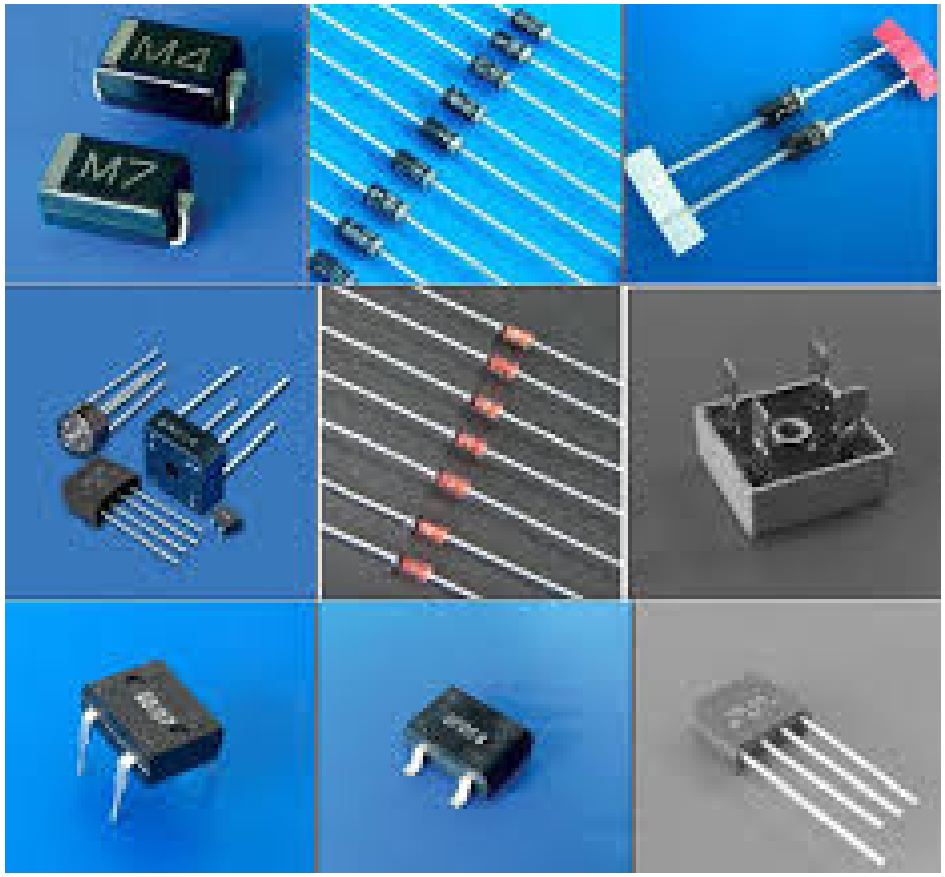
VISOKA ŠKOLA ELEKTROTEHNIKE I RAČUNARSTVA STRUKOVNIH STUDIJA-VIŠER, BEOGRAD

STUDIJSKI PROGRAM: Elektrotehničko inženjerstvo

MASTER STUDIJE 2017/2018

PREDMET: PROJEKTOVANJE ELEKTROENERGETSKIH PRETVARAČA

NEKONTROLISANI AC/DC ENERGETSKI PRETVARAČI (DIODNI ISPRAVLJAČI)

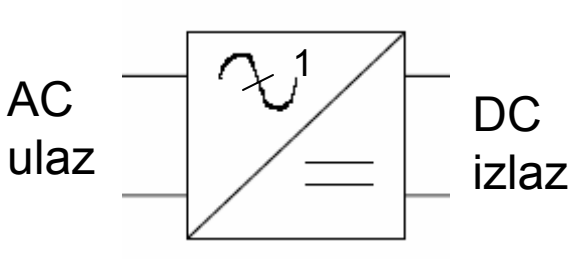


PREDMETNI PROFESOR:
Dr Željko Despotović, dipl.el.inž

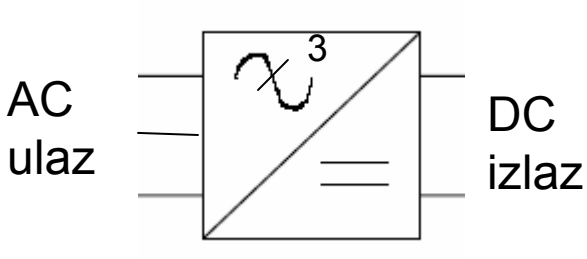
AC/DC pretvarači-ISPRAVLJAČI

AC ulaz: monofazni ili trofazni

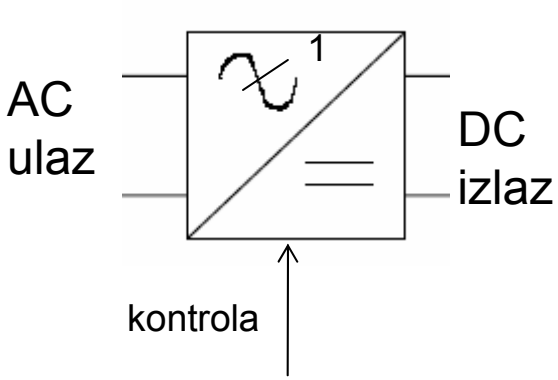
220V, 50/60Hz
3x380/220V, 50/60Hz

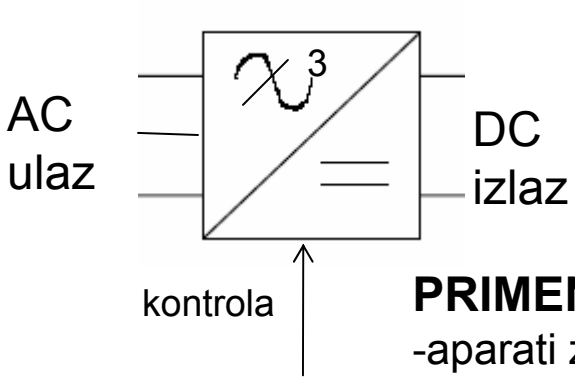


MONOFAZNI DIODNI ISPRAVLJAČI



TROFAZNI DIODNI ISPRAVLJAČI



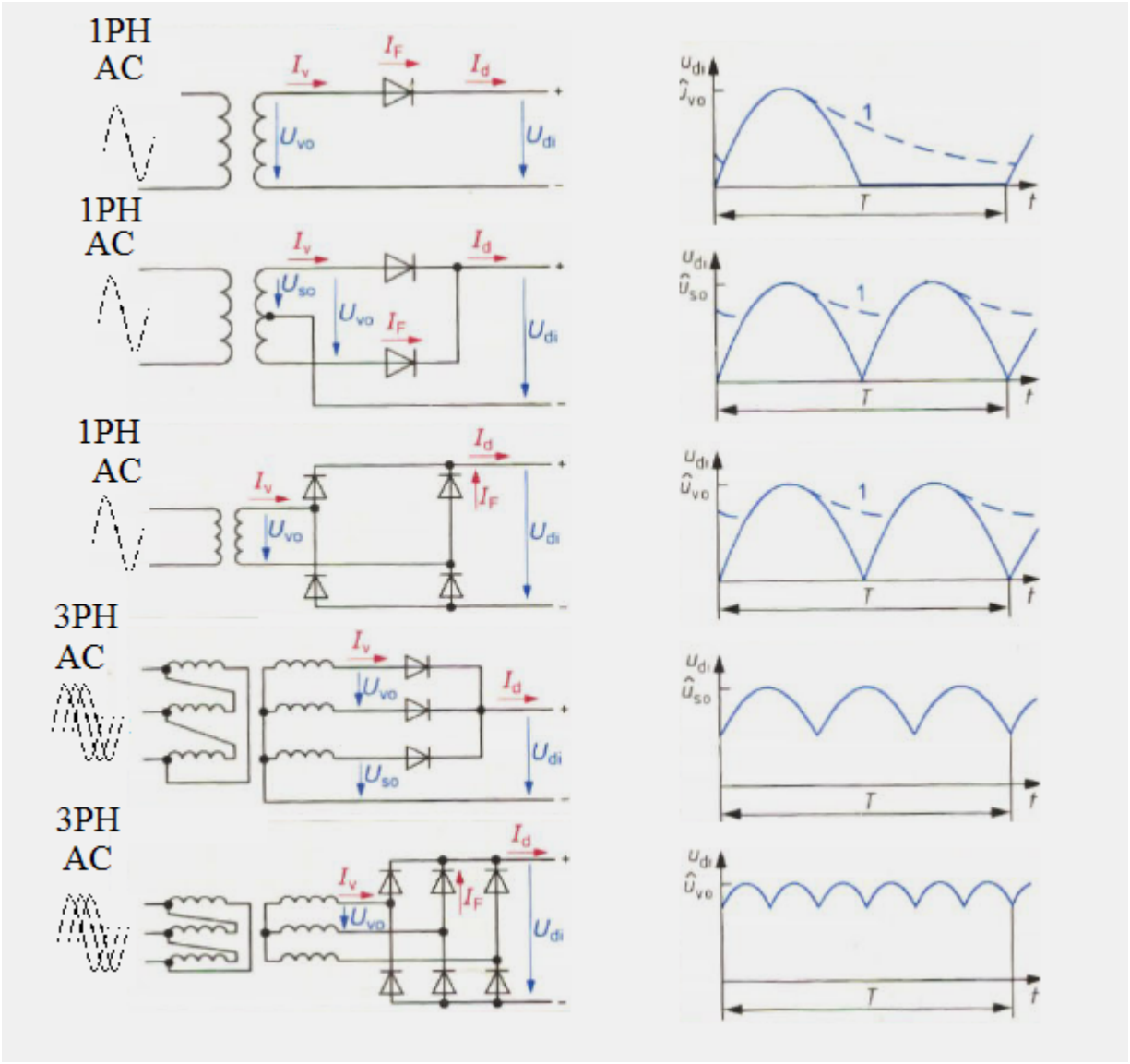


KONTROLISANI
TIRISTORSKI ili TRANZISTORSKI ISPRAVLJAČI
MONOFAZNI , TROFAZNI

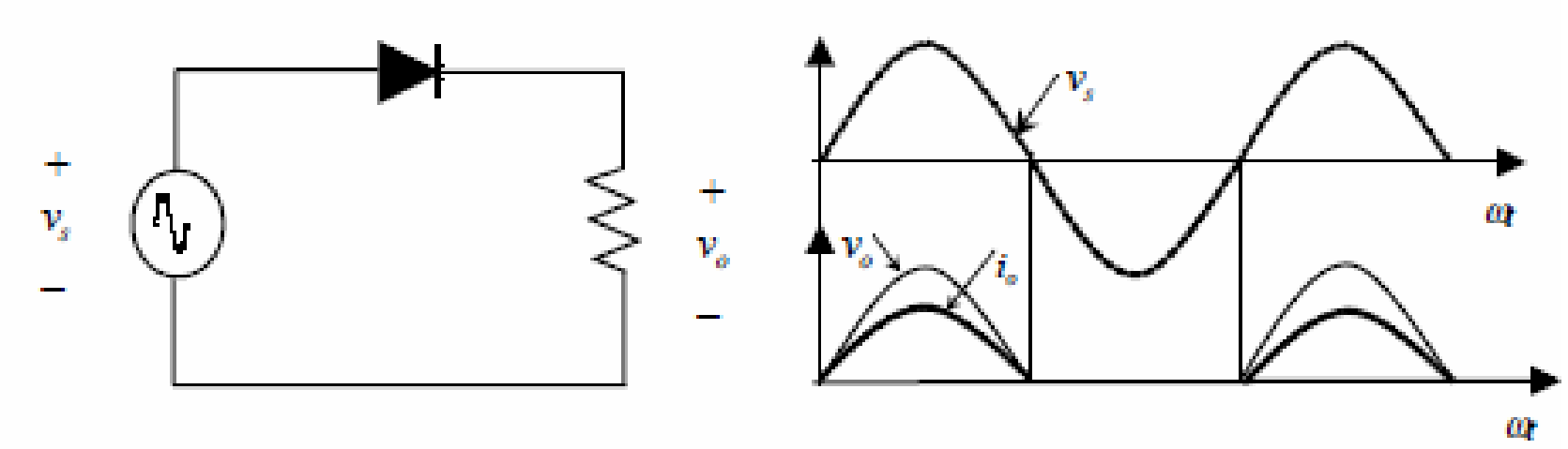
TIRISTORSKI: POLUUPRAVLJIVI I PUNOUPRAVLJIVI

PRIMENA:
-aparati za zavarivanje
-jednosmerni motorni pogoni
-punjači baterija
-jednosmerni izvori napajanja
-visokonaponske DC aplikacije
- DC transmisije dalekovodima

NEUPRAVLJIVI AC/DC PRETVARAČI DIODNI ISPRAVLJAČI



IDEALNI POLUTALSANI ISPRAVLJAČ: MONOFAZNI-OTPORNO OPTEREĆENJE



$$V_o = V_{avg} = V_{DC} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin(\omega t)\, d\omega t$$

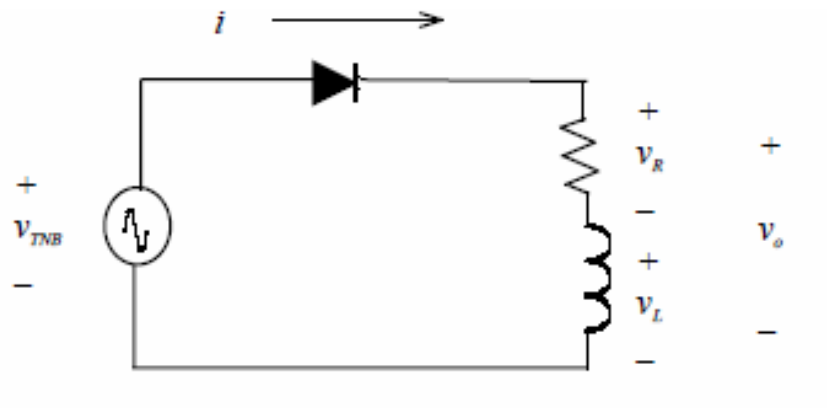
$$= \frac{V_m}{\pi} = 0.318 V_m$$

DC izlazni napon

$$V_{o,rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} (V_m \sin\omega t)^2\, d\omega t} = 0.5 V_m$$

RMS izlazni napon

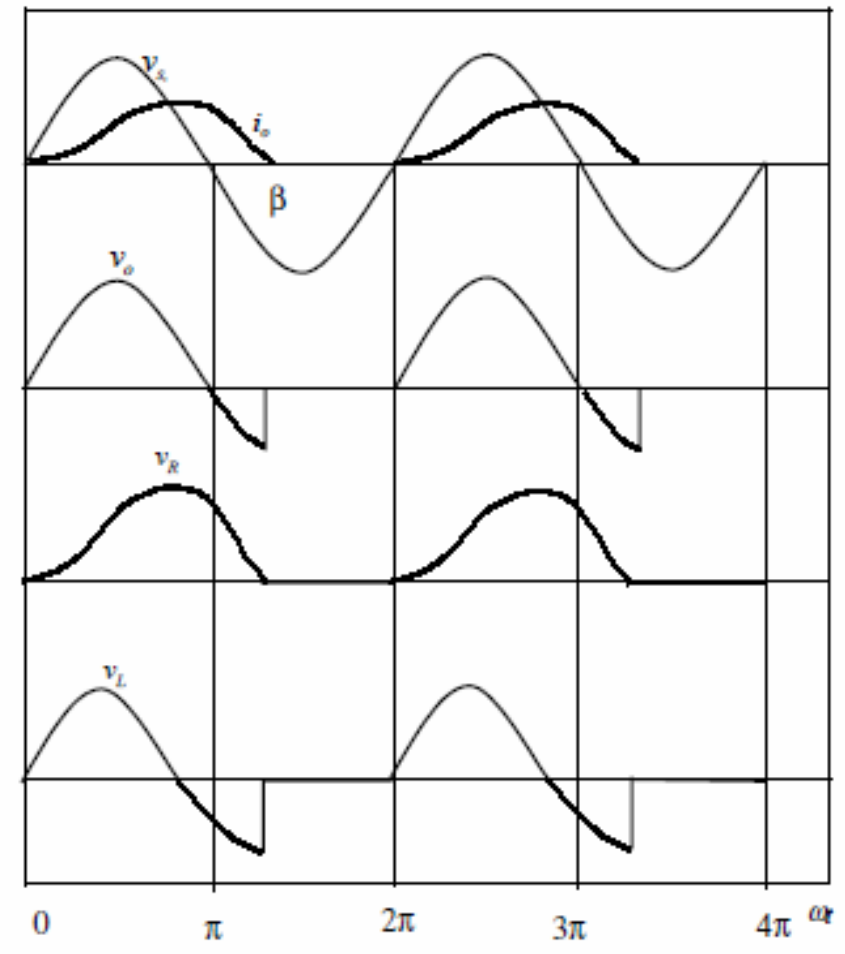
IDEALNI POLUTALSANI ISPRAVLJAČ: MONOFAZNI,R-L OPTEREĆENJE



$$v_s = v_R + v_L$$

$$V_m \sin(\omega t) = i(t)R + L\frac{di(t)}{dt}$$

$$i(\omega t) = \begin{cases} \left(\frac{V_m}{Z}\right)\cdot\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right] & \text{za} \quad 0 \le \omega t \le \beta \\ 0 & \text{za} \quad 2\pi > \omega t > \beta \end{cases}$$



$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

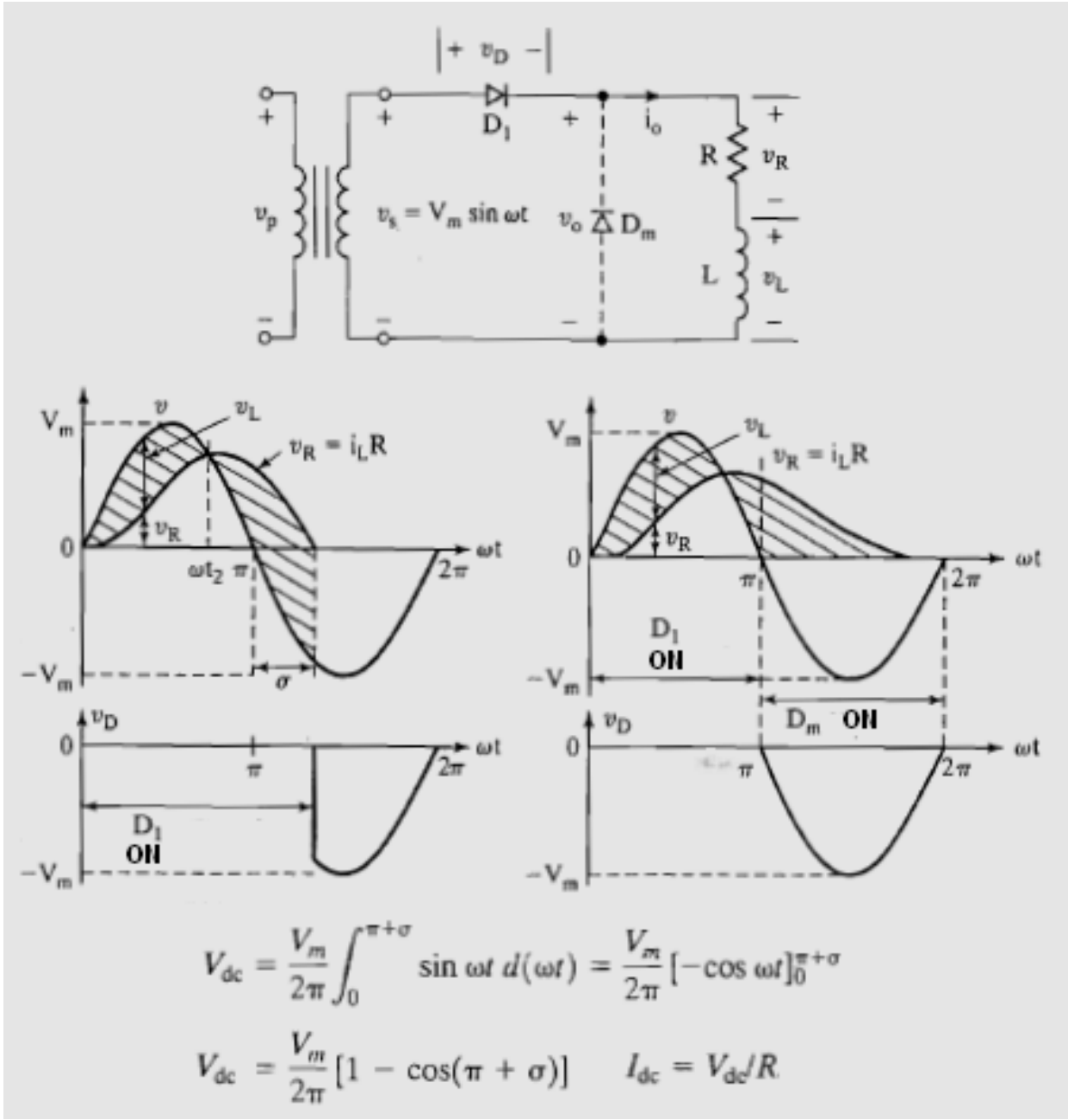
$$\tau = L/R$$

UGAO β ODREĐUJEMO IZ USLOVA:

$$i(\beta) = \left(\frac{V_m}{Z}\right)\cdot\left[\sin(\beta - \theta) + \sin(\theta)e^{-\beta/\omega\tau}\right] = 0$$

$$\sin(\beta - \theta) + \sin(\theta)e^{-\beta/\omega\tau} = 0$$

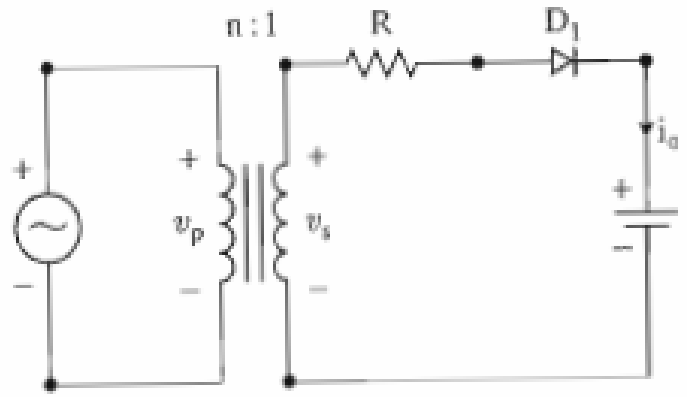
KORIŠĆENJE ZAMAJNE DIODE ZA ELIMINISANJE NEGATIVNOG IZLAZNOG NAPONA



$$V_{dc} = \frac{V_m}{2\pi}\int_0^{\pi+\sigma} \sin \omega t \, d(\omega t) = \frac{V_m}{2\pi}[-\cos \omega t]_0^{\pi+\sigma}$$

$$V_{dc} = \frac{V_m}{2\pi}[1 - \cos(\pi + \sigma)] \qquad I_{dc} = V_{dc}/R$$

IDEALNI POLUTALSANI ISPRAVLJAČ: MONOFAZNI, OPTEREĆENJE: R-DC izvor
(PUNJAČ BATERIJE SA PREDOTPOROM)

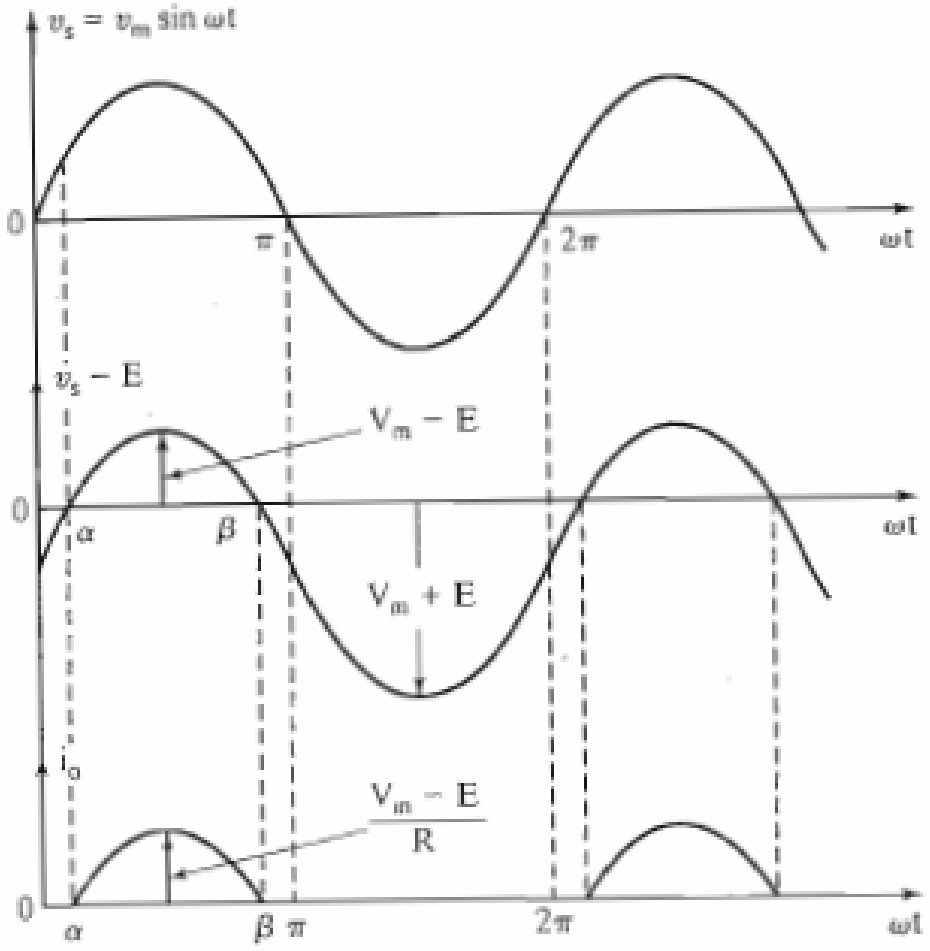


$$V_m \sin\alpha = E \qquad \alpha = \sin^{-1}\frac{E}{V_m}$$

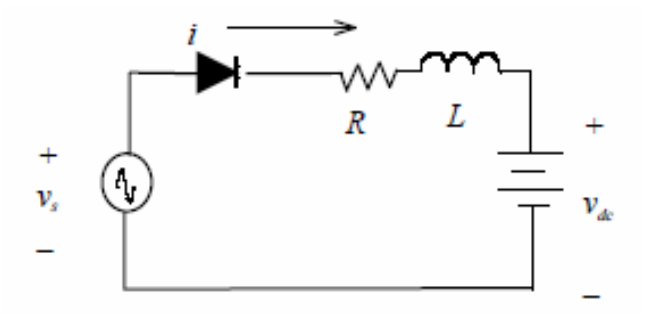
$$\beta = \pi - \alpha$$

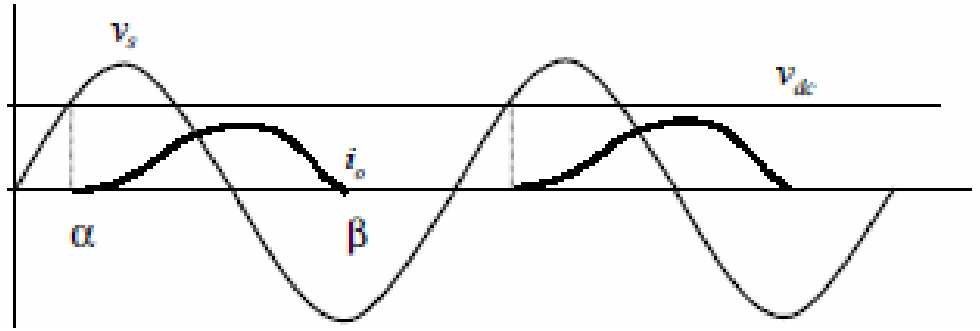
$$i_0 = \frac{v_s - E}{R} = \frac{V_m \sin\omega t - E}{R}$$

$$\alpha < \omega t < \beta$$



IDEALNI POLUTALSANI ISPRAVLJAČ: MONOFAZNI, OPTEREĆENJE: R-L –DC izvor





Početak vođenja diode

$$V_m \sin\omega t = iR(t) + L\frac{di(t)}{dt} + V_{dc}$$

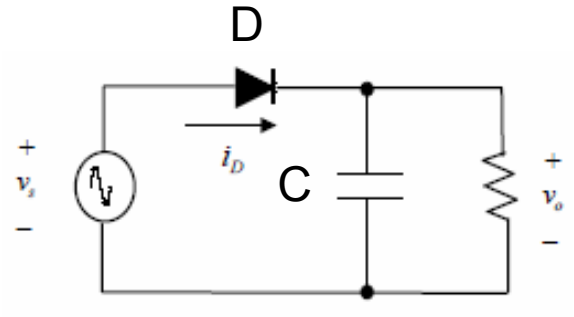
$$V_m \sin\alpha = V_{dc} \qquad \Rightarrow \alpha = \sin^{-1}\left(\frac{V_{dc}}{V_m}\right)$$

$$V_m \sin\omega t = iR(t) + L\frac{di(t)}{dt} + V_{dc}$$

$$i(\omega t) = \begin{cases} \left(\frac{V_m}{Z}\right)\cdot\left[\sin(\omega t - \theta) - \frac{V_{dc}}{R} + Ae^{-\omega t/\omega\tau}\right] & \text{za } \alpha \leqslant \omega t \leqslant \beta \\ 0 & \text{za } 2\pi + \alpha > \omega t > \beta \end{cases}$$

$$A = \left(\frac{V_m}{Z}\sin(\alpha - \theta) + \frac{V_{dc}}{R}\right)e^{\alpha/\omega\tau}$$

IDEALNI POLUTALSANI ISPRAVLJAČ: MONOFAZNI,OPTEREĆENJE: R-C



$$v_o = \begin{cases} V_m \sin(\omega t) & \longrightarrow \text{D on} \\ V_\theta\, e^{-(\omega t-\theta)/\omega RC} & \longrightarrow \text{D off} \end{cases}$$

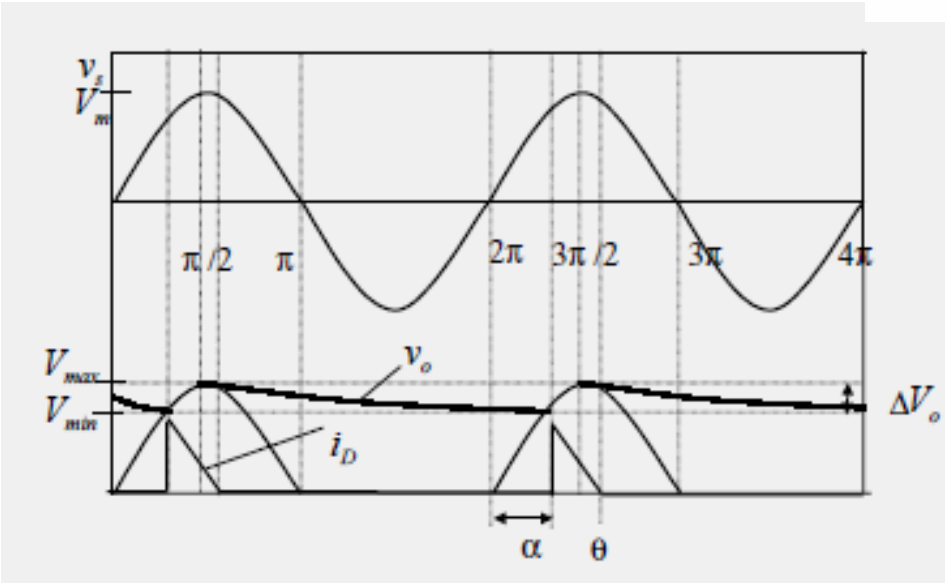
$$V_\theta = V_m \sin\theta$$

$$V_\theta = V_m \qquad \theta = \pi/2 \qquad \alpha \approx \pi/2$$

$$\Delta V_o = V_{\max} - V_{\min} = V_m - V_m \sin(2\pi+\alpha) = V_m - V_m \sin\alpha$$

$$v_o(2\pi+\alpha) = V_m e^{-\left(\frac{2\pi+\pi/2-\pi/2}{\omega RC}\right)} = V_m e^{-\left(\frac{2\pi}{\omega RC}\right)}$$

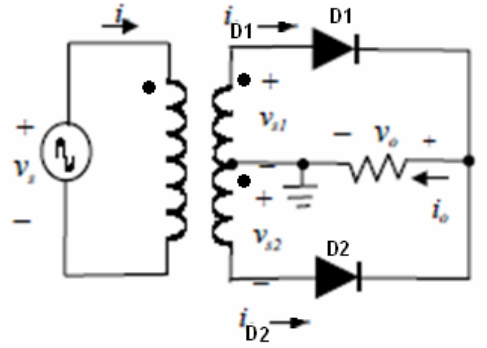
$$\Delta V_o \approx V_m - V_m e^{-\left(\frac{2\pi}{\omega RC}\right)} = V_m\left(1 - e^{-\left(\frac{2\pi}{\omega RC}\right)}\right)$$

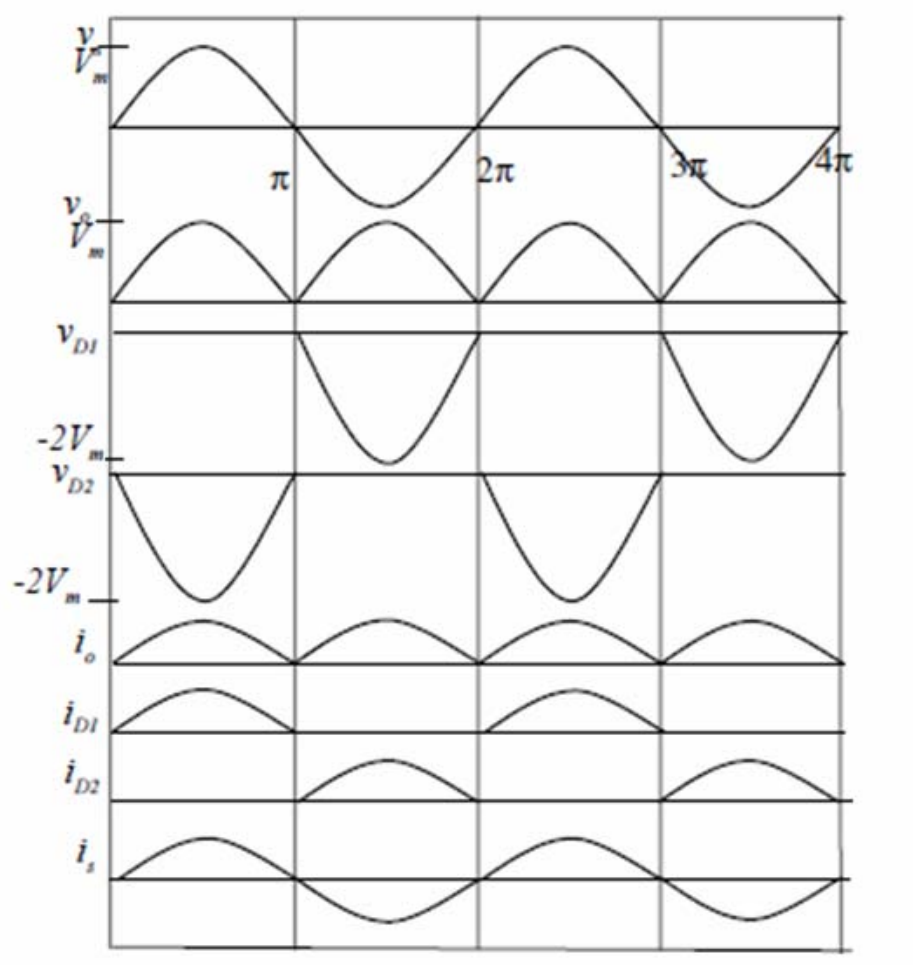


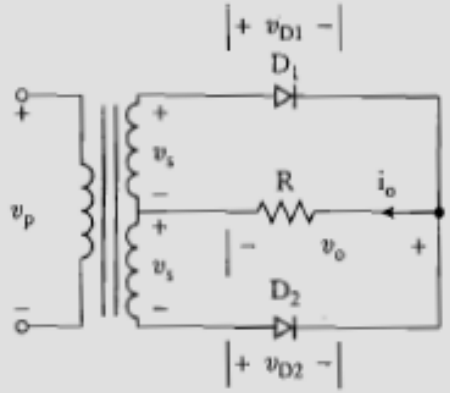
$$1 - e^{-\left(\frac{2\pi}{\omega RC}\right)} = 1 - \frac{2\pi}{\omega RC}$$

$$\Delta V_o = V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}$$

IDEALNI PUNOTALSANI SPRAVLJAČ : MONOFAZNI, OPTEREĆENJE: R (TALASNI OBLICI)





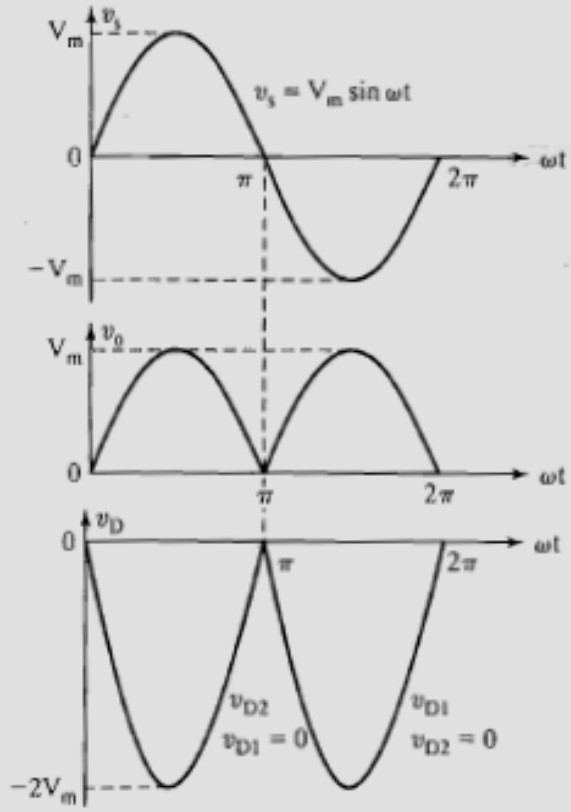


$$V_{dc} = \frac{2}{T}\int_0^{T/2} V_m \sin \omega t \, dt = \frac{2V_m}{\pi} = 0.6366V_m$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{0.6366V_m}{R} \qquad P_{dc} = (0.6366V_m)^2/R$$

$$V_{rms} = \left[\frac{2}{T}\int_0^{T/2} (V_m \sin \omega t)^2 \, dt\right]^{1/2} = \frac{V_m}{\sqrt{2}} = 0.707V_m$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{0.707V_m}{R} \qquad P_{ac} = (0.707V_m)^2/R.$$



EFIKASNOST $\eta = (0.6366V_m)^2/(0.707V_m)^2 = 81\%.$

FORM FACTOR (Faktor Oblika) $FF = 0.707V_m/0.6366V_m = 1.11.$

RIPPLE FACTOR $RF = \sqrt{1.11^2 - 1} = 0.482$ or 48.2%.

MAX INV. diode $PIV = 2V_m$

POWER FACTOR (PF)-faktor snage $PF = \frac{P_{ac}}{VA} = \frac{0.707^2}{2 \times 0.707 \times 0.5} = 0.707$

$1/TUF = 1/0.5732 = 1.75$

TRANSFORMATOR

$V_s = V_m/\sqrt{2} = 0.707V_m.$

$I_s = 0.5V_m/R$

$VA = 2V_sI_s = 2 \times 0.707V_m \times 0.5V_m/R$

$$TUF = \frac{P_{dc}}{VA} = \frac{0.6366^2}{2 \times 0.707 \times 0.5} = 0.5732 = 57.32\%$$

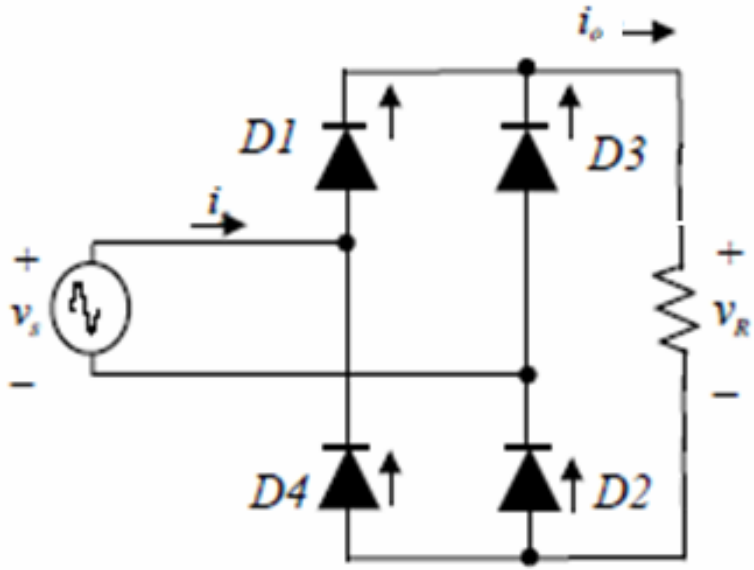
$I_{s(peak)} = V_m/R$

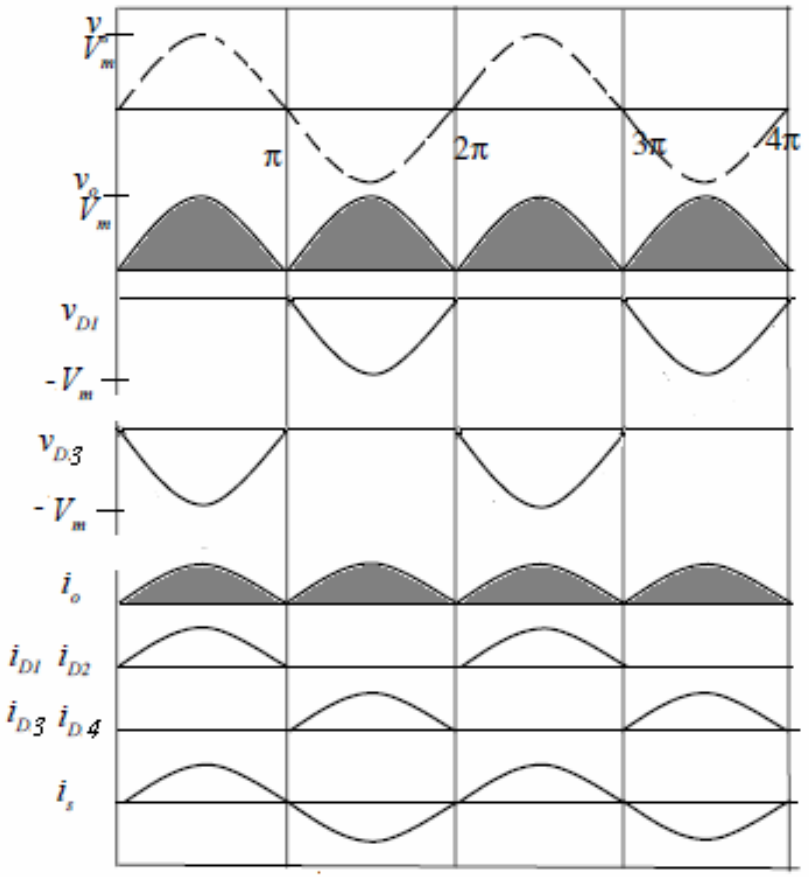
$I_s = 0.707V_m/R$

$CF = I_{s(peak)}/I_s = 1/0.707 = \sqrt{2}.$

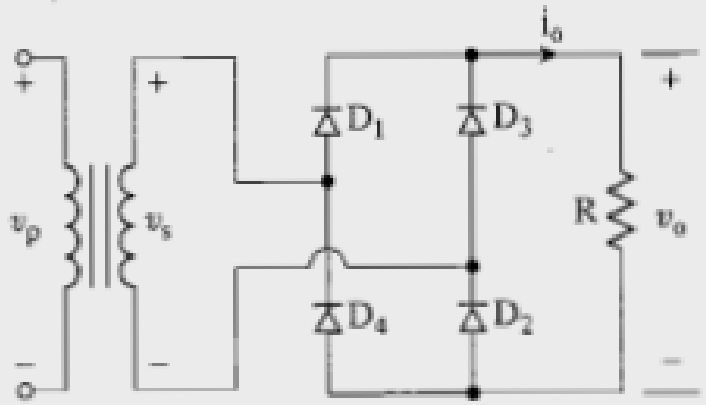
Ovaj transformator mora biti 1.75 puta predimenzionisan u odnosu na klasični sa AC ulazom i izlazom

IDEALNI PUNOTALSANI SPRAVLJAČ U GRECOVOM SPOJU: MONOFAZNI, OPTEREĆENJE: R (TALASNI OBLICI)





IDEALNI PUNOTALSANI SPRAVLJAČ U GRECOVOM SPOJU: PRORAČUNI

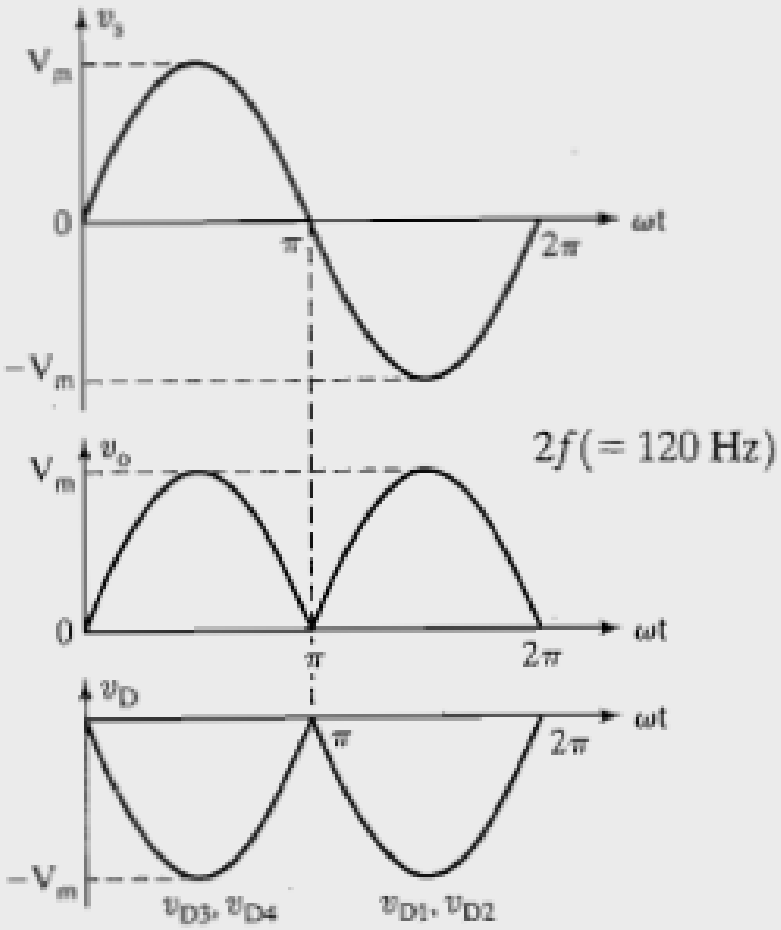


$$V_{dc} = \frac{2}{T}\int_0^{T/2} V_m \sin \omega t \, dt = \frac{2V_m}{\pi} = 0.6366V_m$$

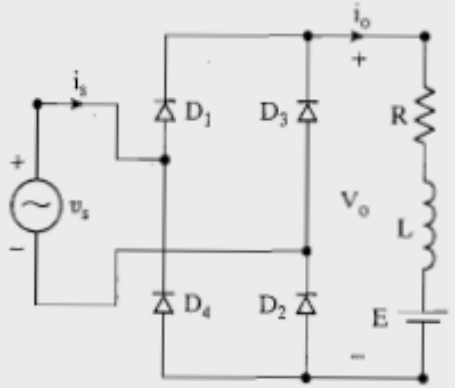
$$I_{dc} = \frac{V_{dc}}{R} = \frac{0.6366V_m}{R} \qquad P_{dc} = (0.6366V_m)^2/R$$

$$V_{rms} = \left[\frac{2}{T}\int_0^{T/2} (V_m \sin \omega t)^2 \, dt\right]^{1/2} = \frac{V_m}{\sqrt{2}} = 0.707V_m$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{0.707V_m}{R} \qquad P_{ac} = (0.707V_m)^2/R.$$



IDEALNI PUNOTALSANI SPRAVLJAČ U GRECOVOM SPOJU: MONOFAZNI, OPTEREĆENJE: R –L-DC izvor (tipična primena za punjače baterija)



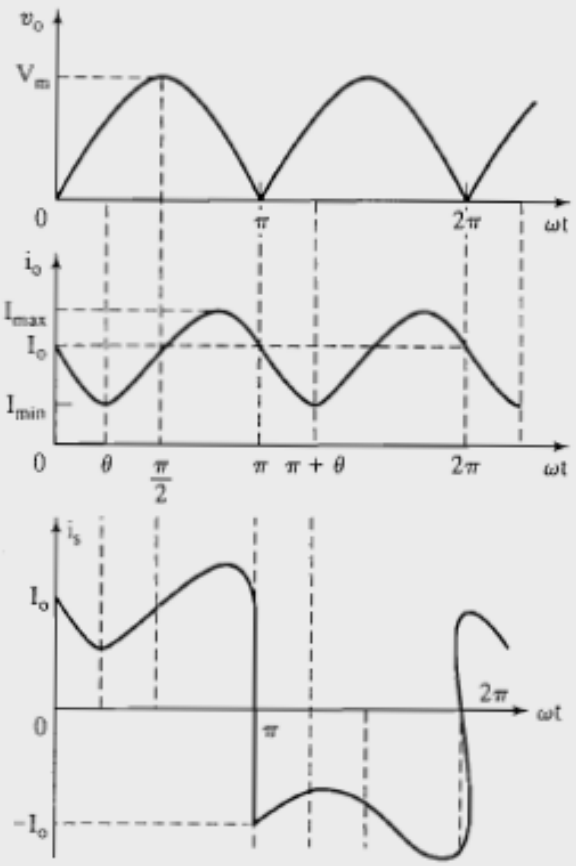
$$L\frac{di_0}{dt} + Ri_0 + E = \sqrt{2}\,V_s \sin \omega t$$

$$i_0 \geq 0$$

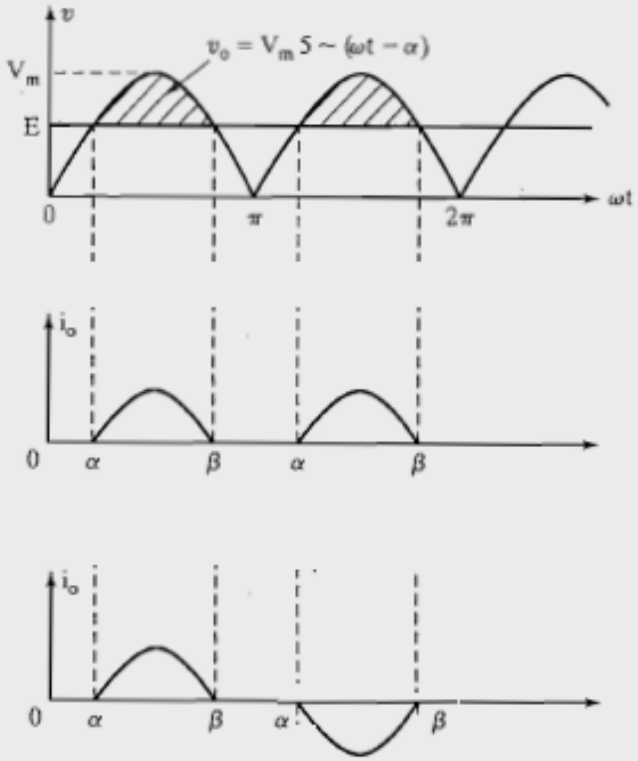
$$i_0 = \frac{\sqrt{2}V_s}{Z}\sin(\omega t - \theta) + A_1 e^{-(R/L)t} - \frac{E}{R}$$

$$Z = [R^2 + (\omega L)^2]^{1/2}$$

$$\theta = \tan^{-1}(\omega L/R)$$



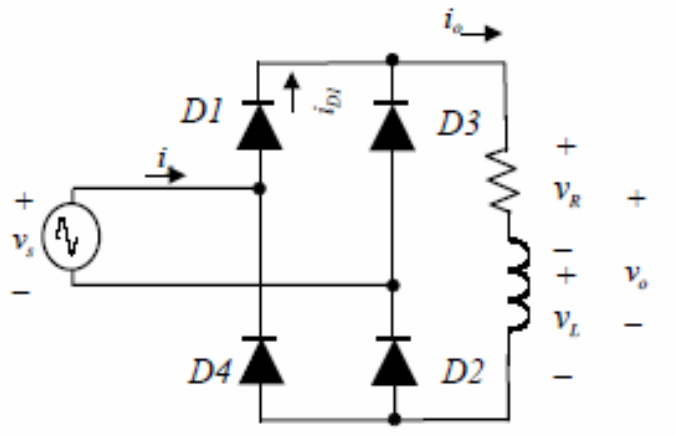
KONTINUALNI REZIM $\omega L/R >> 1$

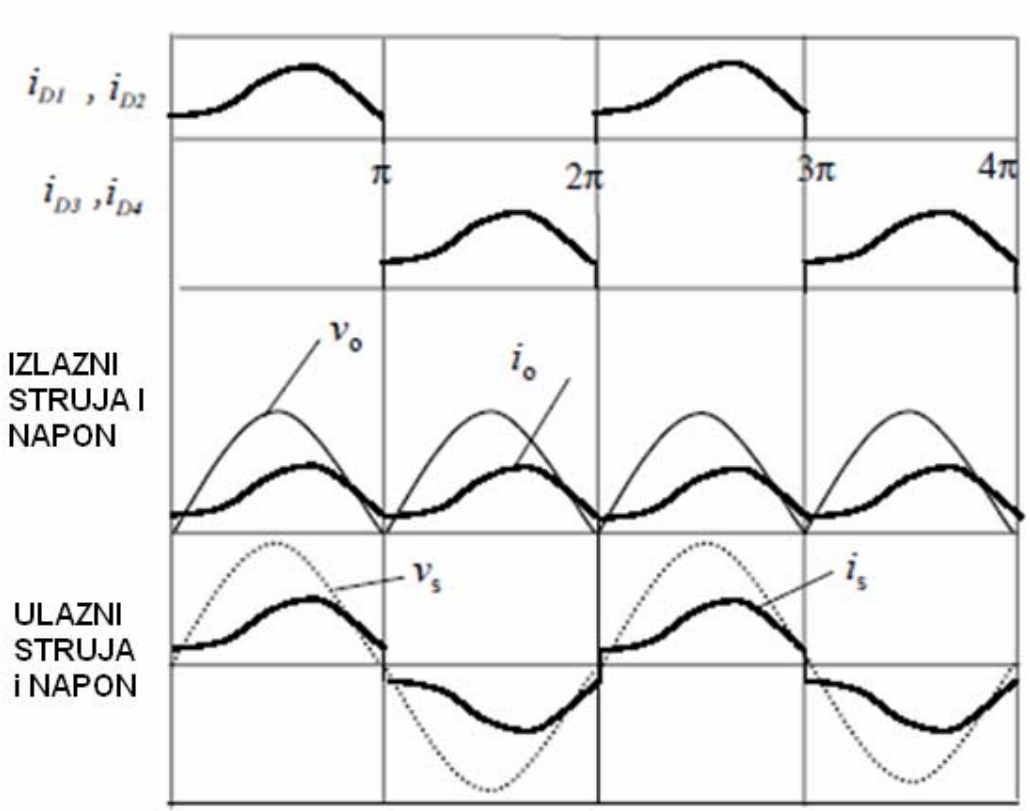


DISKONTINUALNI (PREKIDNI) REZIM

$\omega L/R \approx 1$

IDEALNI PUNOTALSANI SPRAVLJAČ: MONOFAZNI, OPTEREĆENJE: R -L

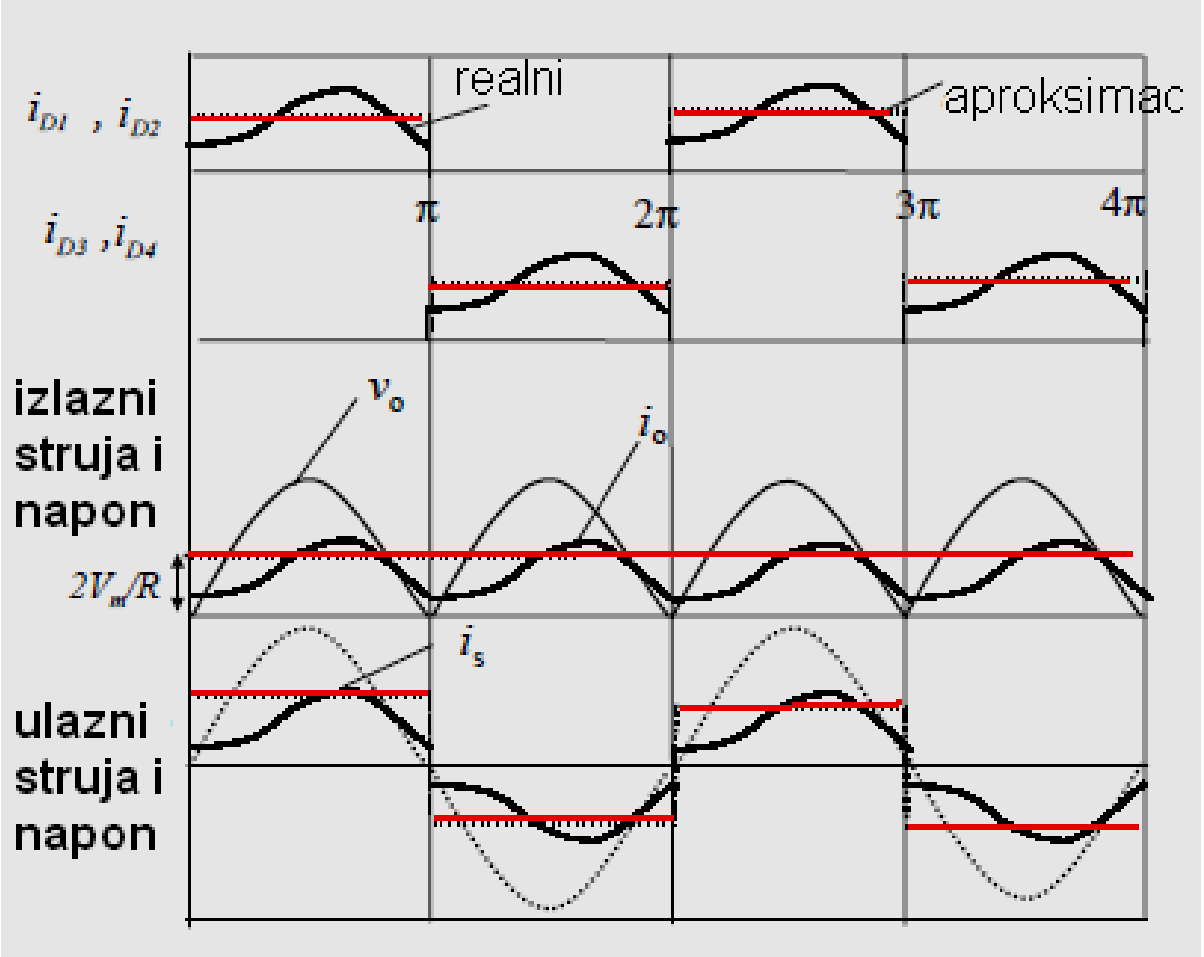




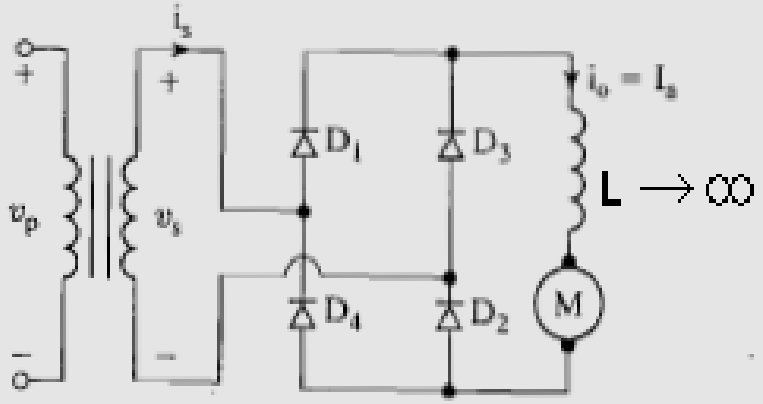
IDEALNI PUNOTALSANI SPRAVLJAČ: MONOFAZNI, OPTEREĆENJE: JAKO INDUKTIVNO

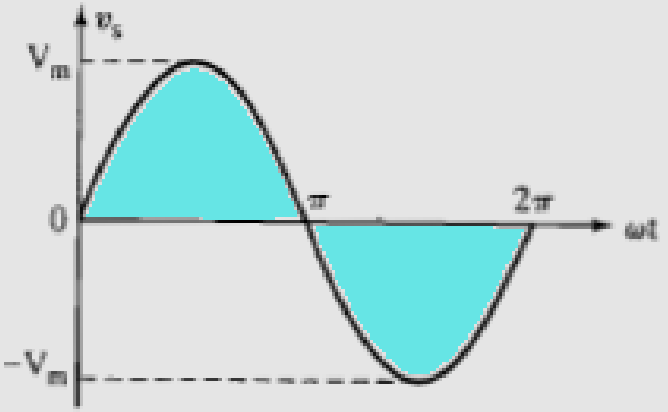
$$i(\omega t) \approx I_o = \frac{V_o}{R} = \frac{2V_m}{R},$$

$$\omega L >> R,$$



IDEALNI PUNOTALSANI SPRAVLJAČ: MONOFAZNI, OPTEREĆENJE: PRETEŽNO INDUKTIVNO (DC MOTOR SA VELIKOM REDNOM PRIGUŠNICOM)





$$i_s(t) = \frac{4I_a}{\pi}\left(\frac{\sin\omega t}{1} + \frac{\sin 3\omega t}{3} + \frac{\sin 5\omega t}{5} + \cdots\right)$$

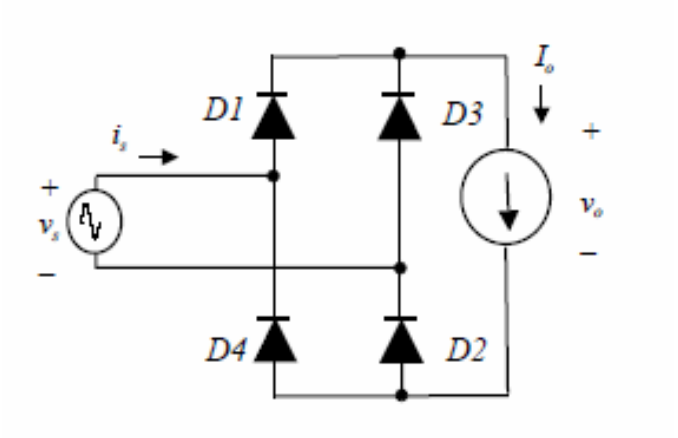
$$I_{s1} = \frac{4I_a}{\pi\sqrt{2}} = 0.90I_a$$

$$I_s = \frac{4}{\pi\sqrt{2}} I_a\left[1 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{7}\right)^2 + \left(\frac{1}{9}\right)^2 + \cdots\right]^{1/2} = I_a$$

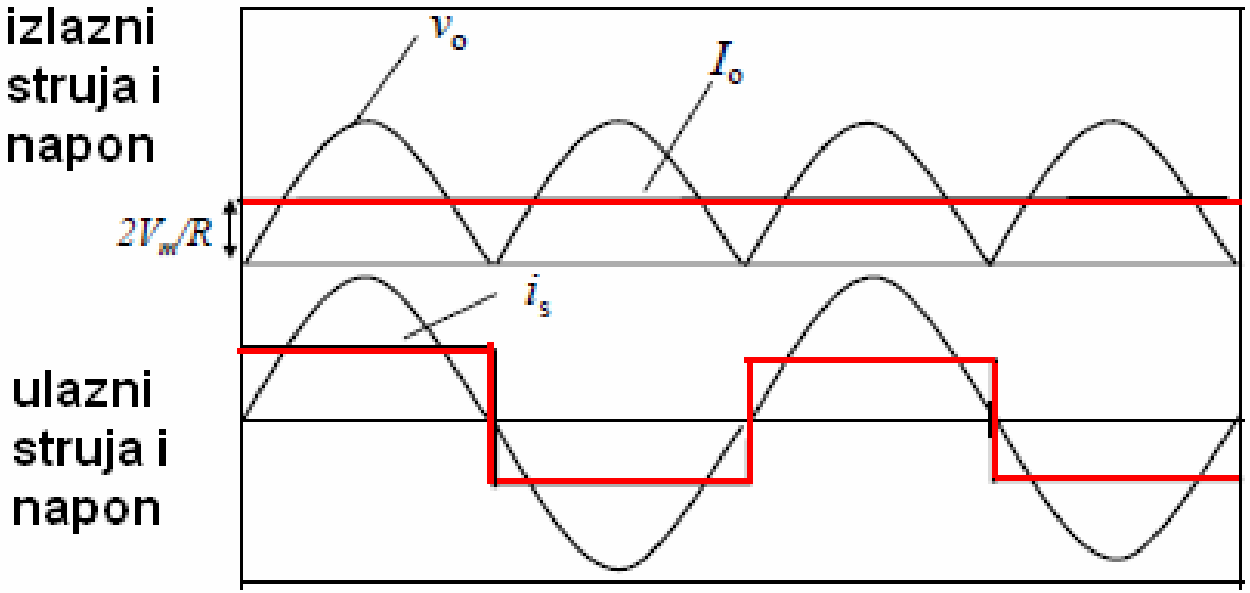
$$\text{PF} = (I_{s1}/I_s)\cos\phi = 0.90 \qquad \text{DF} = \cos\phi = 1$$

$$\text{HF} = \text{THD} = \left[\left(\frac{1}{0.90}\right)^2 - 1\right]^{1/2} = 0.4843$$

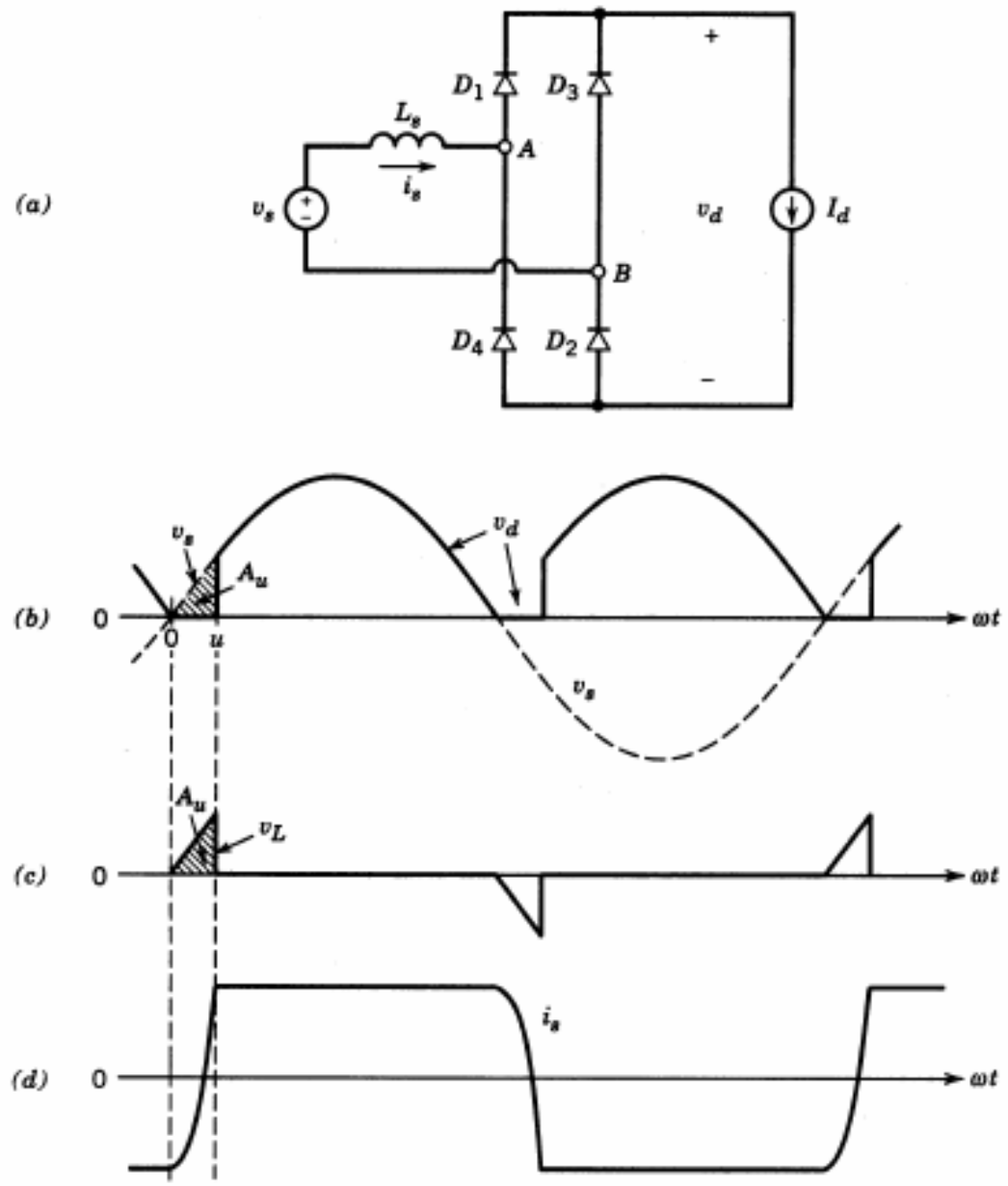
IDEALNI PUNOTALSANI SPRAVLJAČ: MONOFAZNI, OPTEREĆENJE: STRUJNI PONOR



L → ∞



EFEKAT RASIPNE INDUKTIVNOSTI L_s



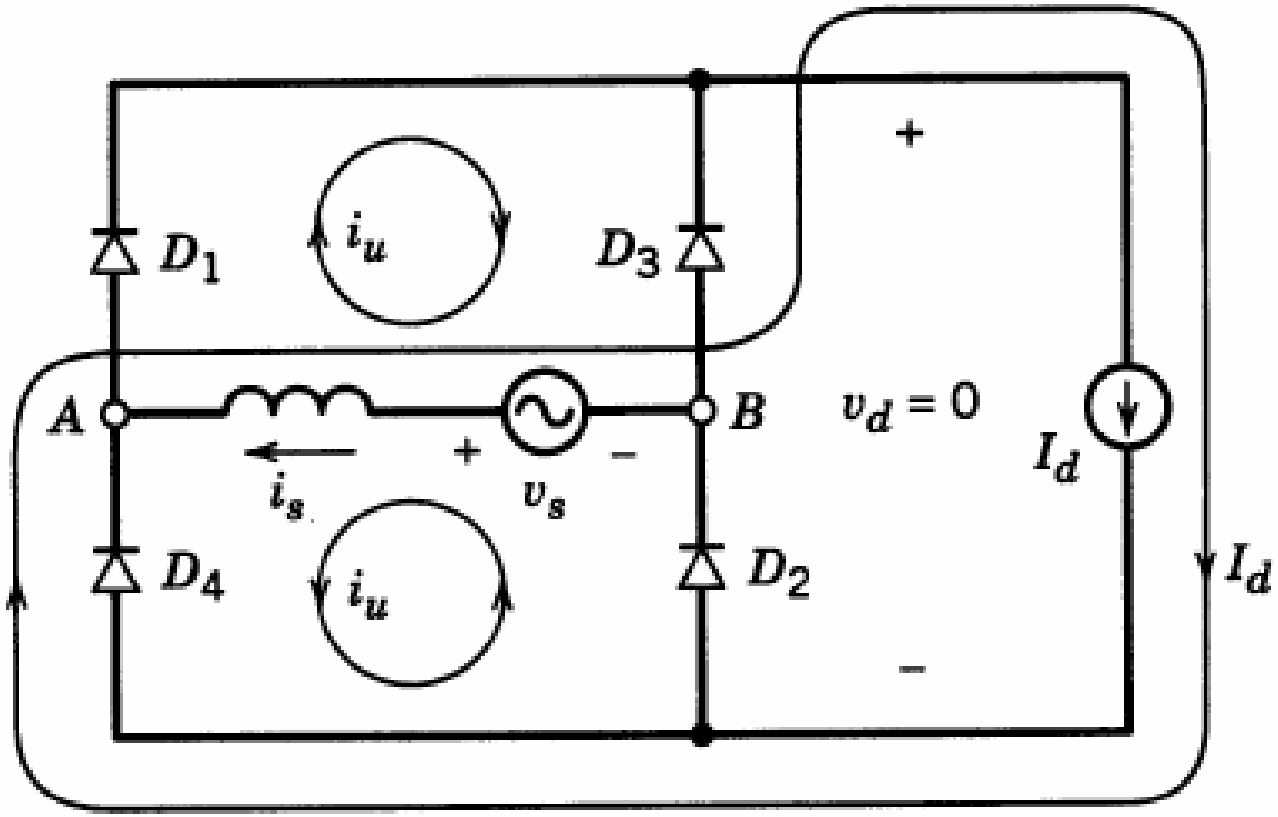
(a)-električna šema

(b)-izlazni napon

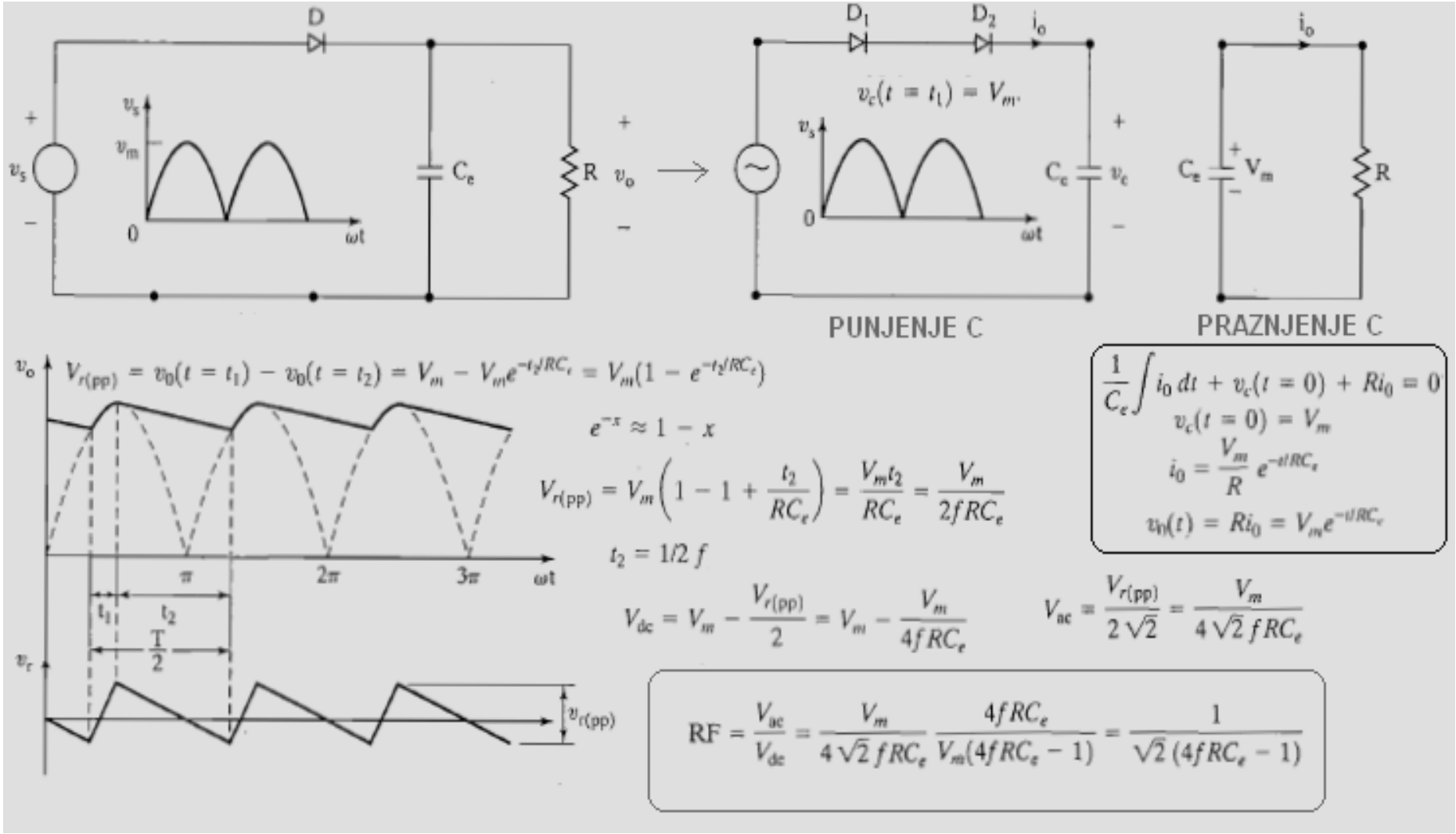
(c)- napon na L_s

(d)- struja L_s

EFEKAT RASIPNE INDUKTIVNOSTI L_s - kolo za analizu



PUNOTALSNI ISPRAVLJAČ SA KAPACITIVNIM FILTROM



$$V_{r(pp)} = v_0(t = t_1) - v_0(t = t_2) = V_m - V_m e^{-t_2/RC_e} = V_m(1 - e^{-t_2/RC_e})$$

$$e^{-x} \approx 1 - x$$

$$V_{r(pp)} = V_m\left(1 - 1 + \frac{t_2}{RC_e}\right) = \frac{V_m t_2}{RC_e} = \frac{V_m}{2fRC_e}$$

$$t_2 = 1/2f$$

$$V_{dc} = V_m - \frac{V_{r(pp)}}{2} = V_m - \frac{V_m}{4fRC_e}$$

$$V_{ac} = \frac{V_{r(pp)}}{2\sqrt{2}} = \frac{V_m}{4\sqrt{2}fRC_e}$$

$$\frac{1}{C_e}\int i_0\,dt + v_c(t = 0) + Ri_0 = 0$$

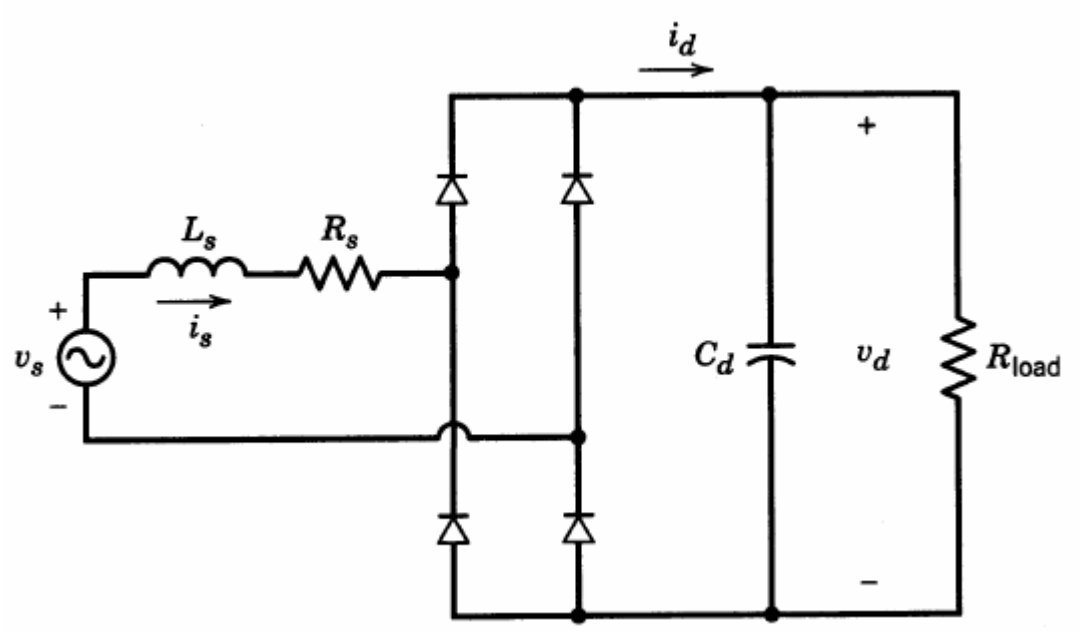
$$v_c(t = 0) = V_m$$

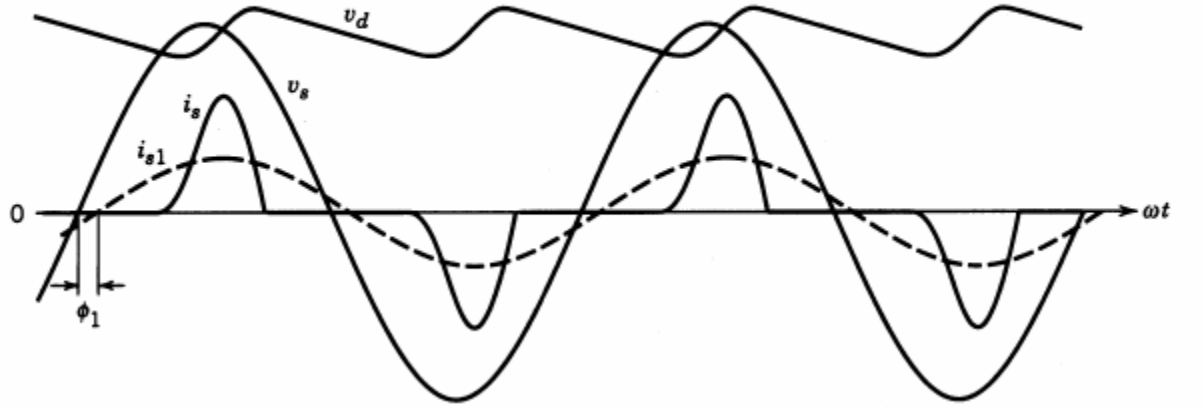
$$i_0 = \frac{V_m}{R}e^{-t/RC_e}$$

$$v_0(t) = Ri_0 = V_m e^{-t/RC_e}$$

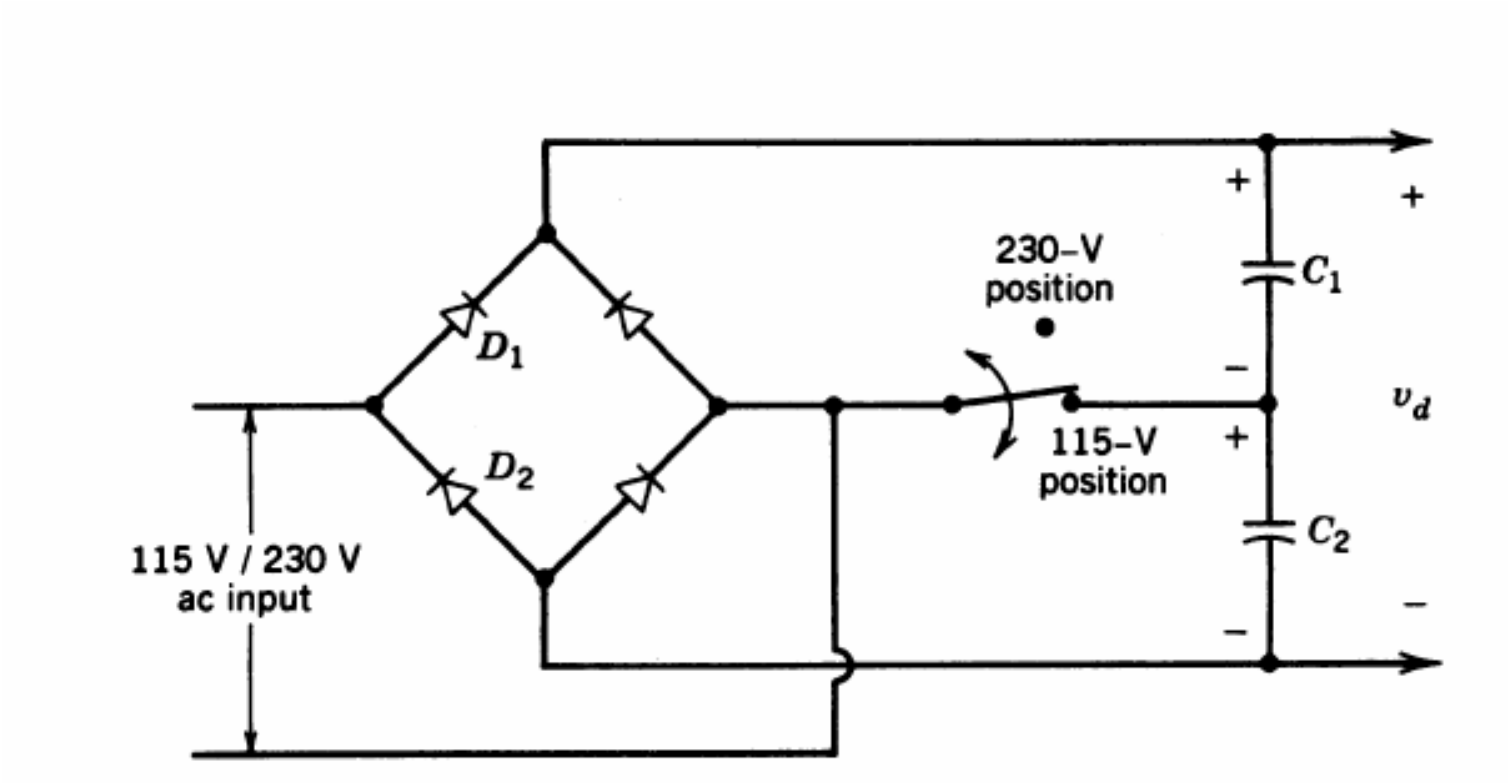
$$RF = \frac{V_{ac}}{V_{dc}} = \frac{V_m}{4\sqrt{2}fRC_e}\,\frac{4fRC_e}{V_m(4fRC_e - 1)} = \frac{1}{\sqrt{2}\,(4fRC_e - 1)}$$

TALASNI OBLICI REALNOG DIODNOG ISPRAVLJAČA SA RC FILTROM



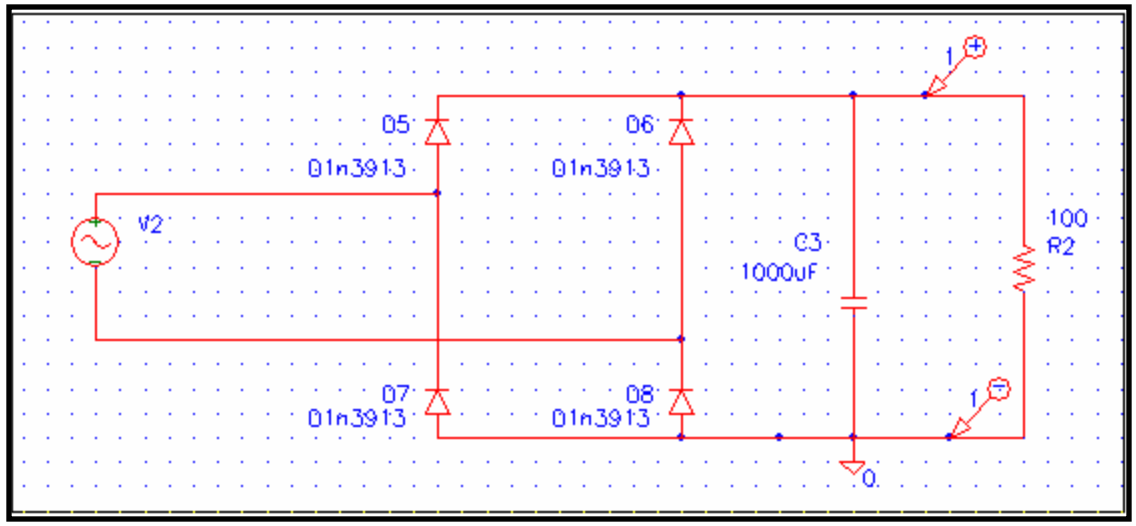


UNIVERZALNI ISPRAVLJAČ SA NAPAJANJEM 110/220V AC

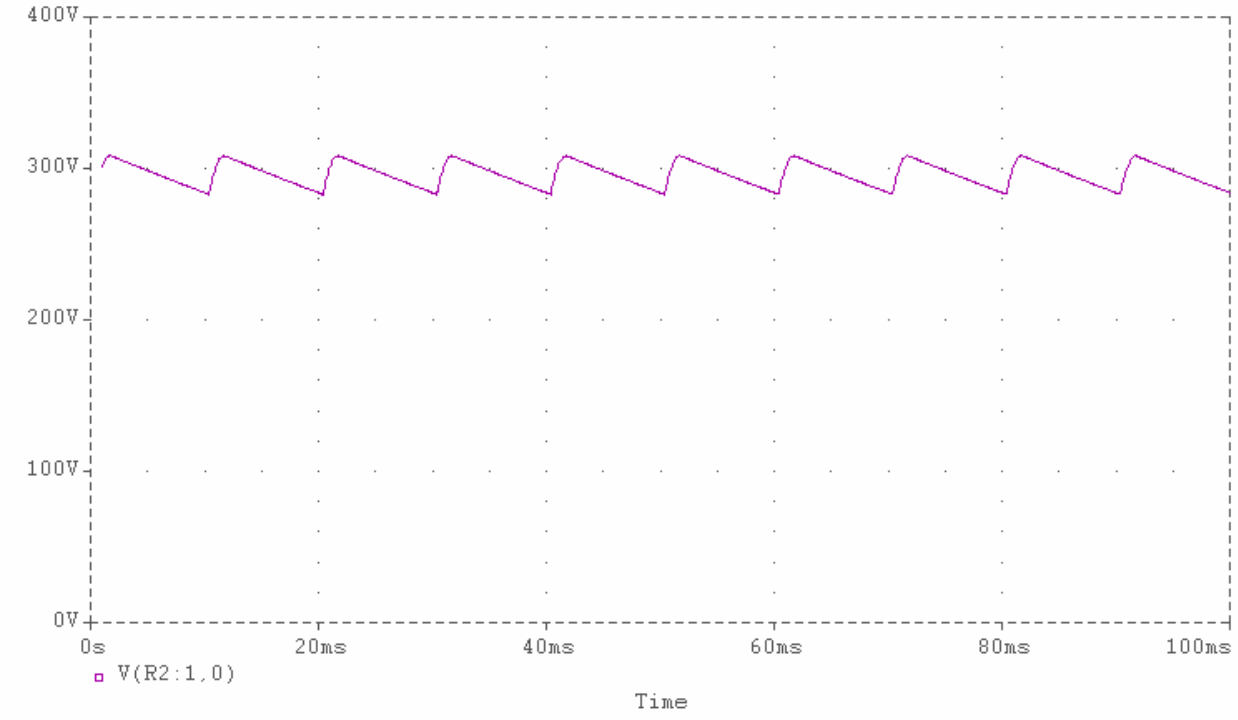


PRIEMENA U MREŽNIM NAPAJANJIMA – *ATX napajanje*

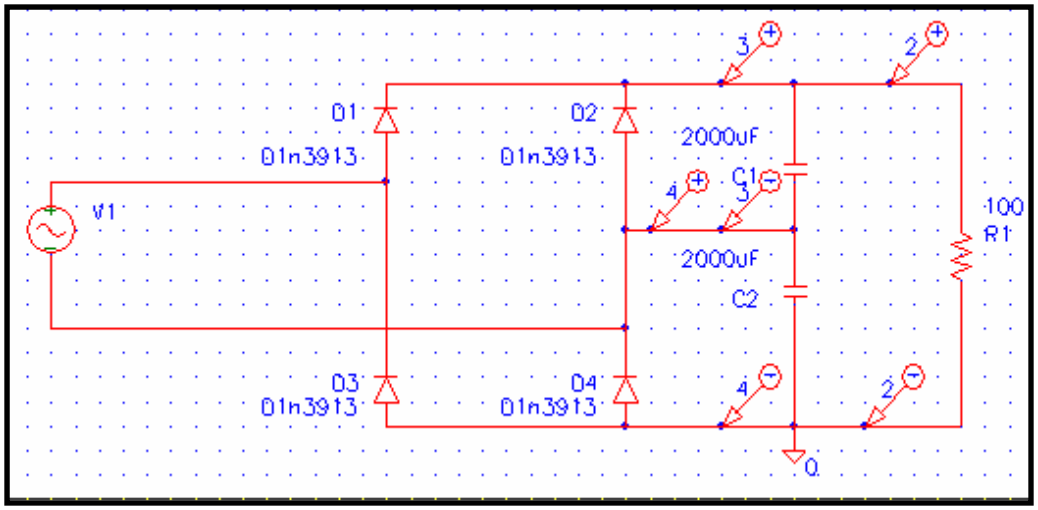
PUNOTALSNI ISPRAVLJAČ U GRECOVOM SPOJU: SIMULACIJA



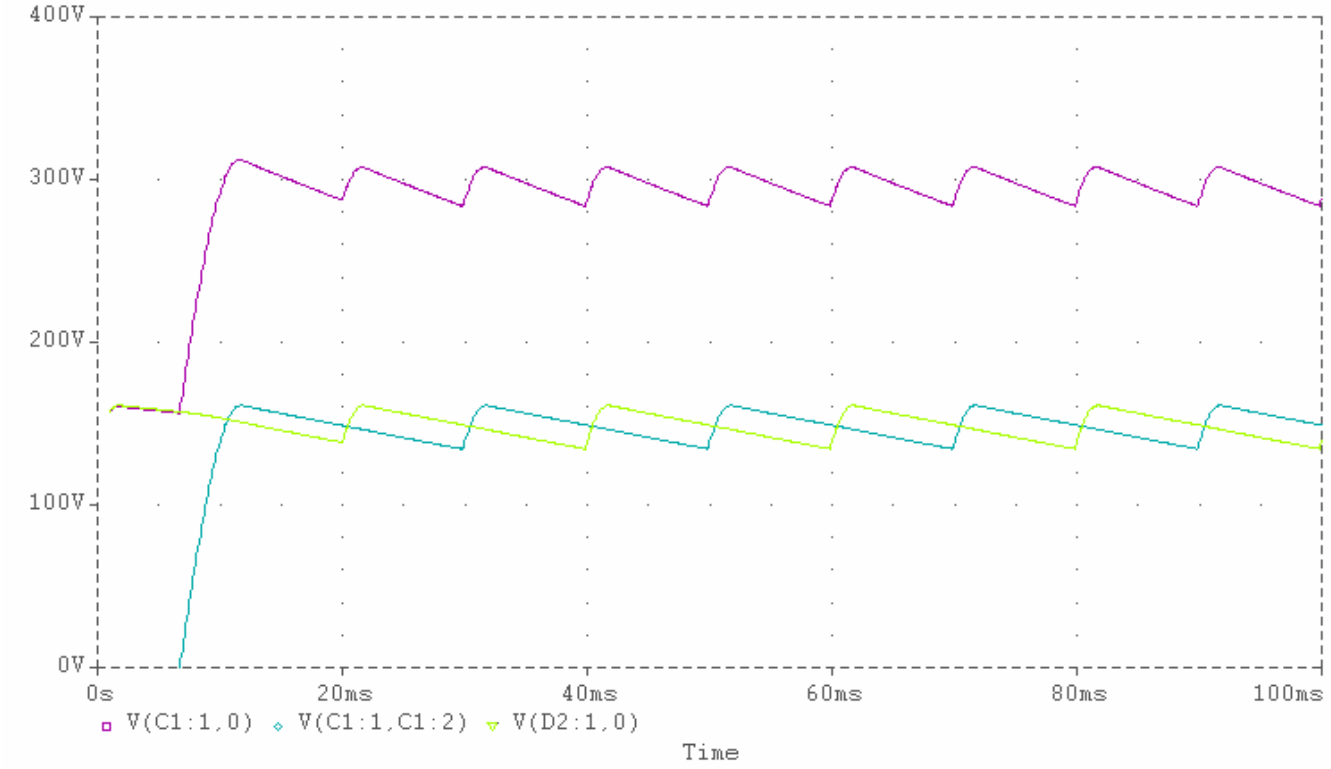
NAPAJANJE
220V,50Hz



PUNOTALSNI ISPRAVLJAČ U GRECOVOM SPOJU: SIMULACIJA

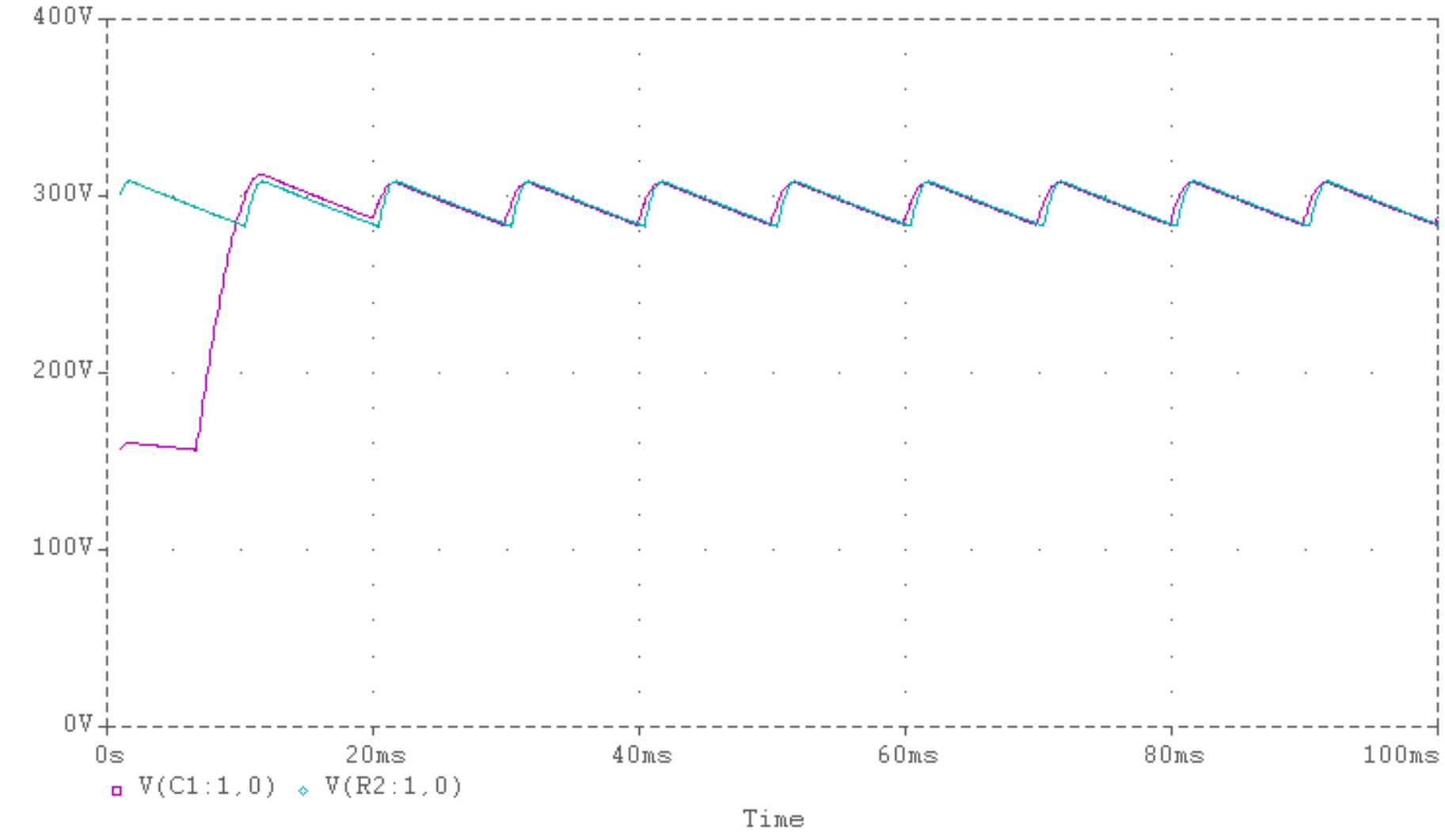


115V, 50Hz
NAPAJANJE



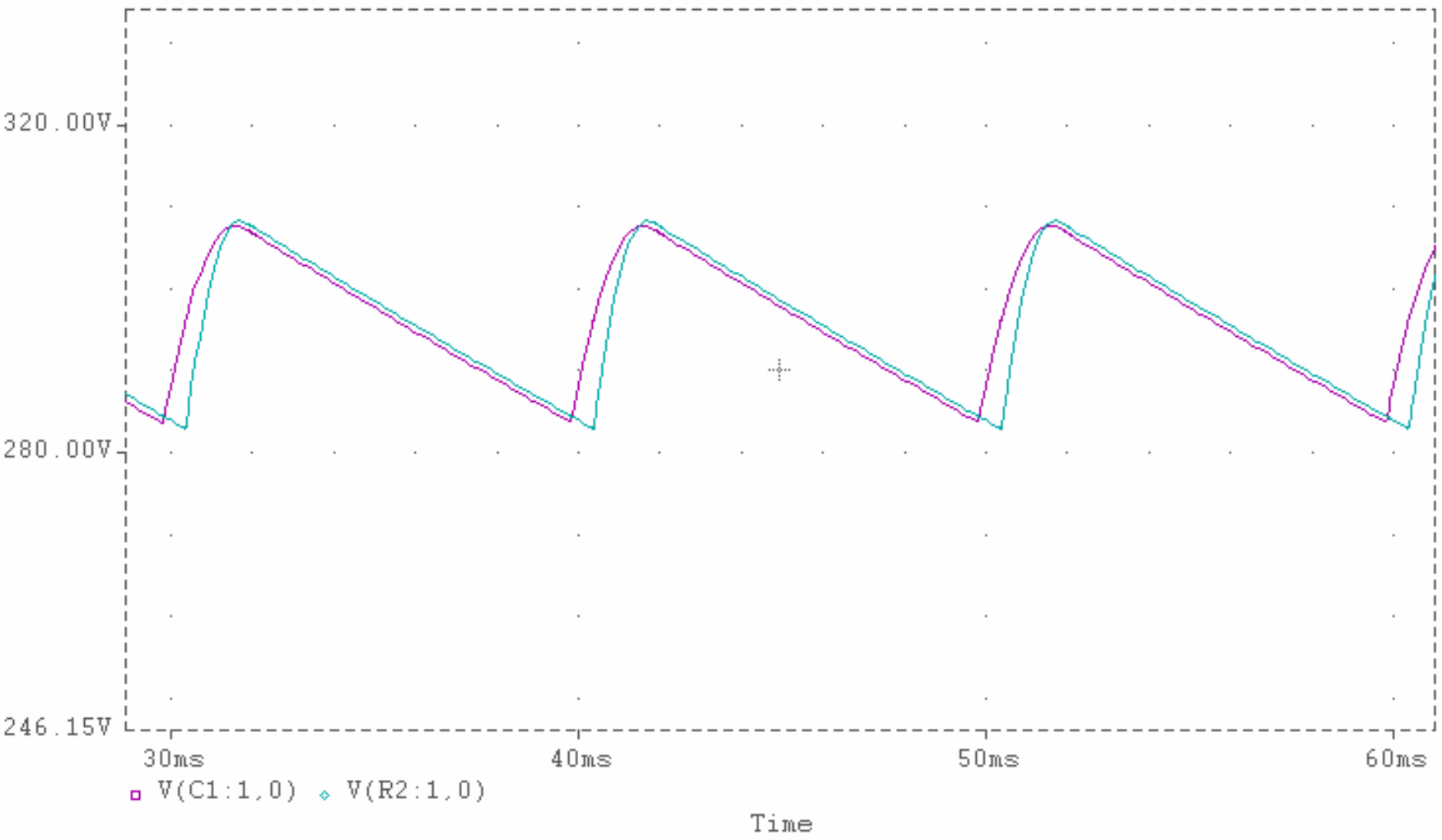
PUNOTALSNI ISPRAVLJAČ U GRECOVOM SPOJU: SIMULACIJA

POREĐENJE IZLAZNOG NAPONA PRI **115V, 50Hz i 230V,50Hz**

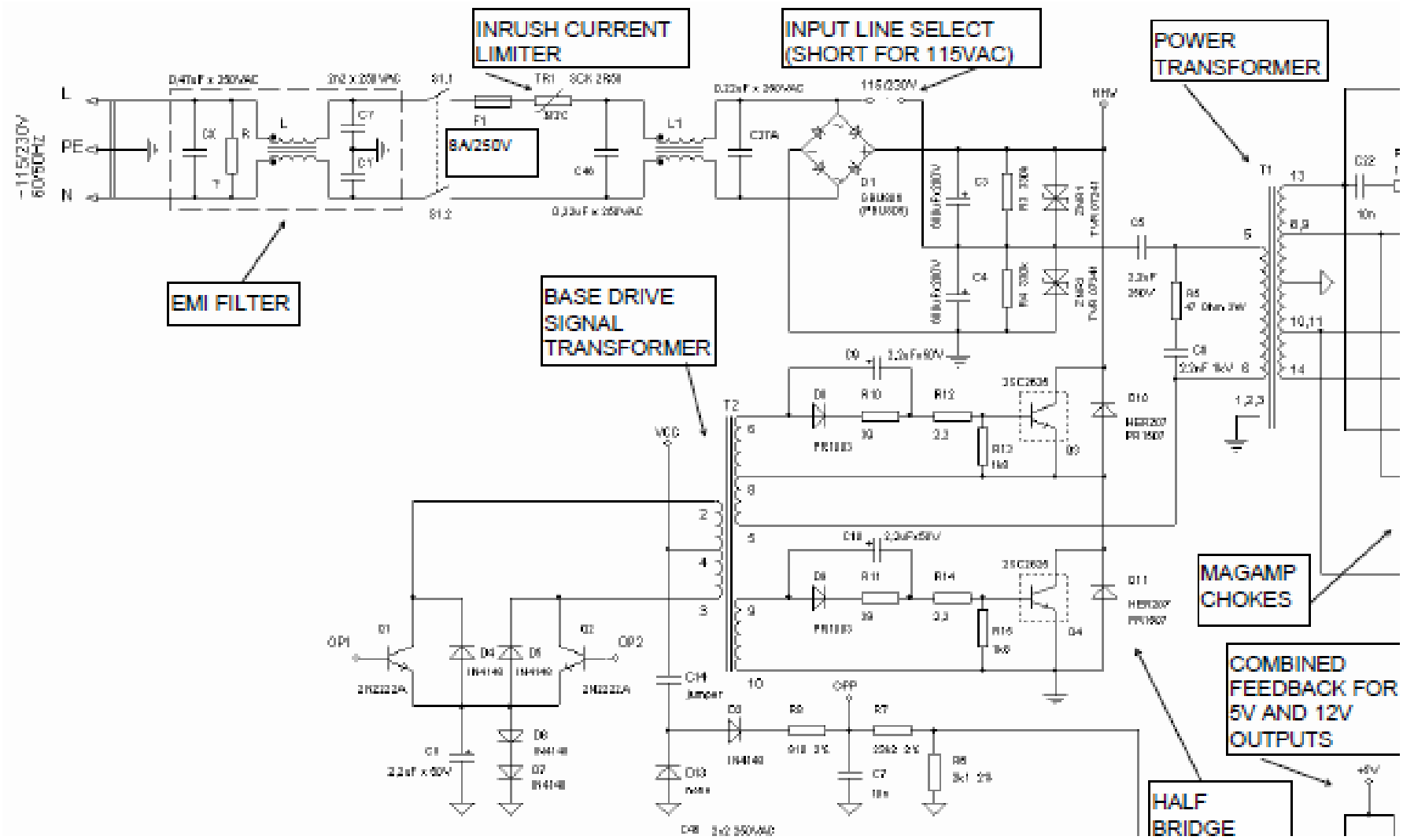


PUNOTALSNI ISPRAVLJAČ U GRECOVOM SPOJU: SIMULACIJA

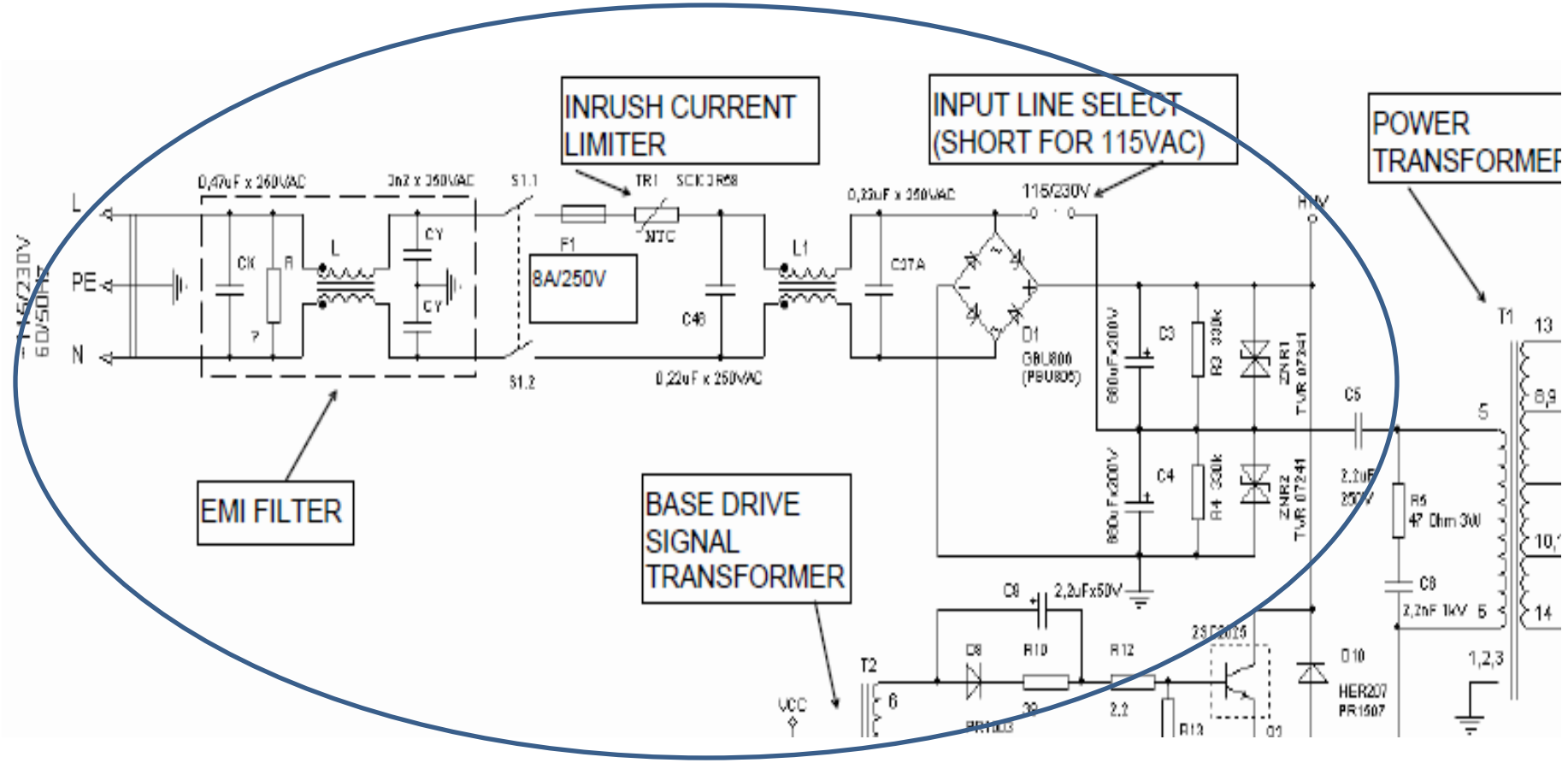
DETALJAN PRIKAZ IZLAZNOG NAPONA PRI **230V, 50Hz i 115V,50Hz**



ELEKTRIČNA ŠEMA TIPIČNOG ATX-napajanja

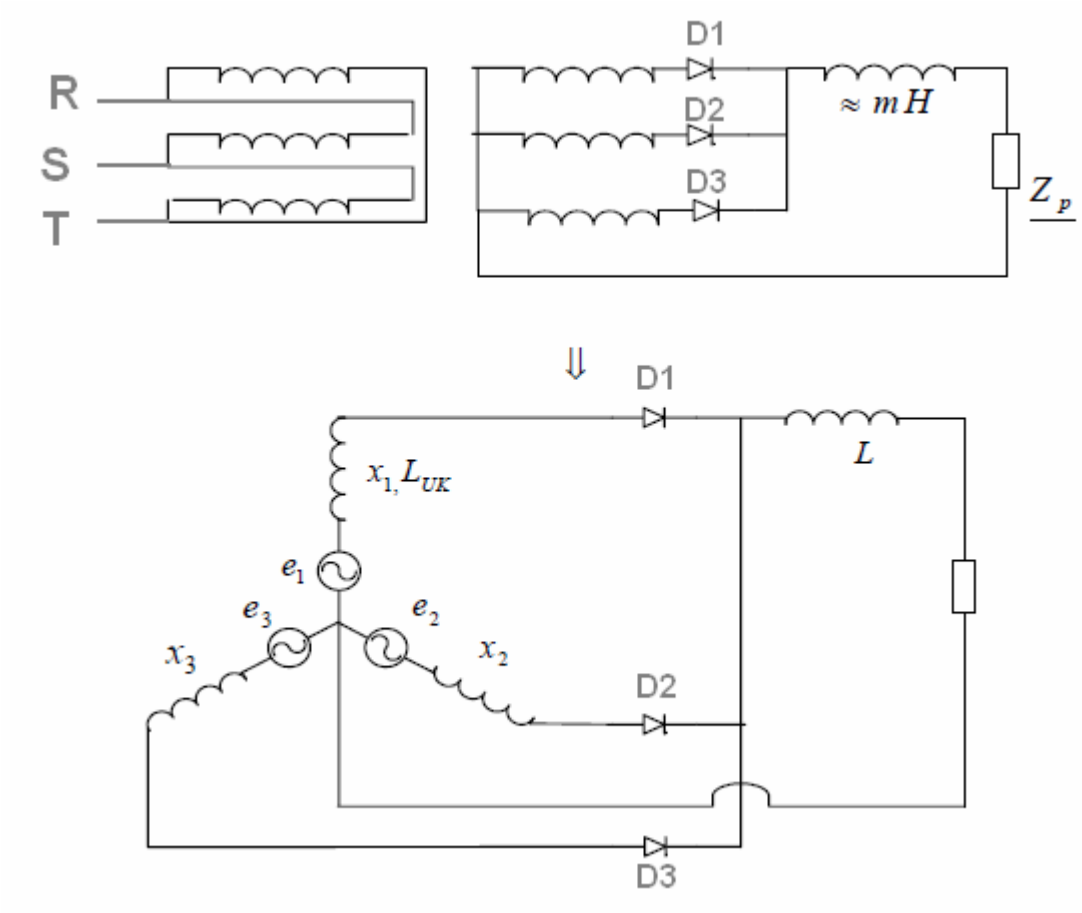


ATX napajanje-DETALJ ULAZNOG ISPRAVLJAČA

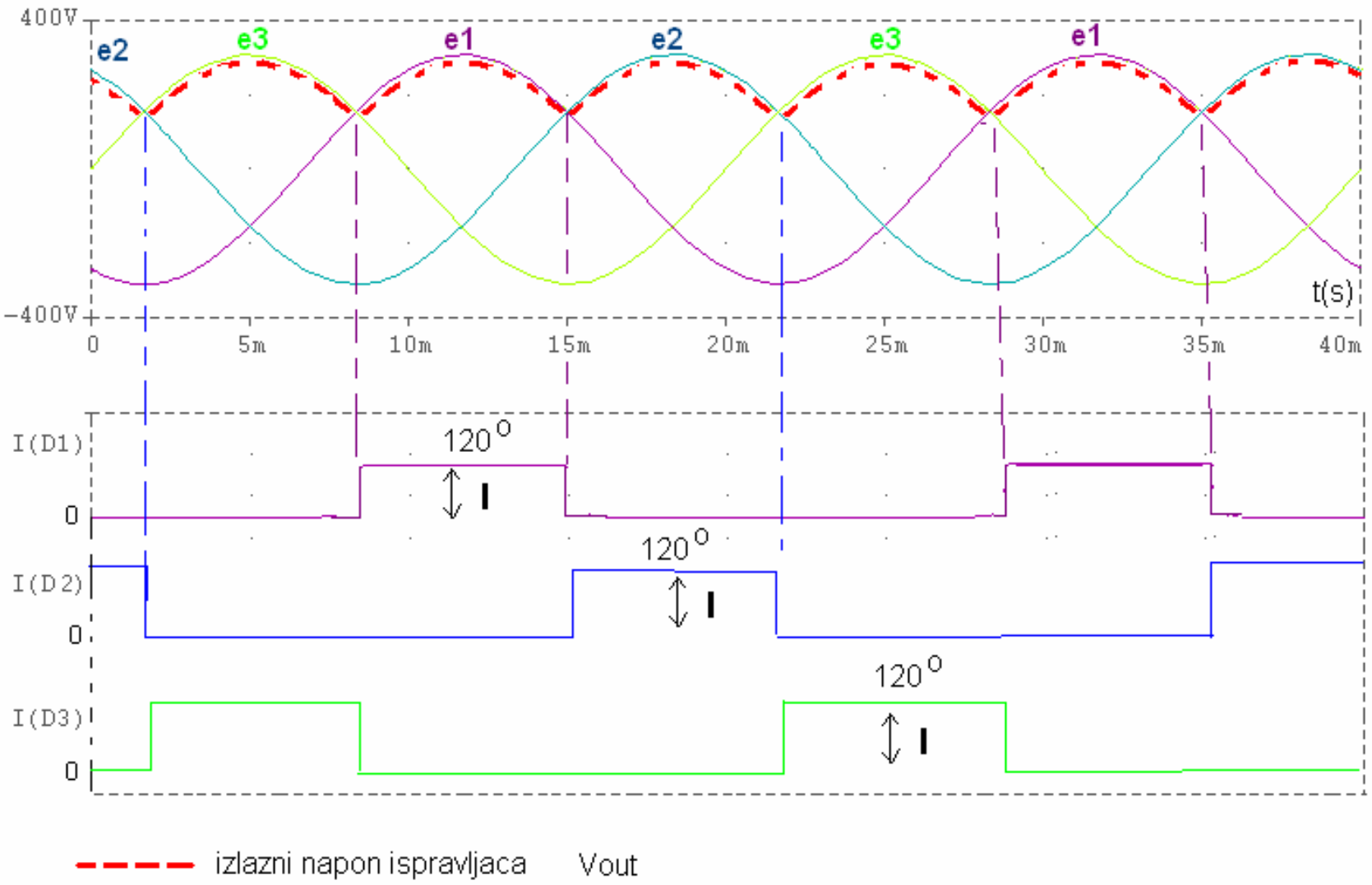


TROFAZNI AC-DC pretvarači (MREŽOM VOĐENI ISPRAVLJAČI)

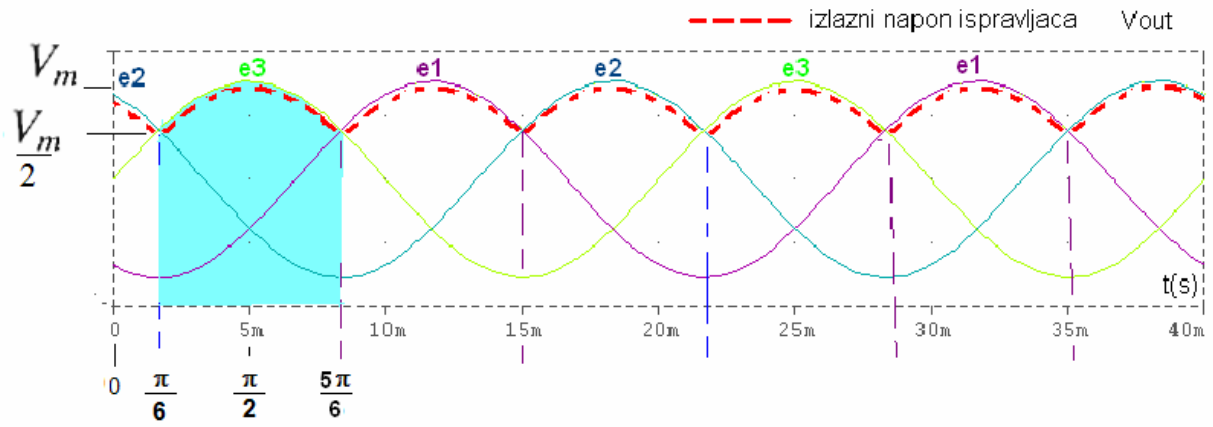
Trofazni ispravljač sa transformatorom sa srednjom tačkom



TROFAZNI ISPRAVLJAČ SA TRANSFORAMATOROM SA SREDNJOM TAČKOM- talasni oblici



TROFAZNI ISPRAVLJAČ SA TRANSFORAMATOROM SA SREDNJOM TAČKOM- proračuni



SREDNJA VREDNOST IZLAZNOG NAPONA

$$V_{dc} = \frac{2}{2\pi/3}\int_0^{\pi/3} V_m \cos \omega t \, d(\omega t) = V_m \frac{3}{\pi} \sin\frac{\pi}{3}$$

$$V_{dc} = 0.827 V_m$$

TALASNOST

$$\Delta V = V_m - \frac{V_m}{2}$$

$$\Delta V = 0.5 V_m$$

EFEKTIVNA VREDNOST IZLAZNOG NAPONA

$$V_{rms} = \left[\frac{2}{2\pi/3}\int_0^{\pi/q} V_m^2 \cos^2 \omega t \, d(\omega t)\right]^{1/2}$$

$$V_{rms} = V_m \left[\frac{3}{2\pi}\left(\frac{\pi}{3} + \frac{1}{2}\sin\frac{2\pi}{3}\right)\right]^{1/2}$$

$$V_{rms} = 0.5945 V_m$$

EFEKTIVNA VREDNOST STRUJE DIODE (SEKUNDARNE STRUJE TRANSFORMATORA)

$$I_m = V_m / R$$

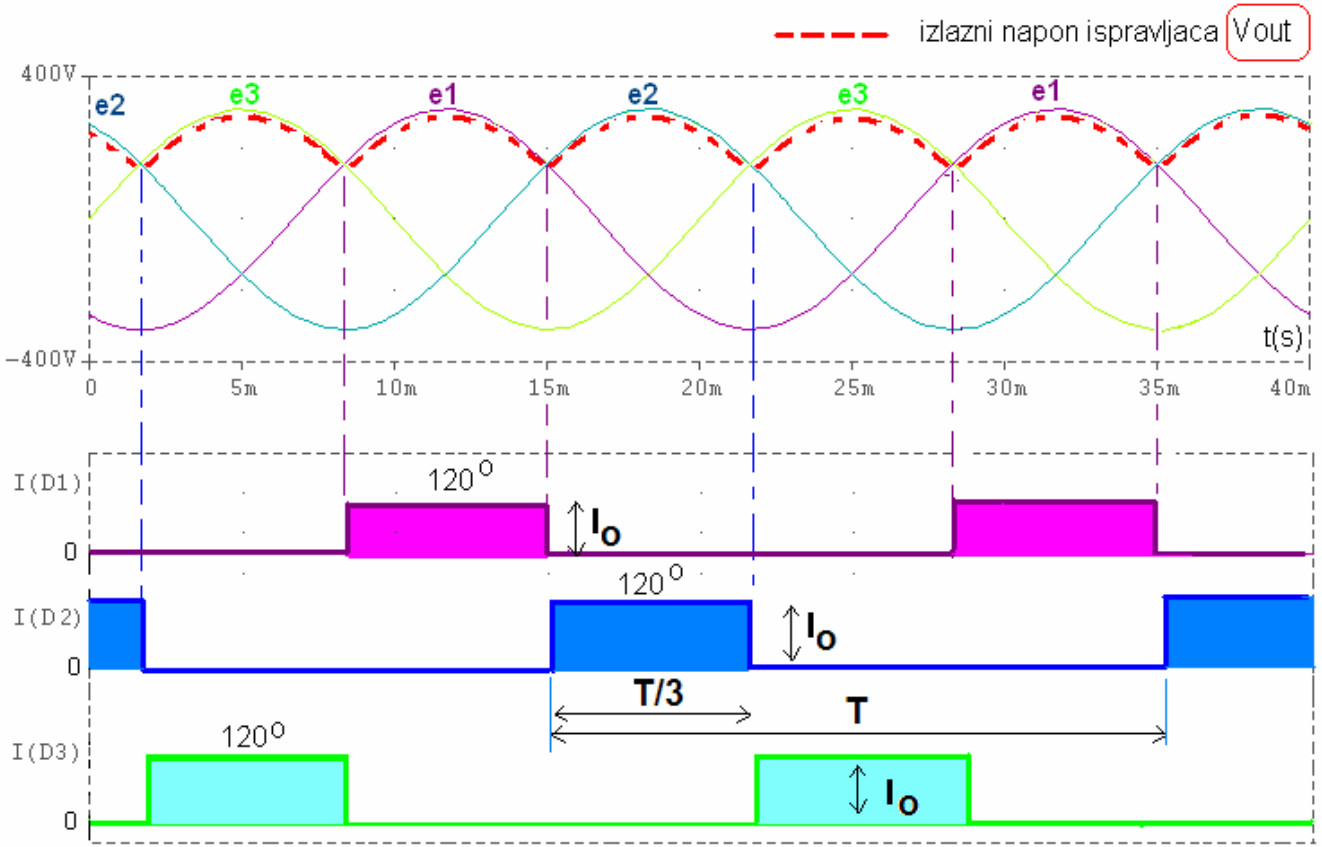
$$I_s = \left[\frac{2}{2\pi}\int_0^{\pi/q} I_m^2 \cos^2 \omega t \, d(\omega t)\right]^{1/2}$$

$$I_s = I_m \left[\frac{1}{2\pi}\left(\frac{\pi}{q} + \frac{1}{2}\sin\frac{2\pi}{q}\right)\right]^{1/2} = \frac{V_{rms}}{R}$$

OTPORNO OPTEREĆENJE!!!!

ŠTA KAD JE OPTEREĆENJE STRUJNI PONOR???

TROFAZNI ISPRAVLJAČ SA TRANSFORAMATOROM SA SREDNJOM TAČKOM- proračuni kad je opterećenje strujni ponor



SREDNJA VREDNOST STRUJE DIODE (SEKUNDARA TRANSFORMATORA)

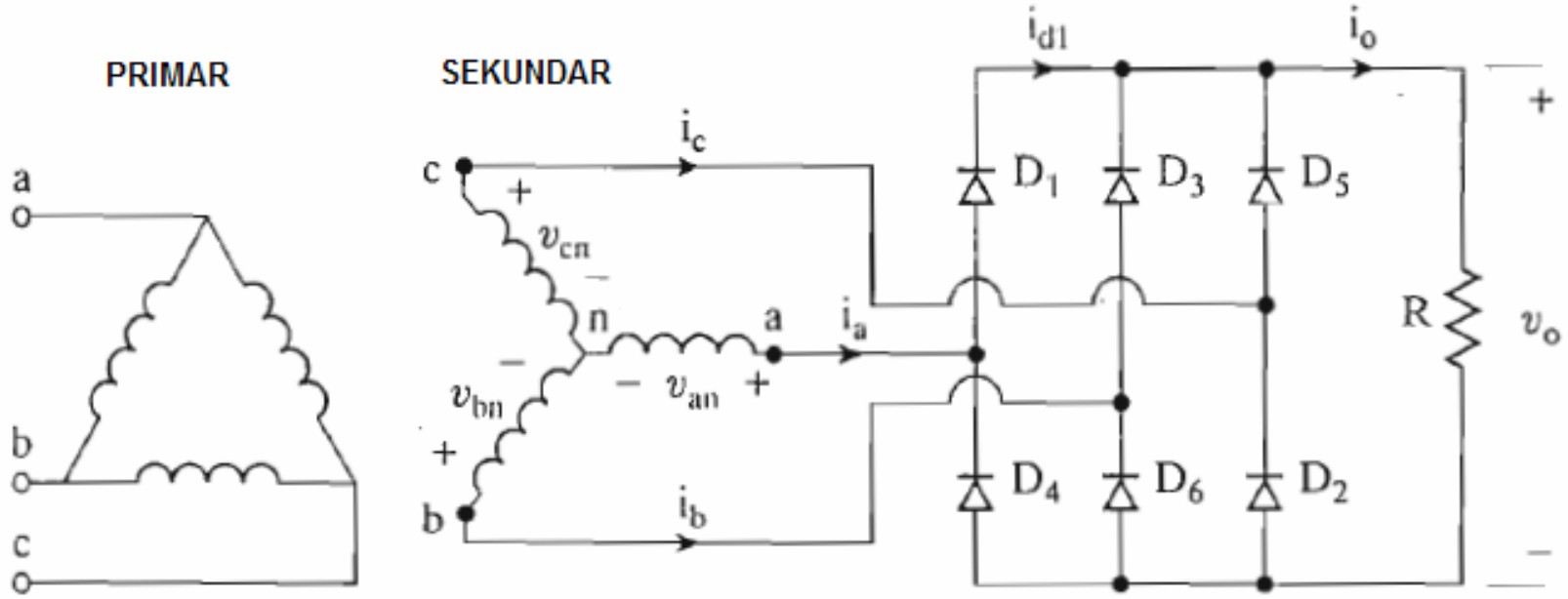
$$I_{avg} = \frac{1}{T}\int_0^T i(t)\,dt = I_o\frac{T/3}{T} = \frac{I_o}{3}$$

EFEKTIVNA VREDNOST STRUJE DIODE (SEKUNDARA TRANSFORMATORA)

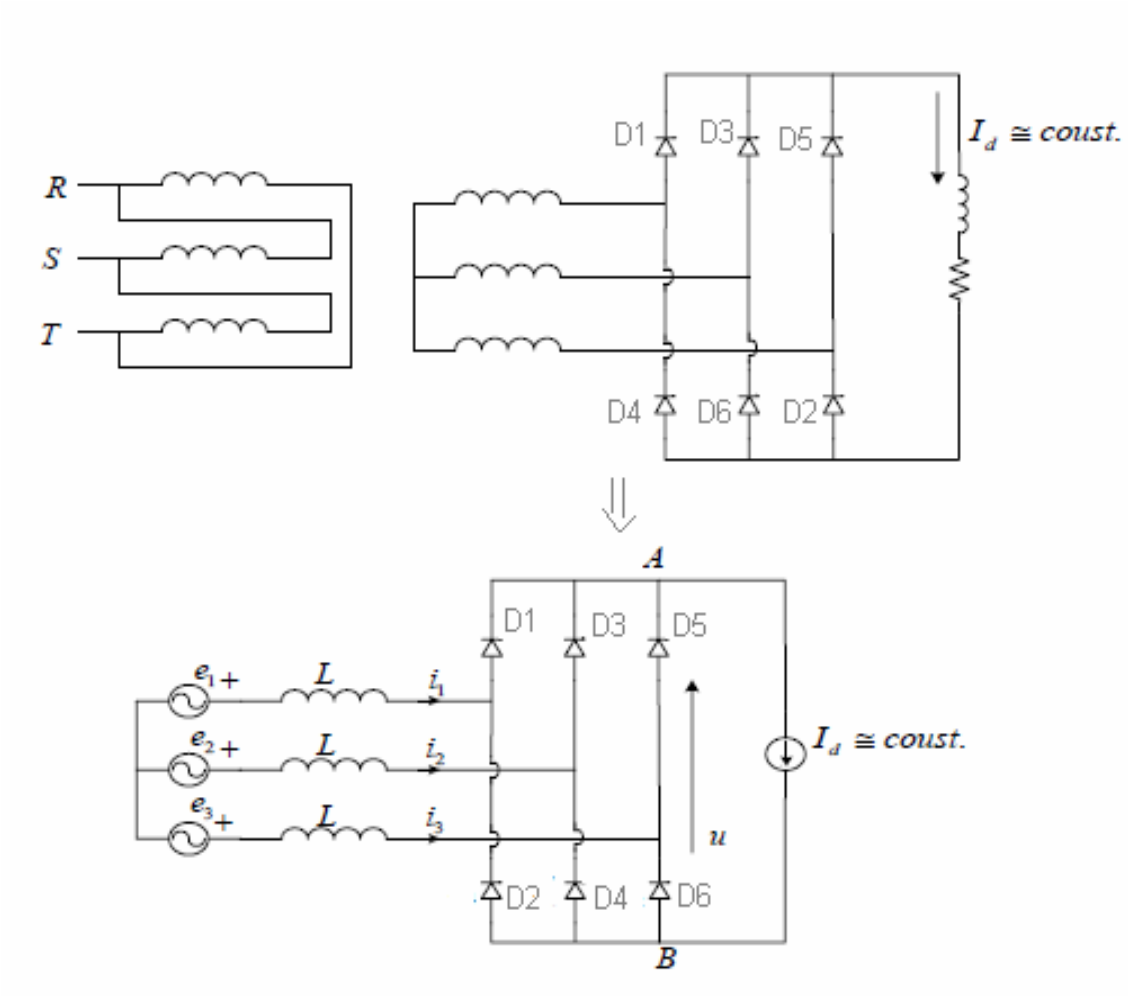
$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} = \sqrt{I_o^2\frac{T/3}{T}} = \frac{I_o}{\sqrt{3}}$$

TROFAZNI MOSNI ISPRAVLJAČ-otporno opterećenje

$$v_{an} = V_m \sin(\omega t) \quad v_{bn} = V_m \sin(\omega t - 120°) \quad v_{cn} = V_m \sin(\omega t - 240°)$$

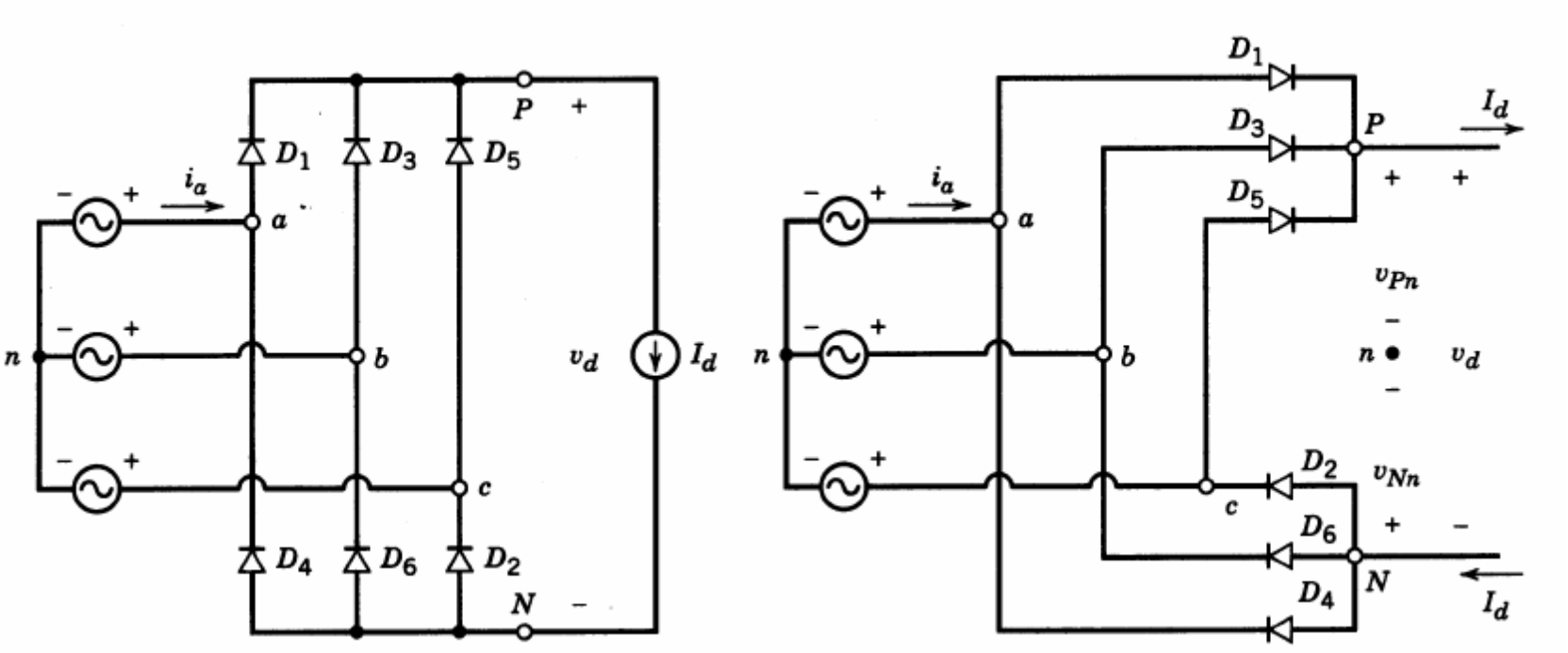


TROFAZNI MOSNI ISPRAVLJAČ-opterećenje strujni ponor Io

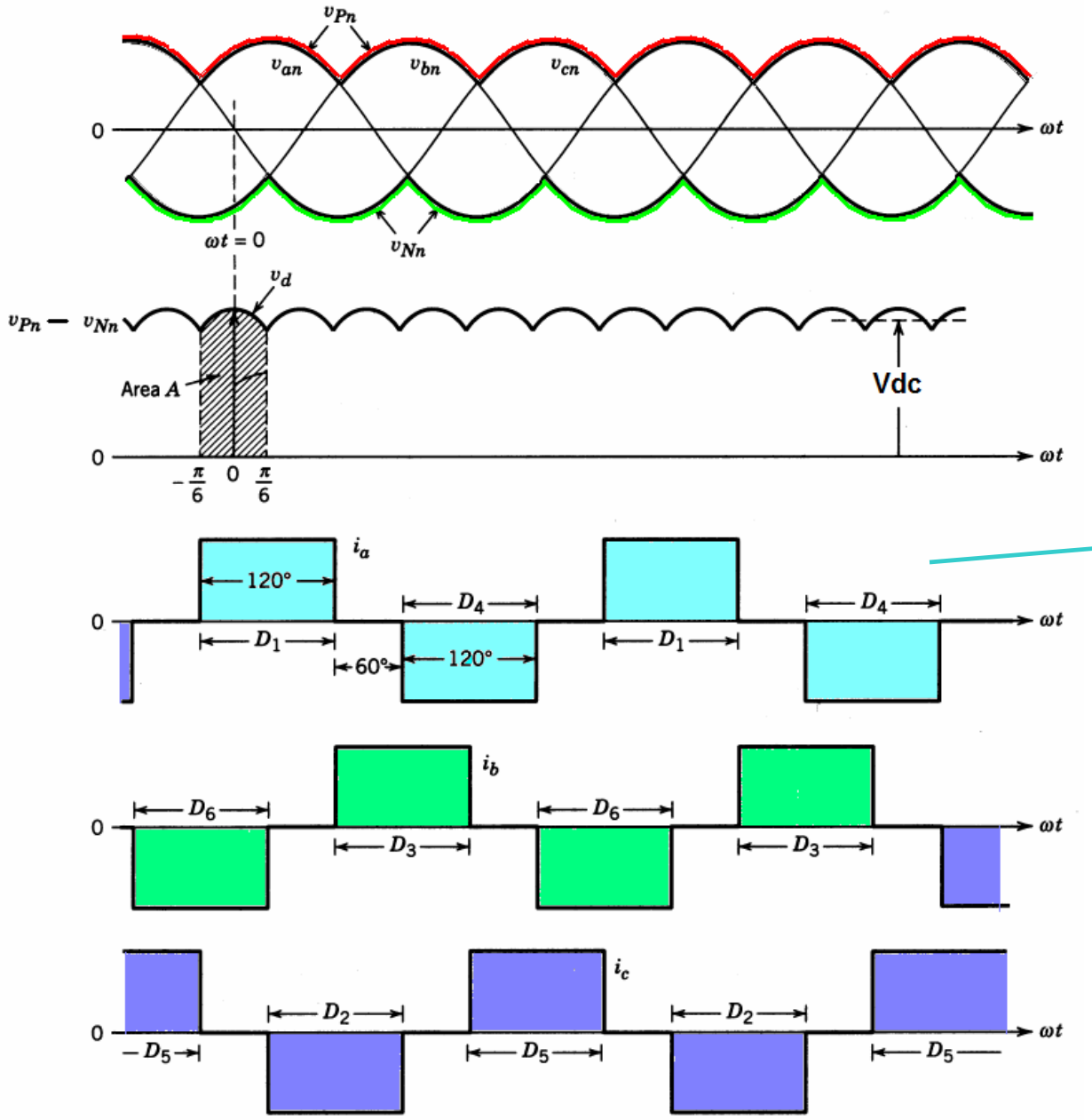


TROFAZNI MOSNI ISPRAVLJAČ

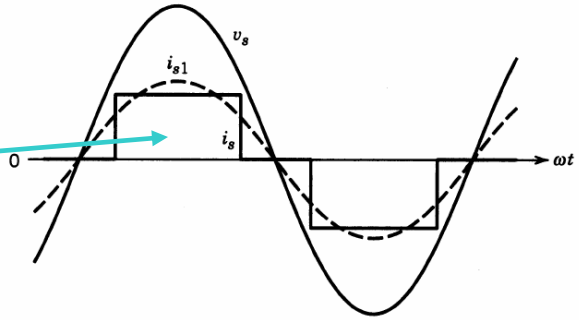
EKVIVALENCIJA SA DVA TROFAZNA POLUMOSNA ISPRAVLJAČA

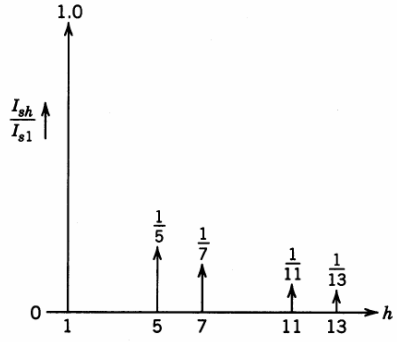


TROFAZNI MOSNI ISPRAVLJAČ-talasni oblici



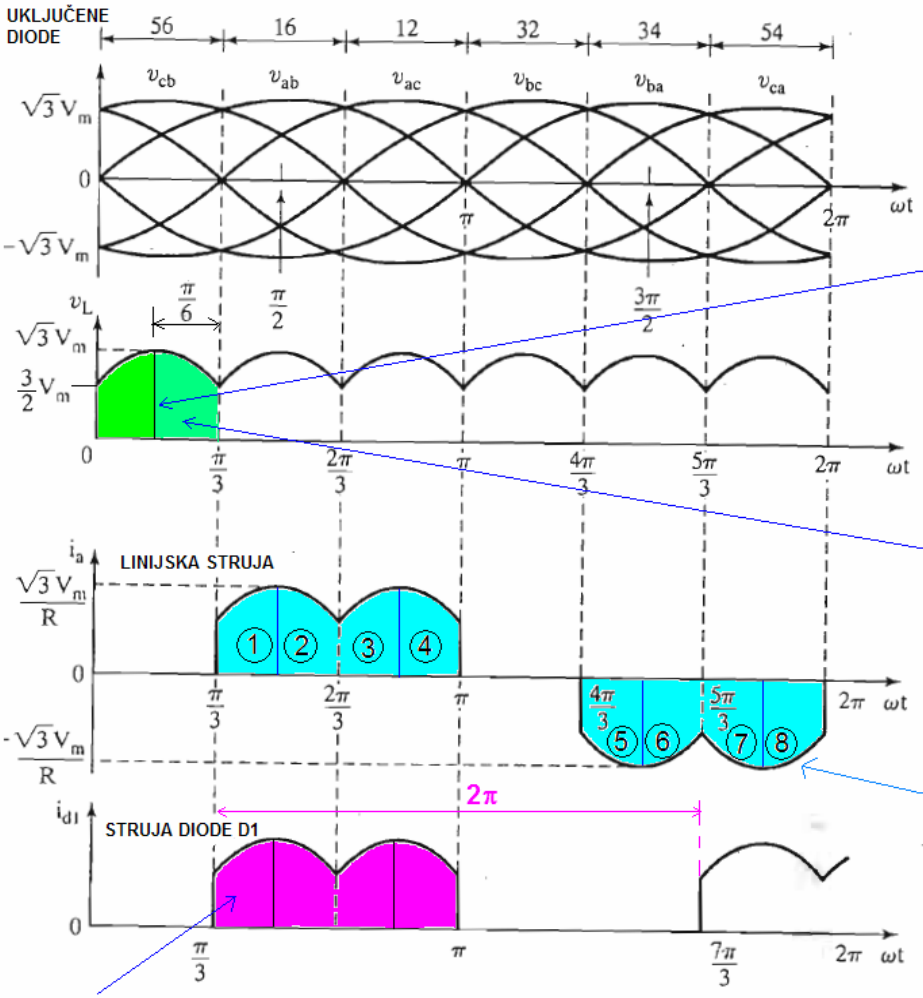
LINIJSKI NAPON I
LINIJSKA STRUJA





HARMONIJSKI SASTAV

TROFAZNI MOSNI ISPRAVLJAČ- izračunavanje srednjih i efektivnih vrednosti



$$v_{ab} = \sqrt{3}\,V_m \sin(\omega t + 30°) \quad v_{bc} = \sqrt{3}\,V_m \sin(\omega t - 90°)$$

$$v_{ca} = \sqrt{3}\,V_m \sin(\omega t - 210°)$$

SREDNJA VREDNOST NAPONA NA OPTEREĆENJU (IZLAZNI NAPON)

$$V_{dc} = \frac{2}{2\pi/6}\int_0^{\pi/6} \sqrt{3}\,V_m \cos\omega t\, d(\omega t)$$

$$V_{dc} = \frac{3\sqrt{3}}{\pi} V_m = 1.654 V_m$$

TALASNOST

$$\triangle V = \sqrt{3}V_m - \frac{3}{2}V_m = 0.23 V_m$$

EFEKTIVNA VREDNOST NAPONA NA OPTEREĆENJU (IZLAZNI NAPON)

$$V_{rms} = \left[\frac{2}{2\pi/6}\int_0^{\pi/6} 3V_m^2 \cos^2\omega t\, d(\omega t)\right]^{1/2}$$

$$V_{rms} = \left(\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}\right)^{1/2} V_m = 1.6554 V_m$$

EFEKTIVNA VREDNOST LINIJSKE STRUJE

$$I_s = \left[\frac{8}{2\pi}\int_0^{\pi/6} I_m^2 \cos^2\omega t\, d(\omega t)\right]^{1/2}$$

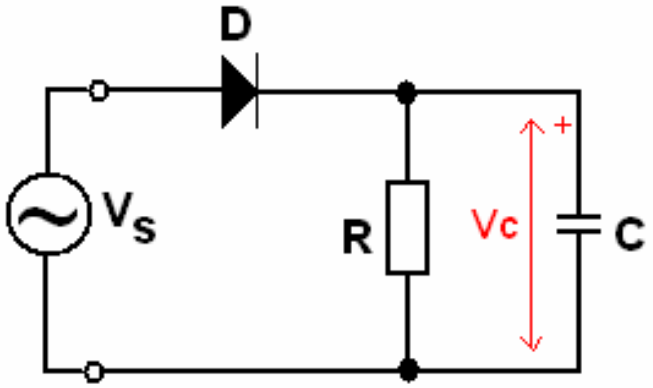
$$= I_m\left[\frac{2}{\pi}\left(\frac{\pi}{6} + \frac{1}{2}\sin\frac{2\pi}{6}\right)\right]^{1/2}$$

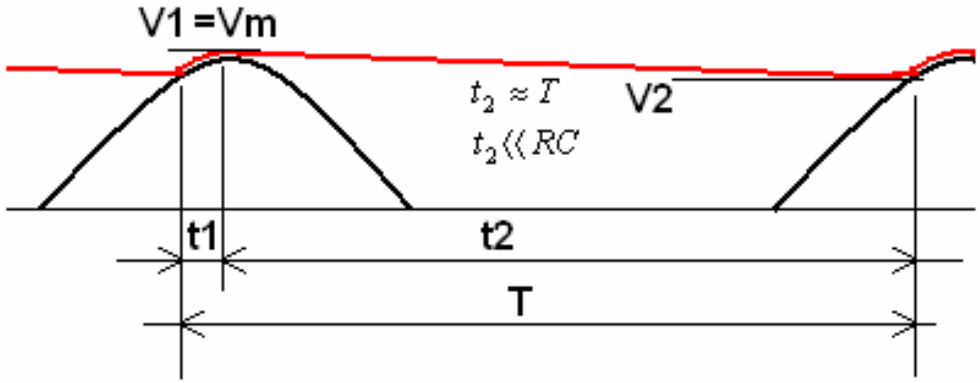
$$I_s = 0.7804 I_m \qquad I_m = V_m/R$$

SREDNJA VREDNOST STRUJE DIODE

$$I_d = = \frac{4}{2\pi}\int_0^{\pi/6} I_m \cos\omega t\, d(\omega t) = I_m \frac{2}{\pi}\sin\frac{\pi}{6} = 0.3183 I_m$$

MONOFAZNI ISPRAVLJAČI (IZBOR KAPACITIVNOG FILTRA) *-rekapitulacija*



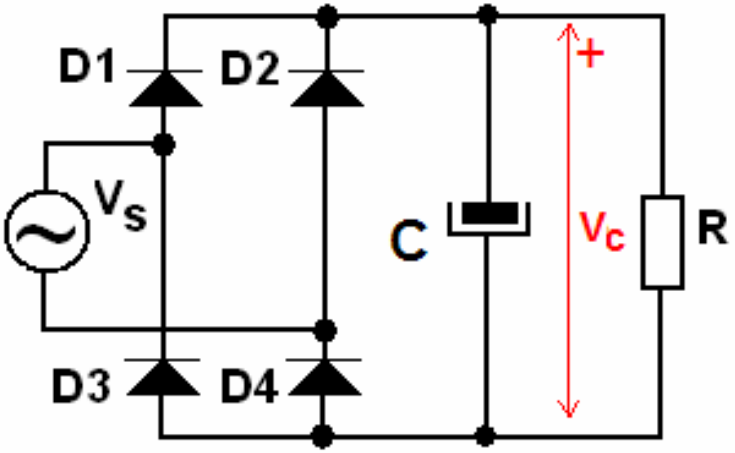


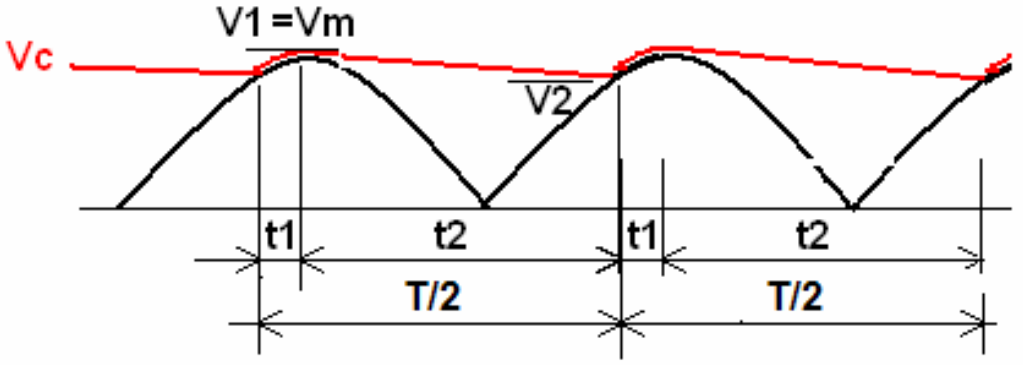
$$\Delta V = V_m \cdot \frac{t_2}{RC} = V_m \cdot \frac{T}{RC} = V_m \cdot \frac{1}{fRC}$$

$$t_2 \approx T$$

$$V_{DC} = V_m - \frac{\Delta V}{2} = V_m - \frac{V_m}{2fRC} = V_m \cdot \frac{2fRC-1}{2fRC}$$

$$\frac{\Delta V}{V_{DC}} = 2 \cdot \frac{1}{2fRC-1}$$





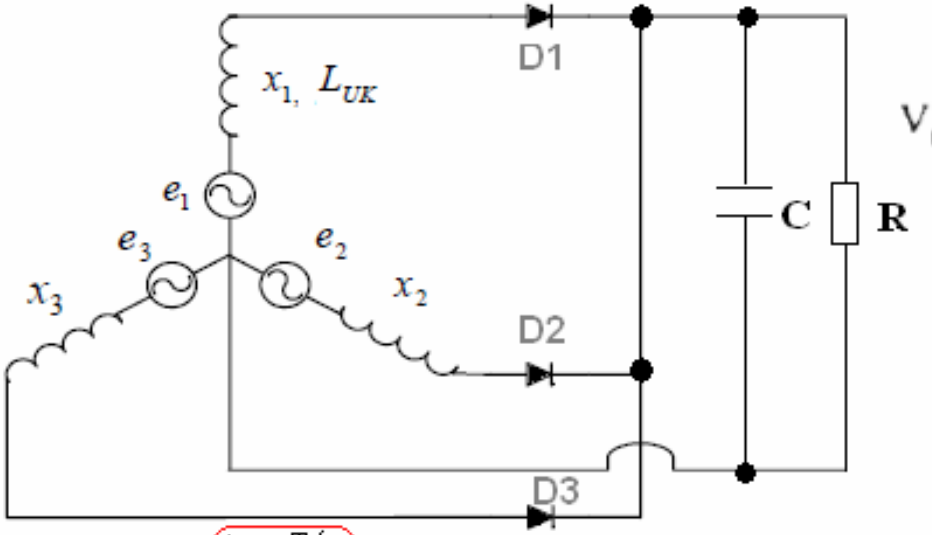
$$\Delta V = V_m \cdot \frac{t_2}{RC} \approx V_m \cdot \frac{T}{2RC} = V_m \cdot \frac{1}{2fRC}$$

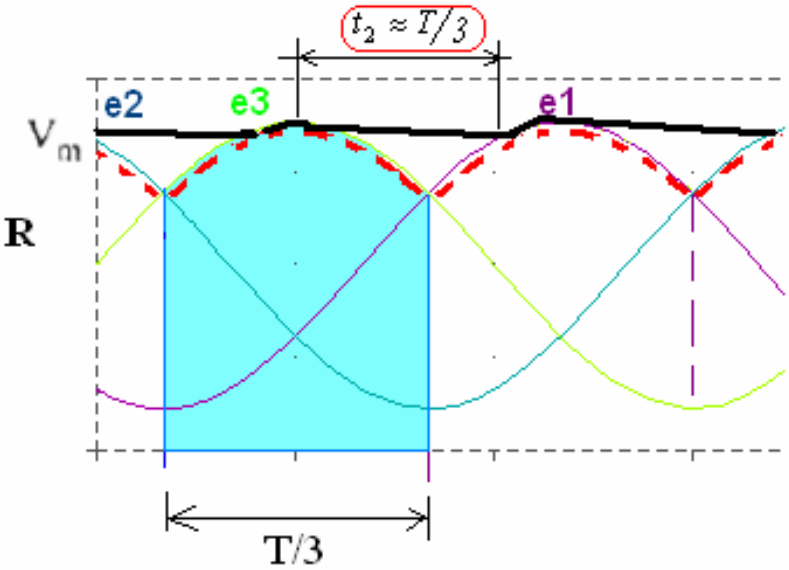
$$t_2 = \frac{T}{2}$$

$$V_{DC} = V_m - \frac{\Delta V}{2} = V_m - \frac{V_m}{4fRC} = V_m \cdot \frac{4fRC-1}{4fRC}$$

$$\frac{\Delta V}{V_{DC}} = 2 \cdot \frac{1}{4fRC-1}$$

TROFAZNI ISPRAVLJAČI (IZBOR KAPACITIVNOG FILTRA) - *rekapitulacija*

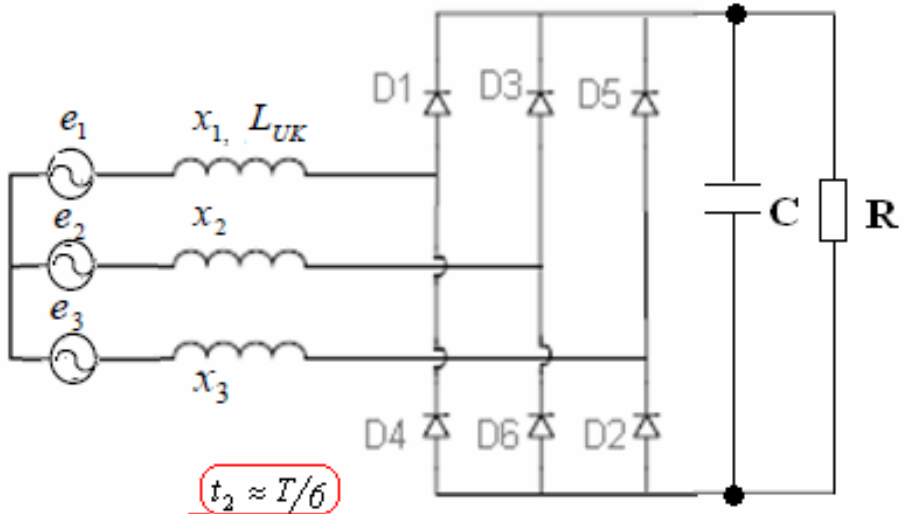


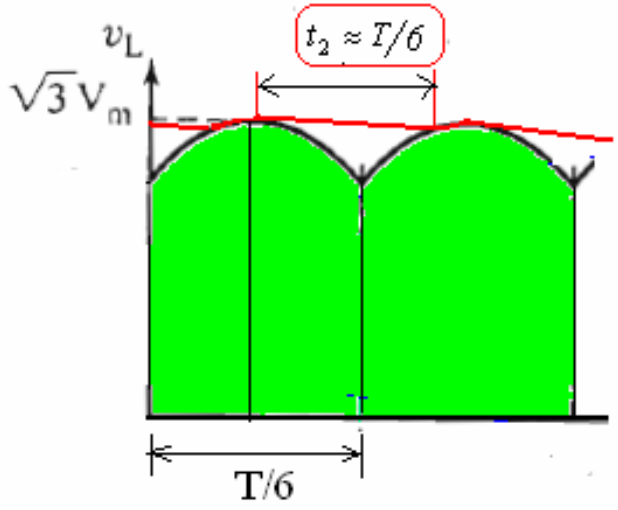


$$\Delta V = V_m \cdot \frac{t_2}{RC} \quad (t_2 \approx T/3) = V_m \cdot \frac{T}{3RC} = V_m \cdot \frac{1}{3fRC}$$

$$V_{DC} = V_m - \frac{\Delta V}{2} = V_m - \frac{V_m}{6fRC} = V_m \cdot \frac{6fRC-1}{6fRC}$$

$$\frac{\Delta V}{V_{DC}} = 2 \cdot \frac{1}{6fRC-1}$$





$$\Delta V = V_m \cdot \frac{t_2}{RC} \quad (t_2 \approx T/6) = V_m \cdot \frac{T}{6RC} = V_m \cdot \frac{1}{6fRC}$$

$$V_{DC} = V_m - \frac{\Delta V}{2} = V_m - \frac{V_m}{12fRC} = V_m \cdot \frac{12fRC-1}{12fRC}$$

$$\frac{\Delta V}{V_{DC}} = 2 \cdot \frac{1}{12fRC-1}$$