{3}}{12}G$$

$$\cancel{G}\left(\frac{3\sqrt{3}}{\cancel{4}} - tg\alpha\right) = \cancel{G}\cdot\frac{5\sqrt{3}}{12}$$

$$\frac{3\sqrt{3}}{4} - tg\alpha = \frac{5\sqrt{3}}{12}$$

$$tg\alpha = \frac{3\sqrt{3}}{4} - \frac{5\sqrt{3}}{12}$$

$$tg\alpha = \frac{9\sqrt{3} - 5\sqrt{3}}{12}$$

$$tg\alpha = \frac{4\sqrt{3}}{12}$$

$$tg\alpha = \frac{\sqrt{3}}{3} \Longrightarrow \boxed{\alpha = 30^\circ}$$